

Multiple Description Quantizer Design Using A Channel Optimized Quantizer Approach

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Abstract—This paper extends the channel optimized quantization scheme of Farvardin and Vaishampayan [1] to two parallel channels. The extended multiple-channel optimized quantizer design (MCOQD) framework is applied to discrete memoryless channels with erasures. The resultant MCOQD subsumes the multiple description scalar quantizer (MDSQ) design of Vaishampayan [2]. While MDSQ is suited to only “on-off” channels, MCOQD accounts for *both* erasure and symbol errors. Performance results based on simulation show that MCOQD provides more robust quantizers than MDSQ.

I. INTRODUCTION

Multiple description coding (MDC) is a method of communicating information from a source over two or more channels such that the information received from any subset of channels can be used for source reconstruction, and the reconstruction quality improves with the size of the subset. Most existing MDC schemes (e.g., [2], [3]) consider only “on-off” channels (i.e., erasure channels), and are not suited to channels with symbol or bit errors. In [4], multiple description scalar quantization is combined with error correcting codes and then applied to channels with bit errors. They reported that the MD codes considered were outperformed by single description codes.

In this paper we consider the more general problem of designing a MDC scheme for *both* erasure errors and symbol errors. The proposed scheme is based on extending the channel-optimized scalar quantization scheme of [1] to multiple parallel channels; hence, we call the new scheme multiple-channel optimized quantizer design (MCOQD). We show in this paper that the multiple description scalar quantizer (MDSQ) design of Vaishampayan [2] is subsumed by MCOQD. MCOQD specializes to MDSQ design when the channel is free of symbol or bit errors. We introduce MCOQD in the next section. As per the usual practice, we gain insights by first studying the two-channel case. In Section 3, the features of MCOQD are compared with MDSQ, in order to determine how they are related. Simulation results are provided in Section 4 for comparing the performance of the two approaches and conclusions are drawn in Section 5.

II. MDC WITH MCOQD

Firstly, we set up the MDC framework by reviewing the MDSQ scheme of Vaishampayan [2], a block diagram of which is shown in Fig. 1. The encoder is shown to comprise two parts. A scalar quantizer maps a source sample x to the

nearest codeword in a codebook such that the quantization error is minimized. The quantizer codeword index is then mapped to two channel codewords, each to be sent over a separate independent erasure channel. If both channel codewords are received, the central decoder reconstructs the source as \hat{x}^0 . If only one codeword is received, a side decoder reconstructs the source as either \hat{x}^1 or \hat{x}^2 . The expected quality of \hat{x}^0 is better than \hat{x}^1 or \hat{x}^2 . In [2], MDSQ is designed by first selecting an assignment of quantizer index values to channel codewords and then optimizing the quantizer codebook for the chosen index assignment (IA).

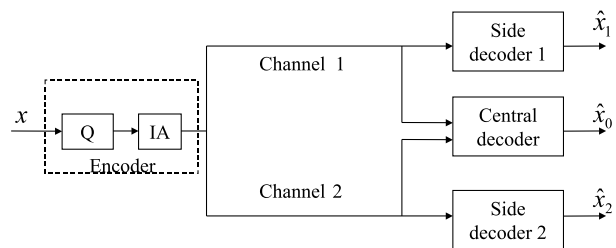


Fig. 1. MDSQ system diagram.

Channel optimized quantizer design (COQD) as originally proposed by Farvardin and Vaishampayan [1] is a joint source channel coding scheme for bit error channels. With COQD, a scalar quantizer is designed to minimize the reconstruction error due to both quantization and channel errors. The design algorithm of COQD is similar in form to the generalized Lloyd algorithm (GLA): alternating between (1) optimizing the encoder for a given decoder, and (2) optimizing the decoder for a given encoder.

The COQD scheme in [1] is readily extended to the case of sending multiple descriptions over multiple channels, where both erasure and symbol errors may occur. A block diagram for two channels is shown in Fig. 2.

