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Chapter xx

LINEAR DISPERSION CODES FOR WIRELESS COMMUNICATIONS

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Abstract

This chapter provides readers a systematic and panoramic view of LDC formulation and various LDC designs under different criteria.

1. LDC

1.1. Introduction

The term, linear dispersion codes (LDC), has been considered as a general terminology for block coding [1–3]. LDC were first proposed by Hassibi and Hochwald as a general framework of arbitrary complex space time codes (CSTC) for block flat-fading channels [1, 2]. LDC subsumes orthogonal space time block codes and Vertical Bell Labs Layered Space-times (V-BLAST) as subclasses [1, 2]. LDC possess high coding rates (the definition of LDC coding rate will be discussed in Section 1.2.2.), and can support arbitrary configuration of transmit and receive antennas. To avoid numerical difficulty of minimizing average pairwise error probability (PEP), LDC design was initially achieved by formulating a power-constrained optimization problem based on mutual information [2]. After the seminal work introduced in [1], the term of LDC has been extensively adopted for high-rate CSTC designs under different design criteria other than capacity criterion. Although initially designed as complex codes in space-time channels, LDC has further been considered

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as a general framework of linear 2-dimensional (2-D) codes in general 2-D communications channels. Note that conventional block error correcting codes can be considered as real integer subclasses of LDC.

This chapter aims to give readers a systematic and panoramic view of LDC formulation and various LDC designs under different criteria. The following notations are used: $(\cdot)^{\dagger}$ denotes matrix pseudoinverse, $(\cdot)^{\mathcal{T}}$ matrix transpose, $(\cdot)^{\mathcal{H}}$ matrix transpose conjugate, $\mathbf{I}_K$ denotes identity matrix of size $K \times K$, $\mathbf{0}_{m\times n}$ denotes zero matrix of size $m \times n$, $[\mathbf{X}]_{a,b}$ denotes the (a, b) entry of matrix $\mathbf{X}$, $\mathbf{A} \otimes \mathbf{B}$ denotes Kronecker (tensor) product of matrices $\mathbf{A}$ and $\mathbf{B}$, $C^{m\times n}$ denotes a complex matrix with dimensions $m \times n$, and $\mathbf{F}_M$ denotes the discrete Fourier transform (DFT) matrix, representing the M-point fast Fourier transform (FFT) with entries $[\mathbf{F}_M]_{a,b} = \left(1/\sqrt{M}\right) \exp\left(-j2\pi(a-1)(b-1)/M\right)$.

1.2. Concepts of LDC

1.2.1. LDC Definition

Assume that an uncorrelated data sequence has been modulated using complex-valued source data symbols chosen from an arbitrary, e.g. r_c-PSK or r_c-QAM, constellation. A $T \times N_t$ LDC matrix codeword $\mathbf{S}_{LDC}$ is transmitted from N_t transmit antennas and occupies T channel uses and encodes Q source data symbols. LDC was originally proposed as a complex space-time matrix coding framework [2]. The matrix codeword $\mathbf{S}_{LDC}$ is expressed as

$$\mathbf{S}_{LDC} = \sum_{q=1}^{Q} \alpha_q \mathbf{A}_q + j\beta_q \mathbf{B}_q, \tag{1}$$

where $\mathbf{S}_{LDC} \in C^{T\times N_t}$ and $\mathbf{A}_q \in C^{T\times N_t}, \mathbf{B}_q \in C^{T\times N_t}, q = 1, ..., Q$ are called dispersion matrices. The complex source data symbols are defined by

$$s_q = \alpha_q + j\beta_q, \quad q = 1, ..., Q. \tag{2}$$

Note that there is another LDC definition with different dispersion matrices, $\mathbf{C}_q$ and $\mathbf{D}_q$, as follows [2]

$$\mathbf{S}_{LDC} = \sum_{q=1}^{Q} s_q \mathbf{C}_q + s_q^* \mathbf{D}_q, \tag{3}$$

where $\mathbf{C}_q = \frac{1}{2}\left(\mathbf{A}_q + \mathbf{B}_q\right)$ and $\mathbf{D}_q = \frac{1}{2}\left(\mathbf{A}_q - \mathbf{B}_q\right)$, $q = 1, ..., Q$.

If the basic encoding units are real and imaginary components of data symbols, the LDC codewords are can be written as

$$\mathbf{S}_{LDC} = \sum_{k=1}^{2Q} \tau_k \mathbf{K}_k, \tag{4}$$

where the components are

$$\tau_k = \begin{cases} \alpha_k, & k = 1, ..., Q \\ \beta_{k-Q}, & k = Q+1, ..., 2Q \end{cases}$$

and the component dispersion matrices are

$$\mathbf{K}_k = \begin{cases} \mathbf{A}_k, & k = 1, ..., Q \\ j\mathbf{B}_k, & k = Q+1, ..., 2Q \end{cases}$$

1.2.2. Coding Rate of LDC

Hassibi and Hochwald have defined the coding rate of LDC in terms of bits as

$$R = \frac{Q}{T} \log_2 r, \tag{5}$$

where r is the size of constellation [2].

