

# Downlink Distributed Beamforming Through Relay Networks

Yi Zheng and Steven Blostein

Department of Electrical and Computer Engineering  
Queen's University, Kingston, Ontario, K7L3N6, Canada

**Abstract**—Beamforming and relaying are two methods of improving wireless system performance. Until now, these techniques have been considered separately. In this paper, a single source (basestation) with multiple antennas broadcasts to multiple single-antenna destinations through a network of single-antenna relays. A transmit precoder is first proposed to optimize the system for a set of fixed relay gains and phases (beamforming weights). Next, by straightforwardly generalizing results from the recent literature, these distributed relay beamforming weights are optimized for this transmit precoder. The process is performed iteratively, and both problems are solved using semidefinite relaxation. For the special case of a single destination, we also provide a closed-form solution for relay optimization using a fixed linear precoder at the transmitter. Simulation results quantify the tradeoffs between the number of antennas at the basestation versus the number of relays, in terms of total relay network transmit power savings.

## I. INTRODUCTION

Communication over wireless channels is subject to channel fading, and results in fluctuation of the received signals. To combat channel fading, diversity techniques are commonly used. Traditionally, to introduce diversity, multiple copies of signals are created that span time, frequency or space [1]. A recent technique to introduce diversity into the system was proposed in [2] [3], in which multiple spatially separated communication devices (nodes) cooperate to improve the quality of communications between source and destination. Such systems can introduce diversity into the system by using cooperating nodes to relay information to the destination after some delay. Several protocols to accomplish the task have been introduced in [2] [4], of which Amplify and Forward (AF) has been shown to achieve full diversity. These systems broadcast signals from a source to a set of relays which then forward onto a destination. In the literature, the broadcast channel has been studied in depth. In [5], the design of linear precoders for fixed MIMO receivers is considered using SINR constraints. The MMSE linear precoding/decoding design has been studied for the uplink in [6], and for the downlink in [7] [8]. The signal processing involved in the uplink beamforming is simpler than that of the downlink [9] [10]. Thus, uplink-downlink duality is an important tool to simplify downlink system design.

Recently, the problem of transmission over a broadcast channel has been applied to the above problem of cooperation, where relays cooperate so as to form a beam towards the receiver [11], known as distributed beamforming. In distributed beamforming, amplitudes and phases of transmitted signals are coherently combined at the receiver [12]. In [13], rate maximization for a parallel relay network with noise correlation is studied. In [14], a distributed beamforming system with a single transmitter and receiver and multiple relay nodes is studied, and second order statistics of the channel are employed to design the optimal relay beamforming weights. In [15], single-antenna source-destination pairs that communicate through a relay network are considered, and the relay weight optimization is formulated in terms of semidefinite programming (SDP) and solved through a semidefinite relaxation technique. In [16], a distributed beamforming scheme with two relays is proposed which has the advantage of limited feedback and improved diversity. In [17], with limited feedback and assuming perfect CSI, beamforming codebook design based on Grassmanian codebook for multiple-input multiple output (MIMO) amplify-and-forward (AF) relay system is proposed.

Most prior research reported in the literature assumes a single antenna at the source, relay and destination. In this paper, we study a scenario where the source has multiple antennas, and determine the optimal precoder for the cooperative system with fixed relay weights. We extend the relay weight optimization developed for single antenna cooperative systems [14] [15] to multiple transmit antenna cooperative systems. We also develop an iterative optimization algorithm to iteratively optimize the precoder and relay weights. The proposed scheme is especially applicable to enhance performance in indoor wireless environments. Here, the channels are slowly varying or static, corresponding to the practical case when a base station broadcasts signals to multiple users through a network of relays.

More specifically, consider a wireless network consisting of an  $N$  antenna source (base station),  $R$  single-antenna relays and  $M$  single antenna destinations (users). The system operates in a half-duplex mode: in the first time slot, the source broadcasts its signals to the relays. In the second time slot, the relays amplify and forward their signals to the destinations. In this approach, the relay beam weights are

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calculated at the source and then transmitted to the relays. First, we separately consider the optimization of (i) the linear precoder for fixed relay weights, and (ii) the relay weights for a fixed linear precoder. For the single-user case, we derive a closed-form solution for the optimum relay weights. We then propose an iterative algorithm to minimize the sum power at the relays and the transmit power at the source.