A source sample x is first quantized with an M -level scalar quantizer, which maps the source $x \in \mathbb{R}$ into one of M values. The quantization effects an encoder mapping $f: \mathbb{R} \rightarrow A_{IP} = \{1, 2, \dots, M\}$, whose output is quantizer index $u \in A_{IP}$, i.e., $u = f(x)$. u is mapped to two descriptions u^1 and u^2 , for transmission over channels 1 and 2, at bit rates of R_1 bits and R_2 bits, respectively, with $2^{R_1+R_2} \geq M$, i.e., $\{1, 2, \dots, M\} \rightarrow \{1, \dots, 2^{R_1}\} \times \{1, \dots, 2^{R_2}\}$. Each channel is an independent discrete memoryless channel (DMC) with erasure

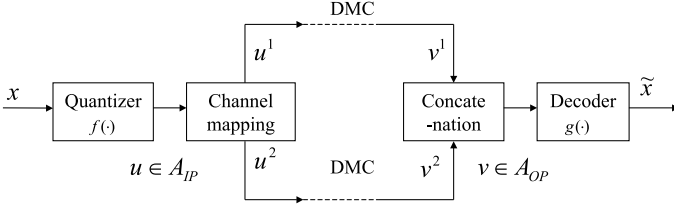


Fig. 2. Two-channel MDC using MCOQD.

output. A DMC model with input alphabet $\{1, \dots, 2^R\}$ and output alphabet $\{1, \dots, 2^R, E\}$ is shown in Fig. 3. Symbol erasure is treated as a channel output, denoted as “E” in the DMC output alphabet. The channel is parameterized by channel transition probability $p(v = j|u = i) = p(j|i)$, $i = 1, \dots, 2^R$, $j = 1, \dots, 2^R, E$. Fig. 3 shows the specific case with $p(j|i) = \epsilon$, $\forall i, j \in \{1, \dots, 2^R\}, i \neq j$ and $p(E|i) = \gamma$, $\forall i \in \{1, \dots, 2^R\}$. γ and ϵ are referred to as symbol erasure probability and symbol error probability, respectively.

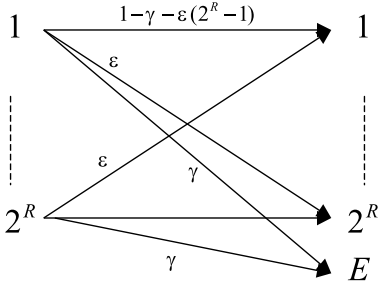


Fig. 3. A discrete memoryless channel model.

The individual channel outputs v^1 and v^2 are concatenated into channel output $v = (v^1, v^2)$ for decoding. The decoder effects a mapping $g : A_{OP} = \{1, \dots, N\} \rightarrow \mathbb{R}$. v is decoded to one of the N possible values in a decoder codebook, i.e., $\tilde{x} = g(v)$. Considering all possible channel outputs, the decoder codebook size is therefore $N = (2^{R_1} + 1)(2^{R_2} + 1)$.

III. COMPARISON BETWEEN MDSQ AND MCOQD

The encoder and decoder in MCOQD are designed based on the channel transition probabilities defined earlier. Necessary conditions for optimality of the encoder (with fixed decoder) and decoder (with fixed encoder) are similar to those in [1]. For readers with interest, the necessary optimality conditions for MCOQD are derived in the appendices.

The MDSQ design in [2] considers only erasure errors while MCOQD accounts for both erasure errors and symbol errors. When the channels are free of symbol errors, i.e., $\epsilon = 0$ in the DMC model, MCOQD is identical to MDSQ design. To show this viewpoint, we compare MDSQ design with MCOQD over five aspects: system structure, performance criteria, necessary conditions for optimal encoder/decoder, design algorithm and initialization.

A. System structure

In MDSQ, three decoders are used to reconstruct the source. If both channels are operational, the central decoder is used to reconstruct the source sample as \hat{x}^0 . If one of the two channels fails, a side decoder is used to reconstruct the source sample as either \hat{x}^1 or \hat{x}^2 . The corresponding central and side expected distortions are D_0 , D_1 , and D_2 .

The equivalence between the three decoders in MDSQ and the decoder of MCOQD can be seen by partitioning the latter decoder codebook. We divide the set of channel outputs A_{OP} in Fig. 2 into four subsets, according to the state of channel erasure

$$A_{OP} = A_{OP}^0 \cup A_{OP}^1 \cup A_{OP}^2 \cup A_{OP}^E.$$

$A_{OP}^0 = \{v = (v^1, v^2), v^1 \neq E, v^2 \neq E\}$ is the set of all channel outputs when both channels are operational. $A_{OP}^1 = \{v = (v^1, v^2), v^1 \neq E, v^2 = E\}$ is the set of all channel outputs when only channel 1 is operational; similar definition applies to A_{OP}^2 . $A_{OP}^E = \{v = (E, E)\}$ contains the channel output when both channels fail. Corresponding to each of the four possible channel states, the MCOQD decoder reconstructs to output variables \tilde{x}^0 , \tilde{x}^1 , \tilde{x}^2 , and \tilde{x}^E . Conditioned on the four channel states, we let the conditional expected distortion of the output variables \tilde{x}^0 , \tilde{x}^1 , \tilde{x}^2 , and \tilde{x}^E , be d_0 , d_1 , d_2 , and d_E , respectively.