At the physical layer of a communication system, symbol coding rate often is useful for comparing the spectrum efficiency between different systems. Corresponding to the bit rate definition in (5), the symbol coding rate is defined as

$$R_{LDC}^{sym} = \frac{Q}{T}. \tag{6}$$

Note that this definition is different from that of conventional error correcting codes (ECC). In the latter case, the symbol coding rate is defined as

$$R_{LDC}^{sym} = \frac{Q}{N_t T}, \tag{7}$$

which may be used for codes in frequency-time channels [3]. In space-time channels, the symbol coding rate can be alternatively defined as

$$R_{LDC}^{sym} = \frac{Q}{T \min\{N_T, N_R\}}. \tag{8}$$

According to (8), $R_{LDC}^{sym} = 1$ is the maximum coding rate for linear zero-forcing decoding.

In the rest of this chapter, the original definitions of coding rate (5) and (6) will be used.

1.2.3. Matrix Form LDC Encoding

A special subclass of LDC In this chapter, we primarily consider a special subclass of dispersion matrices with the constraints

$$\mathbf{A}_q = \mathbf{B}_q, \quad q = 1, ..., Q. \tag{9}$$

Substituting (2) and (9) into (1) yields

$$\mathbf{S}_{LDC} = \sum_{q=1}^{Q} s_q \mathbf{A}_q, \tag{10}$$

which can be transformed into matrix form by using the vec operation. Reordering $\mathbf{S}_{LDC}$ and each matrix $\mathbf{A}_q$ into a $TM \times 1$ column vector, respectively, by $vec(\mathbf{S}_{LDC})$ and $vec(\mathbf{A}_q)$, we obtain

$$vec(\mathbf{S}_{LDC}) = \begin{bmatrix} vec(\mathbf{A}_1) & ... & vec(\mathbf{A}_Q) \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_Q \end{bmatrix}. \tag{11}$$

An example of this special class of LDC codes is shown as follows. The group of square dispersion matrices of this code, referred to as HH Square LDC [2], satisfies constraint (9). More specifically, the dispersion matrices are

$$\mathbf{A}_{N_t(k-1)+l} = \mathbf{B}_{N_t(k-1)+l} = \frac{1}{\sqrt{N_t}} \mathbf{D}^{k-1} \mathbf{\Pi}^{l-1}, \tag{12}$$

where $k = 1, ..., N_t$, $l = 1, ..., N_t$,

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & ... & 0 \\ 0 & e^{j\frac{2\pi}{N_t}} & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & e^{j\frac{2\pi(N_t-1)}{N_t}} \end{bmatrix},$$

and

$$\mathbf{\Pi} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}.$$

Using the definition (6), the symbol coding rate of the above codes is N_t.

A possible zero-forcing method to estimate the data symbol vector in (11) is to calculate the Moore-Penrose pseudo-inverse of LDC encoding matrix

$$\mathbf{G}_{LDC} = [vec(\mathbf{A}_1), ..., vec(\mathbf{A}_Q)]. \tag{13}$$

General matrix form Denote

$$\mathbf{A}_{vec} = \begin{bmatrix} vec(\mathbf{A}_1^{\mathcal{T}}) & vec(\mathbf{A}_2^{\mathcal{T}}) & ... & vec(\mathbf{A}_Q^{\mathcal{T}}) \end{bmatrix}, \tag{14}$$

$$\mathbf{B}_{vec} = \begin{bmatrix} vec(\mathbf{B}_1^{\mathcal{T}}) & vec(\mathbf{B}_2^{\mathcal{T}}) & ... & vec(\mathbf{B}_Q^{\mathcal{T}}) \end{bmatrix}, \tag{15}$$

$$\alpha_{vec} = \begin{bmatrix} \alpha_1 & \alpha_2 & ... & \alpha_Q \end{bmatrix}^{\mathcal{T}}, \tag{16}$$

$$\beta_{vec} = \begin{bmatrix} \beta_1 & \beta_2 & ... & \beta_Q \end{bmatrix}^{\mathcal{T}}, \tag{17}$$

$$\theta_{vec} = \left[\begin{array}{cc} \alpha_{vec}^{\mathcal{T}} & \beta_{vec}^{\mathcal{T}} \end{array} \right]^{\mathcal{T}}, \tag{18}$$

$$\mathbf{G}_{vec} = \left[\begin{array}{cc} \mathbf{A}_{vec} & j\mathbf{B}_{vec} \end{array} \right], \tag{19}$$

$$\mathbf{s}_{vec} = \left[\begin{array}{ccc} s_1 & \cdots & s_Q \end{array} \right]^{\mathcal{T}}. \tag{20}$$

In this general case, we have

$$vec\left(\mathbf{S}_{LD}^{\mathcal{T}}\right) = \mathbf{G}_{vec}\theta_{vec}. \tag{21}$$

With constraint (9), we have

$$vec(\mathbf{S}_{LD}^{\mathcal{T}}) = \mathbf{A}_{vec}\mathbf{s}_{vec}. \tag{22}$$

There is a slight difference between $\mathbf{A}_{vec}$ in (14) and $\mathbf{G}_{LDC}$ in (13), i.e., the transpose operations, and both $\mathbf{A}_{vec}$ and $\mathbf{G}_{LDC}$ can encode LDC in different contexts, respectively. Note that other format of encoding matrices may be obatined by permutating the columns of one of the following matrices: $\mathbf{A}_{vec}$ in (14), $\mathbf{G}_{LDC}$ in (13), and $\mathbf{G}_{vec}$ in (19).