In [12], the corresponding uplink distributed beamforming problem is studied where the optimum linear decoder for fixed relay weights is derived. We note that the downlink distributed beamforming problem is not the dual of [12].

The remainder of the paper is organized as follows: In Section II, we present the system model. Linear precoder optimization assuming known relay weights is developed in Section III, followed by relay weight optimization in Section IV. The single user optimization is presented in Section V, and numerical results are provided in Section VI.

## II. SYSTEM MODEL

We consider a broadcast channel from a single source with multiple antennas to multiple destinations through a relay network as shown in Fig. 1. Data is transmitted from a source to multiple users through relays over two time slots in half-duplex mode. In the first time slot, the source transmits signals to the relay network. In the second time slot, the relays forward signals to the destinations. We assume the source has  $N$  antennas, the multi-relay network has  $R$  single-antenna relays, and there are  $M$  distributed destinations, where  $M \leq N$ , each with a single antenna. It is assumed that since range extension is an intended application, there is no direct link between source and destinations. The channel from the source to the  $r$ th relay,  $1 \leq r \leq R$ , is represented as

$$x_r = \mathbf{h}_r^T \sum_{i=1}^M \mathbf{t}_i s_i + \nu_r \quad (1)$$

where  $\mathbf{h}_r^T$  is the channel from the source to the  $r$ th relay, and  $\mathbf{t}_i$  denotes the source beamforming vector corresponding to signal  $s_i$  intended for the  $i$ th destination.

To model distributed beamforming, the  $i$ th relay multiplies its received signal by a complex weighting coefficient  $w_i^*$ . The vector of the signals  $\mathbf{u}$  transmitted from the relays to the destinations is thus

$$\mathbf{u} = \mathbf{W}^H \mathbf{x}_r \quad (2)$$

where diagonal matrix  $\mathbf{W} = \text{diag}(w_1, w_2, \dots, w_R)$  and  $\mathbf{x}_r = [x_1 \ x_2 \ \dots \ x_R]^T$ . The received signal at the  $i$ th destination is expressed as

$$\begin{aligned} y_i &= \mathbf{g}_i^T \mathbf{u} + n_i \\ &= \underbrace{\mathbf{g}_i^T \mathbf{W}^H \mathbf{H} \mathbf{t}_i s_i}_{\text{Desired Signal}} + \underbrace{\mathbf{g}_i^T \mathbf{W}^H \mathbf{H} \sum_{j=1, j \neq i}^M \mathbf{t}_j s_j}_{\text{interference}} + \underbrace{\mathbf{g}_i^T \mathbf{W}^H \boldsymbol{\nu} + n_i}_{\text{noise}} \end{aligned} \quad (3)$$

where matrix  $\mathbf{H}$  is the combined channel from source to relays,  $\mathbf{g}_i^T$  is the channel from relays to  $i$ th destination, and  $\boldsymbol{\nu} = [\nu_1, \dots, \nu_R]^T$  represents noise at the relays.

## III. TRANSMIT PRECODER OPTIMIZATION

First we consider the problem of finding the optimal transmit precoder while the destinations' quality of service (QoS), expressed in terms of signal-to-interference plus noise ratio (SINR), are kept above pre-defined thresholds for fixed relay weights. It is assumed that channel state information (CSI), in the form of complex channel gains, is fed back to the source from the relays and the destinations. Thus, we aim to solve the following optimization problem:

$$\begin{aligned} & \min_{\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_M} P_T \\ \text{s.t.} \quad & \text{SINR}_k \geq \gamma_k, \quad \text{for } k = 1, 2, \dots, M \end{aligned} \quad (4)$$

where  $P_T$  is the transmit power at the source and  $\text{SINR}_k$  is the SINR at the  $k$ th destination defined as