\tilde{x}^0 is equivalent to the central decoder output variable \hat{x}^0 in MDSQ. \tilde{x}^1 and \tilde{x}^2 are equivalent to the two side decoder output variables \hat{x}^1 and \hat{x}^2 in MDSQ, respectively. The formulation of MDSQ does not consider total erasure, so an equivalent of \tilde{x}^E does not exist in MDSQ. For squared error distortion, \tilde{x}^E is given by (9) in the appendix. \tilde{x}^E is a constant equal to the source mean value $E[X]$; hence, $d_E = \sigma_x^2$ the variance of the source. The performance criterion of MCOQD is compared with MDSQ in the next Section based on the above equivalence.

B. Performance criterion and necessary conditions for optimality

MDSQ design involves minimizing D_0 subject to upper limits on D_1 and D_2 . The optimization objective, from which encoder and decoder optimality criteria are derived, uses two Lagrange multipliers to combine the side with central distortions. From contemporary Lagrangian based rate-distortion optimization theory, the distortion objective minimized in MDSQ can be written as

$$D_{MDSQ} = D_0 + \lambda_1 D_1 + \lambda_2 D_2$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are Lagrange multipliers. Suitable choice of values for λ_1 and λ_2 provides the desired design tradeoff between central and side distortions. In practice, the best choice is found by numerical search over the first quadrant of the (λ_1, λ_2) -plane. Note that channel parameters (i.e., erasure probabilities) are not explicitly included in the design formulation of MDSQ.

In MCOQD, the encoder and decoder are designed to minimize the expected distortion between encoder input and

decoder output. We first consider the case of zero symbol error probabilities. Let erasure probabilities for channel 1 and channel 2 be γ_1 and γ_2 , respectively. The expected end-to-end distortion for MCOQD is

$$D_{MCOQD}^e = (1 - \gamma_1)(1 - \gamma_2)d_0 + \gamma_2(1 - \gamma_1)d_1 + \gamma_1(1 - \gamma_2)d_2 + \gamma_1\gamma_2\sigma_x^2. \quad (1)$$

Divide both sides of (1) by $(1 - \gamma_1)(1 - \gamma_2)$, we obtain

$$D'_{MCOQD} = \frac{D_{MCOQD}^e}{(1 - \gamma_1)(1 - \gamma_2)} = d_0 + \frac{\gamma_2}{(1 - \gamma_2)}d_1 + \frac{\gamma_1}{(1 - \gamma_1)}d_2 + \frac{\gamma_1\gamma_2}{(1 - \gamma_1)(1 - \gamma_2)}\sigma_x^2. \quad (2)$$

Letting $\lambda_1 = \frac{\gamma_2}{1 - \gamma_2}$ and $\lambda_2 = \frac{\gamma_1}{1 - \gamma_1}$, we see that we can sweep over all non-negative values of λ_1 and λ_2 by suitable choice of the erasure probabilities. Note that the last term in the sum above is determined by the source and channel and does not depend on the encoder and decoder being optimized. Hence, designing the encoder and decoder by minimizing (2) results in the same design as by minimizing

$$D''_{MCOQD} = d_0 + \lambda_1 d_1 + \lambda_2 d_2.$$

Thus, the optimization criterion of MCOQD can be made identical to MDSQ design by zeroing the symbol error probabilities and a suitable choice of erasure probability values. Under this restricted condition, the necessary condition for optimal encoder (with fixed decoder) and the necessary condition for optimal decoder (with fixed encoder) for MCOQD and MDSQ are identical. The necessary optimality conditions for MCOQD with erasure and symbol errors are given in the appendix.