In this chapter, under constraint (9), $\mathbf{G}_{LDC}$ is called LDC encoding matrix.

1.2.4. LDC Decoding

Maximum likelihood decoding (MLD) [4,5] and MLD-like decoding, such as sphere decoding (SD) [6–9] have been primarily considered LDC decoding methods in literature. Note that the worst case complexity of both MLD and SD is exponential, which may be prohibitively expensive for practical applications. Although MLD and SD are main methods of LDC decoding, linear LDC decoding has been proposed with lower complexity under some performance loss [3].

1.3. Numerical LDC Designs Based on Capacity Criteria

1.3.1. System Formulation by Hassibi and Hochwald

In [2], Hassibi and Hochwald considered space-time block fading channels. The numbers of transmit and receive antennas are denoted as N_t and N_r, respectively. The basic LDC system was originally formulated as follows [2]:

$$\overline{\mathbf{X}} = \sqrt{\frac{\rho}{N_t}} \sum_{q=1}^{Q} (\alpha_q \mathbf{A}_q + j\beta_q \mathbf{B}_q)\overline{\mathbf{H}} + \overline{\mathbf{V}}, \tag{23}$$

where $\overline{\mathbf{H}} \in C^{N_t \times N_r}$ is the space time MIMO channel matrix, $\overline{\mathbf{X}} \in C^{T \times N_r}$ is the received signal matrix and $\overline{\mathbf{V}} \in C^{T \times N_r}$ is the complex white Gaussian noise. The normalizaton $\sqrt{\frac{\rho}{N_t}}$ ensures that the signal-to-noise-ratio (SNR) at each receive antenna ρ is independent of N_t. A matrix format of (23) can be written as

$$\mathbf{x} = \sqrt{\frac{\rho}{N_t}} \mathcal{H}\theta + \mathbf{v}, \tag{24}$$

where

$$\theta = [\alpha_1, \beta_1, ..., \alpha_Q, \beta_Q]^{\mathcal{T}},$$

$$\mathbf{x} = \left[\mathrm{Re}\left([\overline{\mathbf{X}}]_{:,1}\right), \mathrm{Im}\left([\overline{\mathbf{X}}]_{:,1}\right), ..., \mathrm{Re}\left([\overline{\mathbf{X}}]_{:,N_r}\right), \mathrm{Im}\left([\overline{\mathbf{X}}]_{:,N_r}\right)\right]^{\mathcal{T}},$$

$$\mathbf{v} = \left[\mathrm{Re}\left([\overline{\mathbf{V}}]_{:,1}\right), \mathrm{Im}\left([\overline{\mathbf{V}}]_{:,1}\right), ..., \mathrm{Re}\left([\overline{\mathbf{V}}]_{:,N_r}\right), \mathrm{Im}\left([\overline{\mathbf{V}}]_{:,N_r}\right)\right]^{\mathcal{T}},$$

and

$$\mathcal{H} = \begin{bmatrix} \mathcal{A}_1\underline{\mathbf{h}}_1 & \mathcal{B}_1\underline{\mathbf{h}}_1 & \cdots & \mathcal{A}_Q\underline{\mathbf{h}}_{N_r} & \mathcal{B}_Q\underline{\mathbf{h}}_{N_r} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathcal{A}_1\underline{\mathbf{h}}_{N_r} & \mathcal{B}_1\underline{\mathbf{h}}_{N_r} & \cdots & \mathcal{A}_Q\underline{\mathbf{h}}_{N_r} & \mathcal{B}_Q\underline{\mathbf{h}}_{N_r} \end{bmatrix}, \tag{25}$$

$$\mathcal{A}_q = \begin{bmatrix} \mathrm{Re}(\mathbf{A}_q) & -\mathrm{Im}(\mathbf{A}_q) \\ \mathrm{Im}(\mathbf{A}_q) & \mathrm{Re}(\mathbf{A}_q) \end{bmatrix}, \tag{26}$$

$$\mathcal{B}_q = \begin{bmatrix} -\mathrm{Im}(\mathbf{B}_q) & -\mathrm{Re}(\mathbf{B}_q) \\ \mathrm{Re}(\mathbf{B}_q) & -\mathrm{Im}(\mathbf{B}_q) \end{bmatrix}, \tag{27}$$

$$\underline{\mathbf{h}}_n = \begin{bmatrix} \mathrm{Re}\left([\mathbf{H}]_{:,n}\right) \\ \mathrm{Im}\left([\mathbf{H}]_{:,n}\right) \end{bmatrix}. \tag{28}$$

Remarks:

a) The above LDC system model (23) requires $(N_t \times N_r)$ MIMO block fading channels that are valid only when the channel is constant for at least T channel uses.

b) The matrix model (24) is the same as Eq. (23) in [2] although the notations are different. From (23) and (24), one can see that the knowledge of the channel is required for LDC decoding.