$$\text{SINR}_k = \frac{P_s^k}{P_i^k + P_n^k} \quad (5)$$

and s.t. stands for "subject to". Here  $P_s^k$ ,  $P_i^k$  and  $P_n^k$  denote desired signal power, interference power, and noise power at the  $k$ th destination, respectively. The transmitted power  $P_T$  is given by

$$\begin{aligned} P_T &= E\left\{ \left( \sum_{i=1}^M \mathbf{t}_i s_i \right)^H \left( \sum_{j=1}^M \mathbf{t}_j s_j \right) \right\} \\ &= \sum_{i=1}^M \mathbf{t}_i^H \mathbf{t}_i = \sum_{i=1}^M \text{Tr}\{\mathbf{t}_i \mathbf{t}_i^H\} \end{aligned} \quad (6)$$

where  $\text{Tr}(\cdot)$  stands for trace( $\cdot$ ).

In (5), the desired signal power at the  $k$ th destination is given by

$$\begin{aligned} P_s^k &= E\left\{ (\mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \mathbf{t}_k s_k)^H (\mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \mathbf{t}_k s_k) \right\} \\ &= \mathbf{t}_k^H \mathbf{H}^H \mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \mathbf{t}_k E\{s_k^* s_k\}, \end{aligned} \quad (7)$$

the interference power at the  $k$ th destination is

$$\begin{aligned} P_i^k &= E\left\{ (\mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \sum_{j=1, j \neq k}^M \mathbf{t}_j s_j)^H (\mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \sum_{l=1, l \neq k}^M \mathbf{t}_l s_l) \right\} \\ &= \sum_{j=1, j \neq k}^M \mathbf{t}_j^H \mathbf{H}^H \mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \mathbf{t}_j, \end{aligned} \quad (8)$$

and the noise power at the  $k$ th destination is

$$\begin{aligned} P_n^k &= E\left\{ (\mathbf{g}_k^T \mathbf{W}^H \boldsymbol{\nu} + n_k)^H (\mathbf{g}_k^T \mathbf{W}^H \boldsymbol{\nu} + n_k) \right\} \\ &= \text{Tr}\{\mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H \sigma_\nu^2\} + \sigma_n^2, \end{aligned} \quad (9)$$

where  $\sigma_n^2$  and  $\sigma_\nu^2$  are noise powers at destinations and relays, respectively.

Using (6), (7), (8) and (9), the optimization problem in (4) is expressed as

$$\begin{aligned} & \min_{\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_M} \sum_{i=1}^M \text{Tr}\{\mathbf{t}_i \mathbf{t}_i^H\} \\ \text{s.t. } & \frac{\mathbf{t}_k^H \mathbf{H}^H \mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \mathbf{t}_k}{\sum_{j=1, j \neq k}^M \mathbf{t}_j^H \mathbf{H}^H \mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \mathbf{t}_j + P_n^k} \\ & \geq \gamma_k \quad \text{for } k = 1, 2, \dots, M \end{aligned} \quad (10)$$

Unfortunately, problem (10) is not convex, making optimization difficult. We instead solve a relaxed version of (10). Defining

$$\mathbf{T}_i = \mathbf{t}_i \mathbf{t}_i^H, \quad i = 1, 2, \dots, M \quad (11)$$

(10) becomes

$$\begin{aligned} & \min_{\mathbf{T}_1, \mathbf{T}_2, \dots, \mathbf{T}_M} \sum_{k=1}^M \text{Tr}(\mathbf{T}_k) \\ \text{s.t. } & \text{Tr}(\mathbf{U}_k (\mathbf{T}_k - \gamma_k \sum_{j=1, j \neq k}^M \mathbf{T}_j)) \geq \gamma_k P_n^k, \\ & \mathbf{T}_k \succeq 0 \quad \text{for } k = 1, 2, \dots, M \end{aligned} \quad (12)$$

where  $\mathbf{U}_k = \mathbf{H}^H \mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H \mathbf{H}$ , and where we have dropped the constraints  $\text{rank}(\mathbf{T}_k) = 1, k = 1, 2, \dots, M$ .