Let $d(x, y) = (x - y)^2$ denote squared error distortion. The expected end-to-end distortion for MCOQD for the general case of symbol and erasure errors is evaluated as

$$\begin{aligned} D_{MCOQD} &= E_X[E_{\tilde{X}}[d(X, \tilde{X})]] \\ &= E_X[E_{\tilde{X}}[d(X, \tilde{X})|V \in A_{OP}^0]]P(V \in A_{OP}^0) \\ &\quad + E_X[E_{\tilde{X}}[d(X, \tilde{X})|V \in A_{OP}^1]]P(V \in A_{OP}^1) \\ &\quad + E_X[E_{\tilde{X}}[d(X, \tilde{X})|V \in A_{OP}^2]]P(V \in A_{OP}^2) \\ &\quad + E_X[E_{\tilde{X}}[d(X, \tilde{X})|V \in A_{OP}^E]]P(V \in A_{OP}^E) \\ &= (1 - \gamma_1)(1 - \gamma_2)E_X[E_{\tilde{X}^0}[d(X, \tilde{X}^0)]] \\ &\quad + \gamma_2(1 - \gamma_1)E_X[E_{\tilde{X}^1}[d(X, \tilde{X}^1)]] \\ &\quad + \gamma_1(1 - \gamma_2)E_X[E_{\tilde{X}^2}[d(X, \tilde{X}^2)]] \\ &\quad + \gamma_1\gamma_2E_X[d(X, \tilde{x}^E)] \\ &= (1 - \gamma_1)(1 - \gamma_2)E_X[E_{\tilde{X}^0}[d(X, \tilde{X}^0)]] \\ &\quad + \gamma_2(1 - \gamma_1)E_X[E_{\tilde{X}^1}[d(X, \tilde{X}^1)]] \\ &\quad + \gamma_1(1 - \gamma_2)E_X[E_{\tilde{X}^2}[d(X, \tilde{X}^2)]] \\ &\quad + \gamma_1\gamma_2E_X[d(X, E(X))], \end{aligned} \quad (3)$$

where the last step follows from (9).

For channels free of symbol errors, i.e., $\epsilon = 0$ in the DMC model, the decoder output variables $\tilde{X}^k, k = 0, 1, 2$ (each

conditioned on a channel erasure state) depend only on source but not channel randomness. Therefore,

$$E_X[E_{\tilde{X}^k}[d(X, \tilde{X}^k)]] = E_X[d(X, \tilde{X}^k)] = d_k, \quad k = 0, 1, 2 \quad (4)$$

and (3) can be seen to be reduced to (1).

C. Design algorithm and initialization

The codebook design algorithm for MCOQD is essentially the same as the design algorithm for COQD in [1]. The MCOQD algorithm is summarized below for clarity:

- 1) Set iteration index $n = 0$. Initialize the decoder codebook. Set initial distortion $D^{(0)} = \infty$, and a threshold $\delta > 0$.
- 2) Let $n = n + 1$. Determine the optimal encoder $f^{(n)}$ for a fixed decoder $g^{(n-1)}$ using (5).
- 3) Determine the optimal decoder $g^{(n)}$ for a fixed encoder $f^{(n)}$ using (7).
- 4) Evaluate the distortion $D^{(n)}$ using (6). If $|(D^{(n)} - D^{(n-1)})/D^{(n)}| < \delta$, stop; otherwise, go to step 2.

A variant of the above algorithm targets the case where the symbol error probability ϵ is not known to the encoder but known to the decoder. In such case, the algorithm is run using channel transition probabilities calculated assuming $\epsilon = 0$. Upon termination of the algorithm, step 3 is repeated once using channel transition probabilities calculated with the ϵ known to the decoder.

The codebook design algorithm presented in [2] for MDSQ first finds an encoder index assignment, and then alternates between optimizing the decoder for fixed encoder, and optimizing the encoder for fixed decoder, until the algorithm converges to a local minimum. Thus, both MDSQ and MCOQD follow the generalized Lloyd algorithm design paradigm. Difference exists however in the initialization step. MDSQ design initializes the encoder index assignment, while MCOQD initializes the decoder codebook. For channels with no symbol errors, the decoder codebook for MCOQD can be initialized to be equivalent to the index assignment for MDSQ (see next Section for an example). With no symbol errors and with such equivalent initialization, the two algorithms perform identically step for step.