1.3.2. Optimization Based on Capacity Criteria

Design Criterion 1 *[2]*

a) Select $Q \le \min\{N_t, N_r\} T$. *Assume* $E\left(\theta\theta^{\mathcal{T}}\right) = \frac{1}{2}\mathbf{I}_{2Q}$.

b) Select $\{\mathbf{A}_q, \mathbf{B}_q\}$ *to optimize*

$$\begin{aligned} & C_{LD}(\rho, T, N_t, N_r) \\ &= \max_{\mathbf{A}_q, \mathbf{B}_q, q=1,...,Q} \frac{1}{2T} E \log\det\left(\mathbf{I}_{2N_rT} + \frac{\rho}{N_t}\mathcal{H}\mathcal{H}^{\mathcal{T}}\right) \end{aligned} \tag{29}$$

for a SNR ρ *of interest, subject to one of the following constraints:*

a.

$$\sum_{q=1}^{Q}\left\{\mathrm{tr}\left(\left(\mathbf{A}_q\right)^{\mathcal{H}}\mathbf{A}_q\right)+\mathrm{tr}\left(\left(\mathbf{B}_q\right)^{\mathcal{H}}\mathbf{B}_q\right)\right\}=2TN_t, \tag{30}$$

b.

$$\mathrm{tr}\left(\left(\mathbf{A}_q\right)^{\mathcal{H}}\mathbf{A}_q\right)=\mathrm{tr}\left(\left(\mathbf{B}_q\right)^{\mathcal{H}}\mathbf{B}_q\right)=\frac{TN_t}{Q}, q=1,...,Q, \tag{31}$$

c.

$$\left(\mathbf{A}_q\right)^{\mathcal{H}}\mathbf{A}_q=\left(\mathbf{B}_q\right)^{\mathcal{H}}\mathbf{B}_q=\frac{T_t}{Q}\mathbf{I}_{N_t}, q=1,...,Q. \tag{32}$$

Note that the constraints (30), (31), and (32) are convex in the dispersion matrices since they can be rewritten as

a)

$$\sum_{q=1}^{Q}\left\{\mathrm{tr}\left(\left(\mathbf{A}_q\right)^{\mathcal{H}}\mathbf{A}_q\right)+\mathrm{tr}\left(\left(\mathbf{B}_q\right)^{\mathcal{H}}\mathbf{B}_q\right)\right\}\leq 2TN_t, \tag{33}$$

b)

$$\mathrm{tr}\left(\left(\mathbf{A}_q\right)^{\mathcal{H}}\mathbf{A}_q\right)=\mathrm{tr}\left(\left(\mathbf{B}_q\right)^{\mathcal{H}}\mathbf{B}_q\right)\leq\frac{TN_t}{Q}, q=1,...,Q, \tag{34}$$

c)

$$\left(\mathbf{A}_q\right)^{\mathcal{H}}\mathbf{A}_q\leq\frac{T_t}{Q}\mathbf{I}_{N_t}, \left(\mathbf{B}_q\right)^{\mathcal{H}}\mathbf{B}_q\leq\frac{T_t}{Q}\mathbf{I}_{N_t}, q=1,...,Q, \tag{35}$$

all of which are convex. However, the cost function,

$$f=\frac{1}{2T}E\left(\log\det\left(\mathbf{I}_{2N_rT}+\frac{\rho}{N_t}\mathcal{H}\mathcal{H}^{\mathcal{T}}\right)\right) \tag{36}$$

is neither concave nor convex in the variables $\{\mathbf{A}_q,\mathbf{B}_q\}$, meaning that (29) might have local maxima. In [2], the authors show that if not the global maxima, the local maxima is quite close to actual channel capacity. The local maxima is obtained via constrained-gradient-ascent method. Note that

$$\begin{aligned}\mathcal{Z}&=\mathbf{I}_{2N_rT}+\tfrac{\rho}{N_t}\mathcal{H}\mathcal{H}^{\mathcal{T}}\\&=\mathbf{I}_{2N_rT}+\left(\tfrac{\rho}{N_t}\sum_{q=1}^{Q}\left[\overline{\mathcal{A}}_q\overline{\mathbf{h}}\left(\overline{\mathbf{h}}\right)^{\mathcal{T}}\left(\overline{\mathcal{A}}_q\right)^{\mathcal{T}}\right]\right)\\&+\left(\tfrac{\rho}{N_t}\sum_{q=1}^{Q}\left[\overline{\mathcal{B}}_q\overline{\mathbf{h}}\left(\overline{\mathbf{h}}\right)^{\mathcal{T}}\left(\overline{\mathcal{B}}_q\right)^{\mathcal{T}}\right]\right),\end{aligned} \tag{37}$$

where $\overline{\mathcal{A}}_q = \begin{bmatrix} \mathcal{A}_q & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathcal{A}_q \end{bmatrix}$, $\overline{\mathcal{B}}_q = \begin{bmatrix} \mathcal{B}_q & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathcal{B}_q \end{bmatrix}$, and $\overline{\mathbf{h}} = \begin{bmatrix} \underline{\mathbf{h}}_1 \\ \vdots \\ \underline{\mathbf{h}}_{N_r} \end{bmatrix}$.