The solution to (12) establishes a lower bound to the problem (10). Only when (12) has a solution with  $\mathbf{T}_i, i = 1, \dots, M$  all rank one will (10) achieve the lower bound. Using arguments similar to those found in [18], it can be proven that there always exists at least one solution to (12) with  $\text{rank}(\mathbf{T}_k) = 1, k = 1, \dots, M$ . This can be formalized as:

**Lemma 1** *If the relaxed problem of (10) is feasible, (12) always has at least one minimum power solution where all  $\mathbf{T}_k$  are rank-one.*

A proof of Lemma 1 is outlined as follows: first, Eq. (10) rewritten as

$$\begin{aligned} & \min \sum_{i=1}^M \mathbf{t}_i^* \mathbf{t}_i \\ \text{s.t. } & \mathbf{t}_k^* \mathbf{H}^H \mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \mathbf{t}_k \\ & - \gamma_k \sum_{j=1, j \neq k}^M \mathbf{t}_j^* \mathbf{H}^H \mathbf{W} \mathbf{g}_k^* \mathbf{g}_k^T \mathbf{W}^H \mathbf{H} \mathbf{t}_j \geq \gamma_k P_n^k \\ & k = 1, \dots, M \end{aligned} \quad (13)$$

can be proven to be algebraically equivalent to

$$\begin{aligned} & \min_{\nu_j, \rho_j} \sum_{j=1}^M \rho_j \gamma_j P_n^j \\ \text{s.t. } & \frac{\nu_j^* \rho_j \mathbf{R}_j \nu_j}{\nu_j^* \left( \sum_{i \neq j} \rho_i \gamma_i \mathbf{R}_i + \mathbf{I} \right) \nu_j} \geq 1, \\ & \|\nu_i\|^2 = 1, \quad i = 1, \dots, M \end{aligned} \quad (14)$$

and  $\mathbf{t}_i = \sqrt{\rho_i} \nu_i$ , where  $\mathbf{R}_j = \mathbf{H}^H \mathbf{W} \mathbf{g}_j^* \mathbf{g}_j^T \mathbf{W}^H \mathbf{H}$ . The solution does not depend on the coefficients of  $\rho_j$  in the cost function.

It is obvious that the objective value of the semidefinite relaxation of (13), which is (12), is lower than that of (13) because of the constraint relaxation. On the other hand, the Lagrange dual of the semidefinite relaxation provides a lower bound on the cost function. So by demonstrating that (13) and the Lagrange dual of the semidefinite relaxation of (13) have the same optimum cost, it proves that (13), the semidefinite relaxation of (13) and the Lagrange dual of the semidefinite relaxation are equivalent.

We then note that (14) has the same form as a problem considered in [19], that determines uplink basestation beamformers  $\nu$  and transmit powers  $\rho_j$  at the mobiles. Since in [19] an optimum solution is found, we therefore argue that an optimum solution exists for (12).

#### IV. RELAY WEIGHT OPTIMIZATION

In this section, we first consider the relay weight optimization for a single destination, then we study the optimization for multiple destinations.

##### A. RELAY WEIGHT OPTIMIZATION: SINGLE DESTINATION

For this special case, the signal model in (3) becomes

$$y = \underbrace{\mathbf{w}^H \text{diag}\{\mathbf{g}^T\} \mathbf{H} \mathbf{t} s}_{\text{Desired Signal}} + \underbrace{\mathbf{w}^H \text{diag}\{\mathbf{g}^T\} \nu + n}_{\text{Colored noise}} \quad (15)$$

where  $\mathbf{t}$  is the linear precoder at the source,  $\mathbf{g}$  is the channel from the relays to the user. We remark that the problem in [14] is a special case of (15) since in [14] there is one source antenna ( $N = 1$ ).