IV. PERFORMANCE RESULTS

We consider a memoryless Gaussian source with zero mean and unit variance. For simplicity, the two independent DMCs are assumed to have equal symbol error probability and erasure probability. End-to-end performance is gauged using mean square error (MSE). To fairly compare MDSQ and MCOQD we provide an initial encoder index assignment to MDSQ design and an equivalent decoder codebook for MCOQD. The equivalent initialization is illustrated in Fig. 4 using an example with $R_1 = R_2 = 1$ bps (bit per source symbol). The example shows an MDSQ with no redundancy, as evidenced by the absence of empty cells in the index assignment matrix. The decoder codebook provides for all

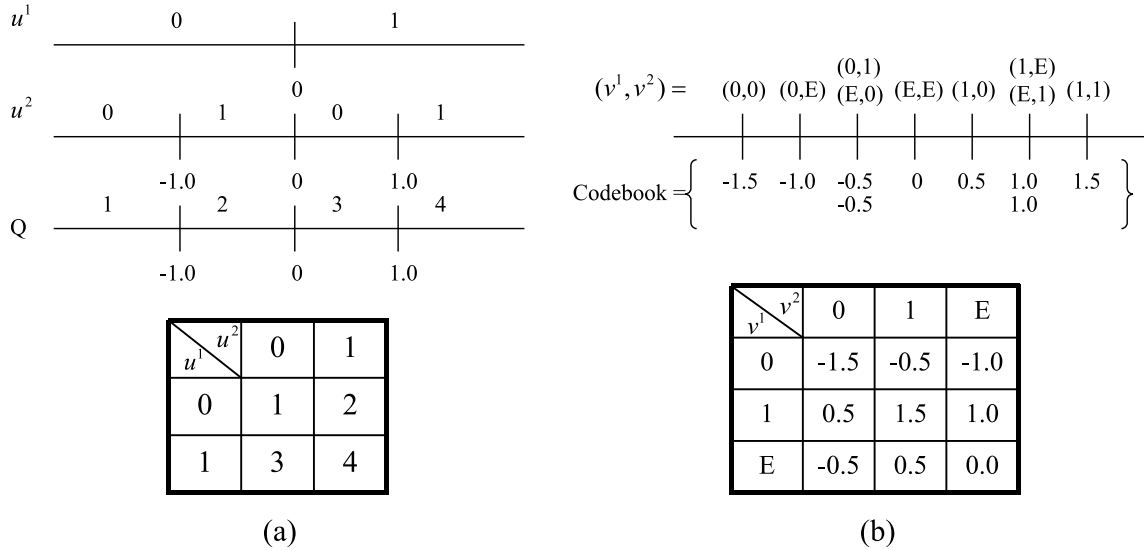


Fig. 4. (a) An index assignment matrix for MDSQ with $R_1 = R_2 = 1$ bps. The encoder quantizer indexes, i.e., 1,2,3,4 in the matrix cells, are mapped to two channel input symbols $u^1 \in \{0, 1\}$ and $u^2 \in \{0, 1\}$. (b) An initial decoder codebook for MCOQD equivalent to the index assignment in (a). Each channel output symbol $(v^1, v^2) \in \{0, 1, E\} \times \{0, 1, E\}$ corresponds to a reconstruction value in the decoder codebook, i.e., the real numbers in the matrix cells. The marked points in (a) are quantizer decision boundary points, and the marked points in (b) are quantizer reconstruction points.

possible channel outputs and hence has one more row and column than the index assignment matrix in MDSQ.

For the performance of MCOQD both offline design and online operation are considered. For offline design the channel error probabilities ϵ are assumed known to both the encoder and decoder. For online operation, ϵ is assumed known only to the decoder. Fig. 5 shows the performance of MDSQ and MCOQD for bit rates $R_1 = R_2 = 1$ bps, and $R_1 = R_2 = 2$ bps. Without surprise, in symbol error-free channel, i.e., $\epsilon = 0$, MCOQD and MDSQ perform the same. As ϵ increases, MCOQD performs better.

MDSQ may operate with redundancy. In such case, the index assignment matrix contains empty cells, which correspond to channel codewords that are never transmitted. This property of MDSQ can be observed in MCOQD. When the MCOQD algorithm terminates, empty cells can be identified by the zero empirical probabilities of non-transmitted channel codewords. By our observation when γ is larger than a certain value some codewords are never transmitted. For instance, in the case with bit rates $R_1 = R_2 = 1$ bps and fixed $\epsilon = 0$, Fig. 6 shows the probability variation of each channel codeword for a series of γ . If $\gamma > 0.2$, the codeword “10” is never used by the encoder. The unused “10” codeword is associated with quantizer index value 3, in the encoder index assignment matrix in Fig. 4(a).