The gradient of the cost function,

$$f = \frac{1}{2T} E\left(\log\left(\det\left(\mathcal{Z}\right)\right)\right), \tag{38}$$

with respect to the spreading matrices $\mathrm{Re}\left(\mathbf{A}_q\right)$, $\mathrm{Im}\left(\mathbf{A}_q\right)$, $\mathrm{Re}\left(\mathbf{B}_q\right)$, $\mathrm{Im}\left(\mathbf{B}_q\right)$ is calculated using

a)

$$\begin{aligned}&\left[\frac{\partial f}{\partial \mathrm{Re}(\mathbf{A}_q)}\right]_{a,b} \\ &= \frac{2\rho}{TN_t}\sum_{n=1}^{N_r}\left(\left[\mathbf{P}_q\right]_{a+(2n-2)T,b+(2n-2)N_t} + \left[\mathbf{P}_q\right]_{a+(2n-1)T,b+(2n-1)N_t}\right)\end{aligned}, \tag{39}$$

b)

$$\begin{aligned}&\left[\frac{\partial f}{\partial \mathrm{Im}(\mathbf{A}_q)}\right]_{a,b} \\ &= \frac{2\rho}{TN_t}\sum_{n=1}^{N_r}\left(\left[\mathbf{P}_q\right]_{a+(2n-1)T,b+(2n-2)N_t} - \left[\mathbf{P}_q\right]_{a+(2n-2)T,b+(2n-1)N_t}\right)\end{aligned}, \tag{40}$$

c)

$$\begin{aligned}&\left[\frac{\partial f}{\partial \mathrm{Re}(\mathbf{B}_q)}\right]_{a,b} \\ &= \frac{2\rho}{TN_t}\sum_{n=1}^{N_r}\left(\left[\mathbf{R}_q\right]_{a+(2n-1)T,b+(2n-2)N_t} - \left[\mathbf{R}_q\right]_{a+(2n-2)T,b+(2n-1)N_t}\right)\end{aligned}, \tag{41}$$

d)

$$\begin{aligned}&\left[\frac{\partial f}{\partial \mathrm{Im}(\mathbf{B}_q)}\right]_{a,b} \\ &= \frac{2\rho}{TN_t}\sum_{n=1}^{N_r}\left(\left[\mathbf{R}_q\right]_{a+(2n-2)T,b+(2n-2)N_t} + \left[\mathbf{R}_q\right]_{a+(2n-1)T,b+(2n-1)N_t}\right)\end{aligned}, \tag{42}$$

where

$$\mathbf{P}_q = E\left(\mathcal{Z}^{-1}\overline{\mathbf{h}}\left(\overline{\mathbf{h}}\right)^{\mathcal{T}}\overline{\mathcal{A}}_q\right), \tag{43}$$

and

$$\mathbf{R}_q = E\left(\mathcal{Z}^{-1}\overline{\mathbf{h}}\left(\overline{\mathbf{h}}\right)^{\mathcal{T}}\overline{\mathcal{B}}_q\right). \tag{44}$$

1.4. Numerical Designs under Both Capacity and Diversity Criteria

1.4.1. System Formulation by Heath and Paulraj

In [10], the space time channel model is expressed as

$$\mathbf{Y} = \sqrt{E_s}\sum_{q=1}^{Q}\mathbf{H}\mathbf{M}_q s_q + \mathbf{V}, \tag{45}$$

where $\mathbf{M}_q = \frac{1}{\sqrt{N_t}}\left[\mathbf{A}_q\right]^{\mathcal{T}} = \frac{1}{\sqrt{N_t}}\left[\mathbf{B}_q\right]^{\mathcal{T}}$ is of size $N_t \times T$, $\mathbf{H}$ is a MIMO channel matrix of size $N_r \times N_t$, $s_n, n = 0, ..., N-1$ are data source symbols, $\mathbf{V}$ is a $N_r \times T$ matrix whose columns represent realizations of an independently and identically-distributed (i.i.d.) circular complex additive white Gaussian noise (AWGN) process with distribution $\mathcal{CN}(\mathbf{0}, N_0\mathbf{I}_{N_r})$.

The model (45) can also be written as [10]

$$\mathbf{y} = \sqrt{E_s}\mathcal{H}\mathcal{X}\mathbf{s} + \mathbf{v}, \tag{46}$$

where $\mathcal{X} = [vec(\mathbf{M}_1), ..., vec(\mathbf{M}_Q)]$, $\mathcal{H} = \mathbf{I}_T \otimes \mathbf{H}$, $\mathbf{y} = vec(\mathbf{Y})$, $\mathbf{v} = vec(\mathbf{V})$, $\mathbf{s} = [s_1, ..., s_Q]^{\mathcal{T}}$, and $\mathbf{R_s} = \mathbb{E}_\mathbf{s}\left(\mathbf{s}\mathbf{s}^{\mathcal{H}}\right) = \mathbf{I}_N$.