1) **RELAY POWER MINIMIZATION:** Using the derivations in the last section, the problem of the sum power minimization of relays with SNR constraints for a fixed linear precoder is:

$$\begin{aligned} & \min_{\mathbf{w}} \mathbf{w}^H \mathbf{D} \mathbf{w} \\ \text{s.t. } & \frac{\mathbf{w}^H \text{diag}\{\mathbf{g}^T\} \mathbf{H} \mathbf{t} \mathbf{t}^H \mathbf{H}^H \text{diag}\{\mathbf{g}^*\} \mathbf{w}}{\sigma_\nu^2 \mathbf{w}^H \text{diag}\{\mathbf{g}^T\} \text{diag}\{\mathbf{g}^*\} \mathbf{w} + \sigma_n^2} \geq \gamma \end{aligned} \quad (16)$$

where

$$\mathbf{D} \triangleq \text{diag}([\mathbf{R}_x]_{1,1}, [\mathbf{R}_x]_{2,2}, \dots, [\mathbf{R}_x]_{R,R}) \quad (17)$$

$$\mathbf{R}_x = \mathbf{H}^H \mathbf{t} \mathbf{t}^H \mathbf{H} + \sigma_\nu^2 \mathbf{I}. \quad (18)$$

By transforming the optimization variable to  $\hat{\mathbf{w}} = \mathbf{D}^{1/2} \mathbf{w}$ , the optimization problem can be written as

$$\begin{aligned} & \min_{\hat{\mathbf{w}}} \|\hat{\mathbf{w}}\|^2 \\ \text{s.t. } & \hat{\mathbf{w}}^H \mathbf{D}^{-1/2} (\mathbf{C} - \gamma \mathbf{E}) \mathbf{D}^{-1/2} \hat{\mathbf{w}} \geq \gamma \sigma_n^2 \end{aligned} \quad (19)$$

where  $\mathbf{C} = \text{diag}\{\mathbf{g}^T\} \mathbf{H} \mathbf{t} \mathbf{t}^H \mathbf{H}^H \text{diag}\{\mathbf{g}^*\}$  and  $\mathbf{E} = \sigma_\nu^2 \text{diag}\{\mathbf{g}^T\} \text{diag}\{\mathbf{g}^*\}$ .

We note that the problem (19) is equivalent to that in [14], Eq. (12). The optimum weight solution of (19) appears as [14], Eq. (19), and the corresponding transmit power is given by (19), Eq. (20).

### B. RELAY SUM POWER MINIMIZATION FOR MULTIPLE DESTINATIONS

We next consider the problem of relay weight optimization for multiple destinations with a fixed precoder, which can be obtained from Section III. As in Section III, we assume full CSI at the source. We reformulate (3) as

$$y_k = \underbrace{\mathbf{w}^H \text{diag}\{\mathbf{g}_k^T\} \mathbf{H} \mathbf{t}_k s_k}_{\text{Desired Signal}} + \underbrace{\mathbf{w}^H \text{diag}\{\mathbf{g}_k^T\} \mathbf{H} \sum_{j=1, j \neq k}^M \mathbf{t}_j s_j}_{\text{Interference}} + \underbrace{\mathbf{w}^H \text{diag}\{\mathbf{g}_k^T\} \boldsymbol{\nu} + n_k}_{\text{Colored noise}} \quad (20)$$

where  $\mathbf{w}$  is a column vector formed by the diagonal elements of  $\mathbf{W}$  such that  $w_i = W_{i,i}$ . We aim to solve the following optimization problem:

$$\min_{\mathbf{w}} P_R \quad (21)$$

s.t.  $\text{SINR}_k \geq \gamma_k$  for  $k = 1, \dots, M$

where  $P_R$  is the sum transmit power at the relays given as

$$\begin{aligned} P_R &= E\{\mathbf{u}^H \mathbf{u}\} \\ &= \text{Tr}\{\mathbf{W}^H E\{\mathbf{x}\mathbf{x}^H\} \mathbf{W}\} \\ &= \mathbf{w}^H \mathbf{D} \mathbf{w}, \end{aligned} \quad (22)$$

where  $\mathbf{D}$  is as in (17) and

$$\mathbf{R}_x = \mathbf{H}^H \left( \sum_{k=1}^M \mathbf{t}_k \mathbf{t}_k^H \right) \mathbf{H}^H + \sigma_{\nu}^2 \mathbf{I}. \quad (23)$$