We examine the robustness of MDSQ and MCOQD when actual channel error probabilities differ from the probabilities used for design, for bit rates $R_1 = R_2 = 1$ bps. In the first case, with results in Fig. 7, the design channel is better than the actual channel. In the second case, with results in Fig. 8, the design channel is worse than the actual channel. In both cases, MCOQD is more robust than MDSQ.

V. CONCLUSIONS

We have extended the COQD in [1] to MCOQD. Comparing with MDSQ design for “on-off” channels, the proposed MCOQD is better suited to more general bit error channels with erasures. For channels free of symbol errors, MCOQD specializes to MDSQ. Simulation results demonstrate that the proposed MCOQD provides more robust quantizers than MDSQ for general DMCs.

APPENDIX

A. Optimal encoder $f(\cdot)$ for a fixed decoder $g(\cdot)$

Our objective is to minimize $E[d(X, \tilde{X})]$. Let the channel input alphabet be $A_{IP} = \{1, \dots, M\}$ and channel output alphabet be $A_{OP} = \{1, \dots, N\}$. Note that $N > M$ since additional channel output symbols corresponding to symbol erasures must be defined. For each source sample x , the encoder searches for the best channel input symbol u^* from A_{IP} , for fixed decoder g , where

$$\begin{aligned}
 u^* &= \arg \min_{i \in A_{IP}} E[d(x, \tilde{X}) | U = i] \\
 &= \arg \min_{i \in A_{IP}} \sum_{j=1}^N p(V = j | U = i) E[d(x, \tilde{X}) | U = i, V = j] \\
 &= \arg \min_{i \in A_{IP}} \sum_{j=1}^N p(j|i) d(x, \tilde{x}_j) \\
 &= \arg \min_{i \in A_{IP}} \sum_{j=1}^N p(j|i) (x - \tilde{x}_j)^2
 \end{aligned} \tag{5}$$

where the last step holds for squared error distortion measure, i.e., $d(x, y) = (x - y)^2$.

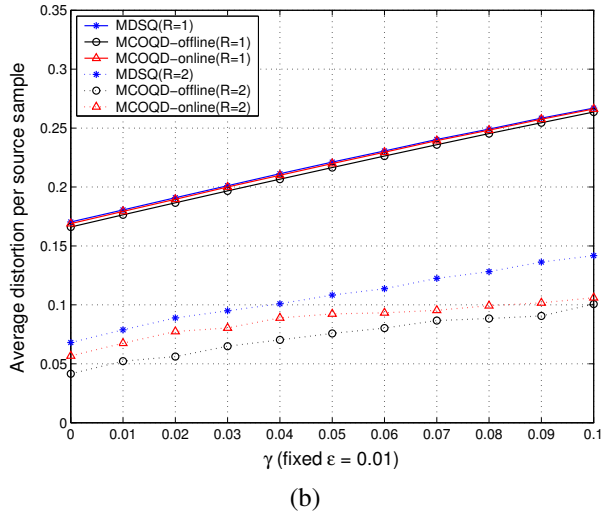
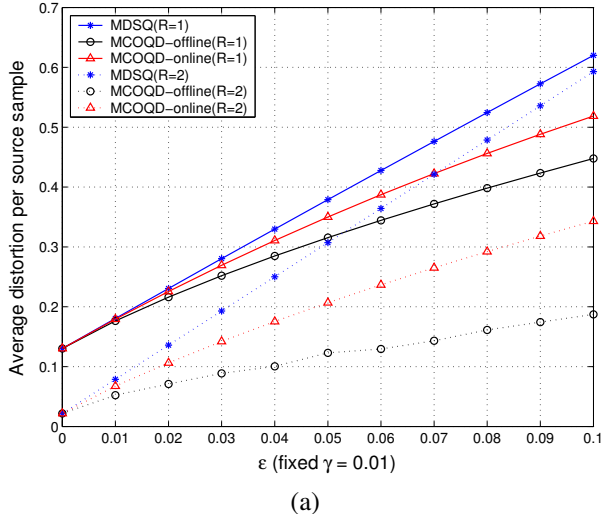


Fig. 5. Performance of MDSQ and MCOQD with $R_1 = R_2 = 1$ bps and $R_1 = R_2 = 2$ bps over two DMCS. (a) Performance with various ϵ for fixed $\gamma = 0.01$. (b) Performance with various γ for fixed $\epsilon = 0.01$.