1.4.2. Optimization under Both Capacity and Diversity Criteria

Capacity based criteria The dispersion matrices should satisfy the power constraints

$$\mathrm{tr}\left\{\sum_{q=1}^{Q}\mathbf{M}_q\left[\mathbf{M}_q\right]^{\mathcal{H}}\right\} = T, \tag{47}$$

and more practically, each dispersion matrix is assume to contain the same power [10], i.e.

$$\mathrm{tr}\left\{\mathbf{M}_q\left[\mathbf{M}_q\right]^{\mathcal{H}}\right\} = \frac{T}{N}. \tag{48}$$

The ergodic capacity of the AWGN system with Rayleigh fading for capacity-optimum complex LDCs is given by [10]

$$C_c = \max_{\mathrm{Tr}(\mathcal{X}\mathcal{X}^{\mathcal{H}})\le N_r T}\frac{1}{T}E_\mathbf{H}\left[\log\det\left(\mathbf{I}_{N_t T} + \frac{E_s}{N_o}\mathcal{H}\mathcal{X}\mathcal{X}^{\mathcal{H}}\mathcal{H}^{\mathcal{H}}\right)\right] \tag{49}$$

We have the following design criterion based on optimal capacity.

Theorem 1 *[10] Let $Q = N_t T$. Any $\mathcal{X}$ which satisfies $\mathcal{X}\mathcal{X}^{\mathcal{H}} = \frac{1}{N_T}\mathbf{I}_{N_T T}$ is a capacity-optimal LDC.*

In the case of $Q < N_t T$, capacity may not necessarily be optimal , and the criterion in Theorem 1 may be rewritten as [10]

$$\mathcal{X}^{\mathcal{H}}\mathcal{X} = \frac{T}{Q}\mathbf{I}_Q. \tag{50}$$

In the case of $Q < N_t T$, the capacity can be bounded by

$$I_{c|\mathcal{X}} = \frac{1}{T} E_{\mathbf{H}} \left[\log\det \left(\mathbf{I}_{N_r T} + \frac{E_s}{N_o} \mathcal{H}\mathcal{X}\mathcal{X}^{\mathcal{H}}\mathcal{H}^{\mathcal{H}} \right) \right], \tag{51}$$

where $I_{c|\mathcal{X}} \le C_c$.

Theorem 2 *[10] The mutual information achieved by using any frame-based $\mathcal{X}$ is bounded by*

$$\begin{array}{l} \frac{1}{T} E \sum\limits_{q=1}^{Q} \left[\log\left(1 + \frac{E_s}{N_t N_o} \lambda_{k+N_t T-Q}^2\right)\right] \le \\ I_{c|\mathcal{X}} \le \frac{1}{T} E \sum\limits_{q=1}^{Q} \left[\log\left(1 + \frac{E_s}{N_t N_o} \lambda_k^2\right)\right] \end{array}, \tag{52}$$

where λ_k is the k-th singular value of $\mathcal{H}$, and the expectation is taken over the distribution of the singular values.

diversity based criteria LDCs proposed in [2] and discussed in Section 1.4. only optimize the ergodic capacity, and thus does not necessarily lead to good error performance. In [10], both capacity and error probability criteria have been considered. In a vector AWGN channel, maximum-likelihood (ML) decoding rule is written as

$$\widehat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\mathcal{S}} \left\| \mathbf{y} - \sqrt{E_s}\mathcal{H}\mathcal{X}\mathbf{s} \right\|_2^2, \tag{53}$$

where $\mathcal{S}$ is the set of all possible vector symbols.

For high SNR, the Chernoff upper bound on the average probability that matrix codeword $\mathbf{S}$ is mis-decoded as $\widehat{\mathbf{S}}$ is

$$P\left(\mathbf{S} \to \widehat{\mathbf{S}}\right) < \frac{1}{\left(\frac{E_s}{4N_o}\right)^{rank(\mathbf{R}_s)N_r} \prod\limits_{n=1}^{rank(\mathbf{R}_s)} (\lambda_n)^{N_r}}, \tag{54}$$

where $\mathbf{R}_s = \left(\mathbf{S} - \widehat{\mathbf{S}}\right)\left(\mathbf{S} - \widehat{\mathbf{S}}\right)^{\mathcal{H}}$, and $\{\lambda_n\}$ is the eigenvalue of $\mathbf{R}_s$.