The SINR constraints for the  $k$ th user can be expressed as

$$\frac{\mathbf{w}^H \mathbf{E}_k \mathbf{w}}{\mathbf{w}^H \mathbf{F}_k \mathbf{w} + \sigma_n^2} \geq \gamma_k \quad (24)$$

where

$$\begin{aligned} \mathbf{E}_k &= \text{diag}(\mathbf{g}_k^T) \mathbf{H} \mathbf{t}_k \mathbf{t}_k^H \mathbf{H}^H \text{diag}(\mathbf{g}_k^*) \\ \mathbf{F}_k &= \text{diag}(\mathbf{g}_k^T) \left( \mathbf{H} \left( \sum_{j=1, j \neq k}^M \mathbf{t}_j \mathbf{t}_j^H \right) \mathbf{H}^H + \sigma_{\nu}^2 \mathbf{I} \right) \text{diag}(\mathbf{g}_k^*). \end{aligned} \quad (25)$$

Using (22) and (24) we can rewrite (21) as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{D} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^H (\mathbf{E}_k - \gamma_k \mathbf{F}_k) \mathbf{w} \geq \gamma_k \sigma_n^2 \\ & \text{for } k = 1, 2, \dots, M. \end{aligned} \quad (26)$$

We remark that with the above definitions, the problem (22) has been transformed into that appearing in [15], Eq. (17). We note, however, that for the multiple antenna source

considered here, the matrices  $\mathbf{F}_k, k = 1, 2, \dots, M$  differ considerably from their counterparts in [15]. As these problems are equivalent, the discussion surrounding [15], Eq. (18) applies and we employ semi-definite relaxation by solving a relaxed version of (26). That is, by defining  $\mathbf{Z} = \mathbf{w}\mathbf{w}^H$ , we drop the constraint  $\text{rank}(\mathbf{Z}) = 1$ . The modified (26) becomes

$$\begin{aligned} \min_{\mathbf{Z}} \quad & \text{Tr}(\mathbf{Z}\mathbf{D}) \\ \text{s.t.} \quad & \text{Tr}(\mathbf{Z}(\mathbf{E}_k - \gamma_k \mathbf{F}_k)) \geq \gamma_k \sigma_n^2 \\ & \text{and } \mathbf{Z} \succeq 0 \\ & \text{for } k = 1, 2, \dots, M. \end{aligned} \quad (27)$$

### C. JOINT DETERMINATION OF LINEAR PRECODER AND RELAY WEIGHTS

To jointly optimize the linear precoder and relay beamforming weights, the following alternating algorithm is proposed:

1. Initialize the relay beamforming vector as  $\mathbf{w} = c * \text{vec}(\mathbf{v})$ , where constant  $c$  is chosen to be large relative to  $\sigma_n^2$ , and  $\mathbf{v}_i = e^{j\theta_i}$ , where  $\theta_i$  is a random variable, uniformly distributed over  $[0, 2\pi]$ .
2. Check the feasibility of the constraints of (12). If not feasible, then modify the SINR constraints and go back to Step 1.
3. Apply (12) to optimize the precoder with the current relay weights fixed.
4. Solve (27) to minimize the relay sum power. In case there is no rank one solution, apply the randomization method in [20].
5. If the relay sum power is sufficiently close to a fixed point, or else if a predetermined number of iterations is exceeded, then stop. Otherwise go back to Step 3.

We remark that since the transmit powers of the source and relays are both lower-bounded, and in each of Steps 3 and 4 the power reduces, the algorithm converges to a fixed point. This is apparent by observing that the optimizations in Steps 3 and 4 each have SINR constraints that are the same.