B. Optimal decoder $g(\cdot)$ for a fixed encoder $f(\cdot)$

Let Q_i be the set of source values which are encoded as i , i.e., $f(x) = i$. Our objective is to minimize the distortion

$$\begin{aligned}
 D &= E[d(X, \tilde{X})] \\
 &= \sum_{i=1}^M E[d(X, \tilde{X})|X \in Q_i]p(X \in Q_i) \\
 &= \sum_{i=1}^M \sum_{j=1}^N E[d(X, \tilde{X})|X \in Q_i, V = j] \\
 &\quad \cdot p(V = j|X \in Q_i)p(X \in Q_i) \\
 &= \sum_{i=1}^M \sum_{j=1}^N E[d(X, \tilde{x}_j)|X \in Q_i, V = j] \\
 &\quad \cdot p(V = j|U = i)p(X \in Q_i) \\
 &= \sum_{i=1}^M \sum_{j=1}^N p(j|i)E[d(X, \tilde{x}_j)|X \in Q_i]p(X \in Q_i) \\
 &= \sum_{i=1}^M \sum_{j=1}^N p(j|i)E[(X - \tilde{x}_j)^2|X \in Q_i]p(X \in Q_i) \quad (6)
 \end{aligned}$$

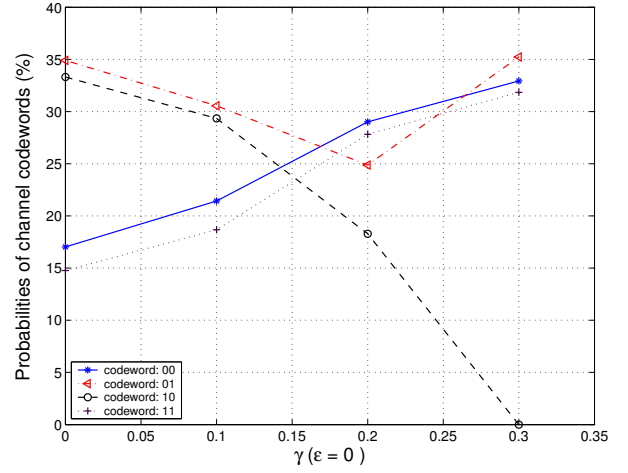


Fig. 6. Probabilities of channel codewords for different channel erasure probabilities γ (fixed $\epsilon = 0$) with bit rates $R_1 = R_2 = 1$ bps.