Denote the transmitted sequence $\{s_q\}$ corresponding to $\mathbf{S}$ and the erroneous received sequence $\{r_q\}$ corresponding to $\widehat{\mathbf{S}}$. The matrix $\mathbf{R}_s$ is derived as

$$\begin{aligned} \mathbf{R}_s &= \left(\sum_{q=1}^{Q} \mathbf{M}_q (s_q - r_q)\right)^* \left(\sum_{q=1}^{Q} \mathbf{M}_q (s_q - r_q)\right)^{\mathcal{T}} \\ &= \sum_{q_1=1}^{Q} \sum_{q_2=1}^{Q} [\mathbf{M}_{q_1}]^* \mathbf{M}_{q_2} (e_{q_1})^* e_{q_2}, \end{aligned} \tag{55}$$

where $e_q = s_q - r_q$.

Theorem 3 *[10] The diversity order of a linear code is less than or equal to* $N_r \min\{N_t, T\}$.

Design Criterion 2 *High SNR Near-Optimal LDCs: For a code, choose* $\{\mathbf{M}_q\}_{q=1}^{Q}$ *to satisfy the tight frame relationship in (50). Within this class of codes, search for the design that maximizes the minimum rank and product of nonzero singular values of matrix* $\mathbf{R}_s$.

For $T \leqslant N_t$, under the conditions (30), (31), and (32), the design constraints for Design Criterion 2 are

a)

$$\text{tr}\left\{\sum_{q=1}^{Q}\mathbf{M}_q\left[\mathbf{M}_q\right]^{\mathcal{H}}\right\} = T, \tag{56}$$

b)

$$\text{tr}\left\{\mathbf{M}_q\left[\mathbf{M}_q\right]^{\mathcal{H}}\right\} = \frac{T}{N}, q = 1, ..., Q, , \tag{57}$$

c)

$$\left[\mathbf{M}_q\right]^{\mathcal{H}}\mathbf{M}_q = \frac{1}{N}I_T, q = 1, ..., Q. \tag{58}$$

Numerical code design Under capacity criterion, frame based code designs aim to find matrices that satisfies (50), and two relevant methods are provided in [10]:

a) Projection: For an $N_tT \times Q$ matrix $\mathbf{A}$, frame $\mathcal{X}$ is constructed using

$$\mathcal{X} = \sqrt{\frac{T}{Q}}\mathbf{A}\left(\mathbf{A}^{\mathcal{H}}\mathbf{A}\right)^{-1/2}. \tag{59}$$

Note that, if $\mathbf{A} = \mathbf{QR}$ is the QR-decomposition of $\mathbf{A}$, $\mathbf{Q}$ follows $\mathbf{Q} = \mathbf{A}\left(\mathbf{A}^{\mathcal{H}}\mathbf{A}\right)^{-1/2}$.

b) Householder transformation:

Denote

$$\mathbf{v}^{(m)} = \left[0, ..., 0, 1, v_1^{(m)}, ..., v_{N_tT-m}^{(m)}\right] \tag{60}$$

for $m = 1, ..., N_tT$, the corresponding Householder reflection is

$$\mathbf{V}_m = \mathbf{I}_{N_tT} - \frac{2\mathbf{v}^{(m)}\left(\mathbf{v}^{(m)}\right)^{\mathcal{H}}}{\left\|\mathbf{v}^{(m)}\right\|^2}. \tag{61}$$

Denote $\mathbf{U} = \underline{\mathbf{D}}\mathbf{V}_1\mathbf{V}_2...\mathbf{V}_{N_tT}$, where $\underline{\mathbf{D}}$ is a diagonal matrix of arbitrary complex exponentials, and let $\mathbf{Z} = \begin{bmatrix}\mathbf{I}_Q \\ \mathbf{0}_{(N_tT-Q)\times Q}\end{bmatrix}$, the frame $\mathcal{X}$ is constructed using $\mathcal{X} = \sqrt{\frac{T}{Q}}\mathbf{UZ}$.

According to one of the above frame construction procedures, dispersion matrices are optimized using the frame-constraint with respect to the rank and determinant criteria:

a) Global Optimization: Let $\widehat{\mathbf{D}}$ and $\left\{\widehat{\mathbf{v}}^{(m)}\right\}_{m=1}^{Q}$ be an initial set of Householder parameters with the structure in (60). Let $K \leqslant \min\left\{N_t, T\right\}$ be the minimum required rank of the LDC. Find $\left\{\mathbf{v}^{(m)}\right\}_{m=1}^{Q}$ such that

$$J\left(\mathcal{X}\right) = \min_{\mathbf{e}\in\mathcal{E}} \prod_{i=0}^{K} \lambda_i(\mathbf{R}_s(\mathbf{e})) \tag{62}$$

is maximized subject to the constraint

$$rank\left(\mathbf{R}_s(\mathbf{e})\right) \geqslant K \tag{63}$$

for $\forall \mathbf{e} \in \mathcal{E}$, where

$$\mathcal{E} = \left\{ \mathbf{s}_k - \mathbf{s}_l \left| \begin{array}{l} \mathbf{s}_k \in \mathcal{S}, \mathbf{s}_l \in \mathcal{S}, \\ k \neq l \end{array} \right. \right\}.$$

The convergence of this optimization procedure is not guaranteed since

a. the cost function is a nonlinear function of the Householder parameters;
b. it is non-convex due to maximizing the minimum taken over a discrete space.

b) Basis Selection: Let $\mathcal{I}$ be the set of all possible subsets of Q columns of a $N_tT \times N_tT$ unitary matrix. By construction, $|\mathcal{I}| = \begin{pmatrix} N_tT \\ Q \end{pmatrix}$. Denote the matrix formed by the i-th subset in $\mathcal{I}$ as $\mathcal{X}_i$. The basis selection algorithm is chosen such that (62) is maximized subject to the constraint (63).