### V. SIMULATION RESULTS

For the linear precoder optimization, we assume that the channel coefficients from source to relays are known at the source and the second order statistics of the channel coefficients from the  $R$  relays to the  $M$  destinations are available to the source. For the optimization of the relay weights, we assume that the full CSI of all the channel matrices is available to the source. The channel coefficient matrix  $\mathbf{H}$  and vector  $\mathbf{g}_k$  are assumed to be independent where  $\mathbf{H}$  represents the set of  $R$  distributed channels from the source to the relays and the  $R \times 1$  vector  $\mathbf{g}_k$  represents the  $R$  channels from the relays to the  $k$ th destination. Without loss of generality, we assume a scenario where channel estimation error is incurred only in the relay to destination link. More specifically,  $\mathbf{g}_k$  can be written as  $\mathbf{g}_k = \bar{\mathbf{g}}_k + \tilde{\mathbf{g}}_k$ ,  $1 \leq k \leq M$ , where  $\bar{\mathbf{g}}_k$  is an unbiased estimate of  $\mathbf{g}_k$  and  $\tilde{\mathbf{g}}_k$  is a zero mean random variable that represents the channel estimation error.

We remark that we have also considered the case of imperfect CSI with known channel error covariance, which will be presented in a future publication. For consistency, we use the same simulation model as [14]: for all simulations, we assume the transmission power from the  $M$  sources to be identical and 10dB above the noise level. The channel estimation noise is set to -20dB. In the following, we consider scenarios with different numbers of relays and receive antennas. Without loss of generality, we assume the same SINR threshold for all destinations, e.g.,  $\gamma = \gamma_1 = \dots = \gamma_M$ .

Fig. 2 compares the minimum sum transmit power at the source versus the SINR threshold  $\gamma$  with  $M = 4$  destinations,  $R$  relays, and  $N$  source antennas. The cases of  $N = R = 4$ ,  $N = R = 6$  and  $N = R = 8$  are shown. It can be seen in Fig. 2 that as the numbers of source antennas and relays increase, the minimum sum transmit power required at the relays decreases. This is as expected since more source antennas and relays result in greater beamforming gain and diversity.

Fig. 3 depicts relay sum power for the same scenario as Fig. 2. We observe that as more source antennas and relays are added, relay sum power decreases. We also notice a threshold effect: when the SINR constraints exceed about 6 or 8 dB for the case of  $N = R = M = 4$ , transmission power increases sharply. This is as expected since as the SINR constraints tighten when the number of source antennas and relays used are only the minimum required, the problem becomes infeasible.

In Fig. 4, the cases of 4 source antennas and 4 destination,  $M = N = 4$ , are compared using deployments of  $R = 6, 8, 12$  relays. It is observed that as more relays are used, the total power consumed by the relay network decreases. Similarly, the corresponding transmission power required at the source decreases. This latter result is not shown here.

Fig. 5 compares algorithm performance as a function of the number of iterations for the case of  $M = N = 4$  and  $R = 6$ . It is shown that after about 5 iterations, a fixed point is being approached. Also, it is noted that the first two iterations result in the largest gain, and performance gain diminishes as the number of iterations increases.

Since the recently proposed distributed beamforming systems as described in [14] [15] have not been designed for multi-antenna basestations, they cannot be directly compared. It is also worth noting that the computational complexity of semidefinite programming is  $O((M + R^2)^{3.5})$  in the worst case. This makes using larger numbers of relays computationally prohibitive. In the system proposed in this paper, these results demonstrate how source basestation complexity, in terms of the number of antennas, can effectively be traded off with relay network size in order to support a network of users in a power efficient manner.

## VI. CONCLUSION

The scenario of wireless broadcasting from a multiple antenna basestation to multiple destinations through a network

of relays is considered. It is shown that the transmit precoder at the source and distributed relay beamformer can be jointly optimized in an iterative fashion. The results in this paper show that significant overall power savings can be obtained as a function of the number of relays and/or number of transmit antennas, even if only a small number of iterations is used.