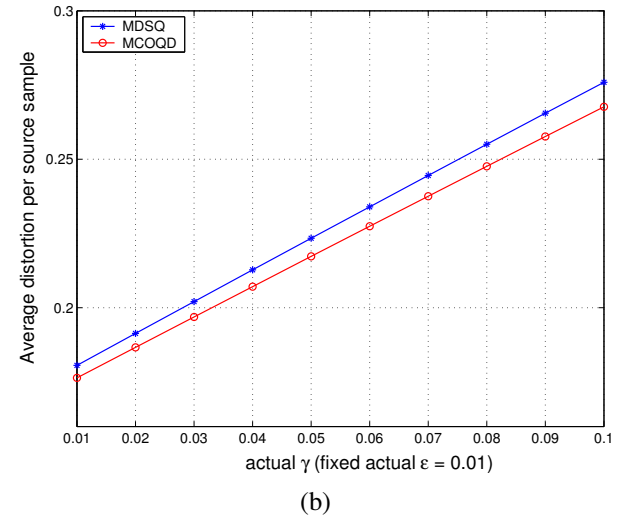
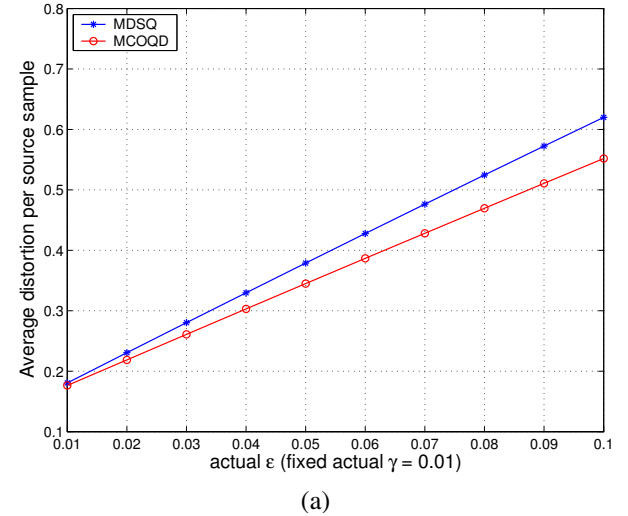
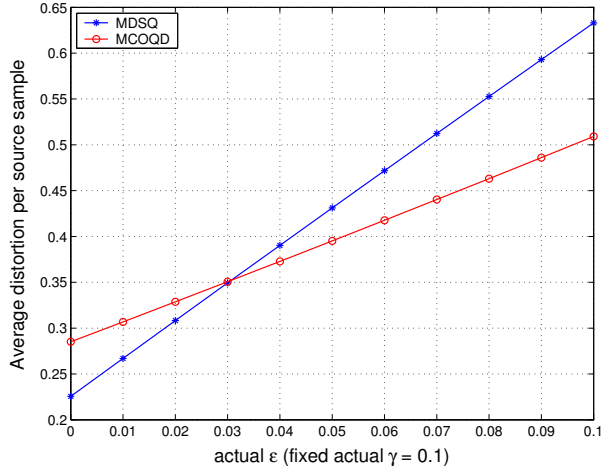
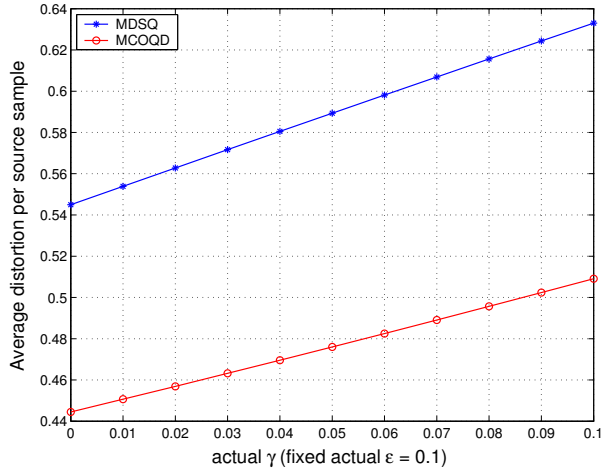


Fig. 7. Performance of MDSQ and MCOQD when design channel is better than the actual channel. $\epsilon_{design} = 0.01$, $\gamma_{design} = 0.01$, $R_1 = R_2 = 1$ bps. (a) Performance for various actual ϵ with fixed actual $\gamma = 0.01$. (b) Performance for various actual γ with fixed actual $\epsilon = 0.01$.



(a)



(b)

Fig. 8. Performance of MDSQ and MCOQD when design channel is worse than the actual channel. $\epsilon_{design} = 0.1$, $\gamma_{design} = 0.1$, $R_1 = R_2 = 1$ bps. (a) Performance for various actual ϵ with fixed actual $\gamma = 0.1$. (b) Performance for various actual γ with fixed actual $\epsilon = 0.1$.

where the last step holds for the squared error distortion measure.

The optimal decoder is to find $\tilde{x}_j^* = \min_{\tilde{x}_j \in \mathbb{R}} D$, $j = 1, \dots, N$. By setting the partial derivative of this equation with respect to \tilde{x}_j to zero, we get

$$\begin{aligned} \frac{\partial D}{\partial \tilde{x}_j} &= -2 \sum_{i=1}^M E[X|X \in Q_i] p(X \in Q_i) p(j|i) \\ &\quad + 2\tilde{x}_j \sum_{i=1}^M p(X \in Q_i) p(j|i) = 0. \end{aligned}$$

This yields

$$\tilde{x}_j^* = \frac{\sum_{i=1}^M E[X|X \in Q_i] p(X \in Q_i) p(j|i)}{\sum_{i=1}^M p(X \in Q_i) p(j|i)} \quad j = 1, \dots, N. \quad (7)$$

Consider the reconstruction value \tilde{x}^E when both channels

fail. In this case,

$$p(V^1 = E, V^2 = E|i) = \gamma_1 \gamma_2. \quad (8)$$

Substituting (8) into (7), we obtain

$$\tilde{x}^E = \frac{\sum_{i=1}^M E[X|X \in Q_i] p(X \in Q_i) \gamma_1 \gamma_2}{\sum_{i=1}^M p(X \in Q_i) \gamma_1 \gamma_2} = E(X). \quad (9)$$

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