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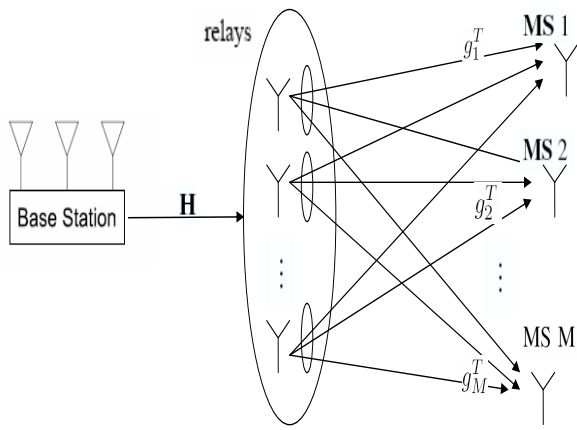


Fig. 1. Downlink distributed beamforming system.

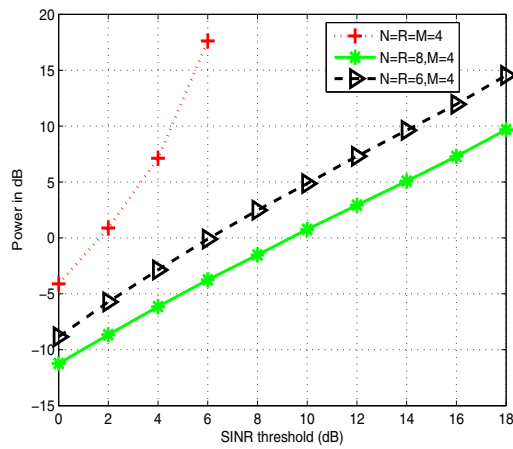


Fig. 2. Comparison of minimum total source transmit power versus SINR threshold  $\gamma$  as a function of network size.

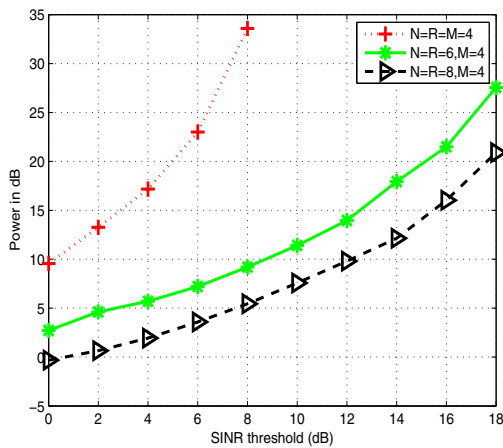


Fig. 3. Comparison of minimum total relay transmit power versus SINR threshold  $\gamma$  as a function of network size.

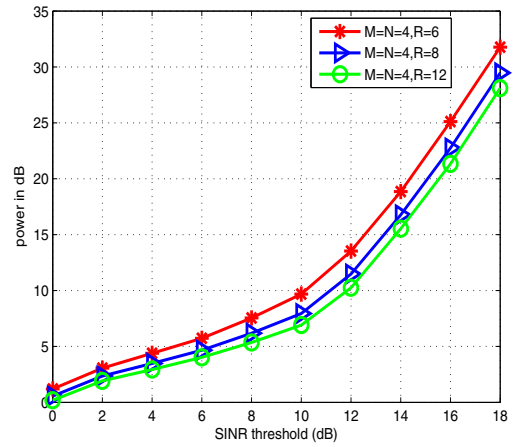


Fig. 4. Comparison of minimum total relay transmit power versus SINR threshold  $\gamma$  for different numbers of relays.

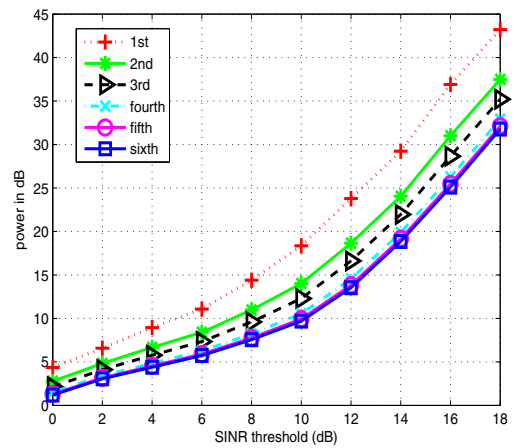


Fig. 5. Comparison of minimum total relay transmit power versus SINR threshold  $\gamma$  for different numbers of iterations.