The advantage basis selection is low complexity. However, it is not useful for $Q = N_tT$, since there is only one possible combination of columns.

c) Random search may improve sampling of the space of frames. Two random search methods are provided in [10]:

Random Search 1 *Generate L realizations of $\mathbf{A}$ from some distribution, for example, the multivariate complex Gaussian distribution. From all the possible $\mathbf{A}$ matrices, calculate $\mathcal{X}$ using (59) such that* (62) *is maximized subject to the constraint (63).*

Random Search 2 *Consider L candidate realizations of a $N_tT \times Q$ random matrix $\mathbf{A}$, and let ε be some stopping value. For each candidate matrix $\mathbf{A}$, calculate $\mathcal{X}$ using (59). Then, extract $\{\mathbf{M}_q\}_{q=1}^{Q}$ from $\mathcal{X}$, and let $\widehat{\mathbf{M}}_q = \mathbf{M}_q\left(\left(\mathbf{M}_q\right)^{\mathcal{H}}\mathbf{M}_q\right)^{-1/2}$ and scale appropriately. Repeat the procedure for $\mathcal{X} = \left[vec(\widehat{\mathbf{M}}_1), ..., vec(\widehat{\mathbf{M}}_Q)\right]$ until $\left\|\mathcal{X}^{\mathcal{H}}\mathcal{X} - \frac{T}{Q}\mathbf{I}_Q\right\| < \varepsilon$. Choose the $\mathcal{X}$ generated from the realizations of $\mathbf{A}$ that maximizes $J\left(\mathcal{X}\right)$ subject to constraint (63).*

Random Search 2 typically converges quickly by enforcing a convex constraint with respect to the dispersion matrices [10].

1.5. Algebraic Designs of LDC

Numerical designs of LDC, as discussed in previous subsections of this chapter, do not have closed algebraic forms. This subsection concentrates on more desirable high-rate algebraic designs of LDC, and presents a systematic survey of algebraic designs of LDC. Prior to the discussion, it is necessary to explain a new term, non-vanishing determinant (NVD), which has recently been introduced as a new method for designing block based complex space-time code (BCSTC) [11–18].

One NVD design should satisfy two requirements

a) full diversity,

b) NVD condition.

The NVD condition is to ensure

$$\Delta_{\min} = \left(\min\det\left(\mathbf{S}_{LDC}\left(\mathbf{S}_{LDC}\right)^{\mathcal{H}}\right)|\forall\mathbf{S}_{LDC}\right) \neq 0, \tag{64}$$

and there exists φ such that $\Delta_{\min} \geqslant \varphi > 0$ for un-normalized data symbols of arbitrary size carved in a finite set. In other words, φ does not depend on the spectral efficiency of the code. Note that (55) is different from product criterion [19] which ensures that

$$\left(\min\det\left(\left(\mathbf{S}_{LDC}^{(a)} - \mathbf{S}_{LDC}^{(b)}\right)\left(\mathbf{S}_{LDC}^{(a)} - \mathbf{S}_{LDC}^{(b)}\right)^{\mathcal{H}}\right)\middle|\forall\mathbf{S}_{LDC}^{(a)}, \mathbf{S}_{LDC}^{(b)}\right) \neq 0 \tag{65}$$

for source data symbol pair $\forall\mathbf{s}^{(a)}, \mathbf{s}^{(b)}, \left(\mathbf{s}^{(a)} - \mathbf{s}^{(b)}\right) \neq \mathbf{0}$. Denote the set containing all LDC codewords as $\mathcal{S}_{LDC}$. The criterion (64) is equivalent to (65) only if

$$\forall\mathbf{S}_{LDC}^{(a)} \in \mathcal{S}_{LDC}, \mathbf{S}_{LDC}^{(b)} \in \mathcal{S}_{LDC} \Rightarrow \left(\mathbf{S}_{LDC}^{(a)} \pm \mathbf{S}_{LDC}^{(b)}\right) \in \mathcal{S}_{LDC}. \tag{66}$$

Thus, the criterion (64) is weaker than the criterion (65), since (66) may not always hold for non-orthogonal LDC. However, (64) may provide a more convenient way to designing good codes using the algebraical approach. Code designs satisfying condition (65) are called infinite code, denote as $\mathcal{C}_\infty$ [16].

NVD designs usually choose QAM or HEX [20] as source symbols [11–18]. The un-normalized QAM or HEX symbols are defined as [18]

$$\mathcal{A}_{QAM} = \left\{a + jb \middle| \begin{array}{l} |a| \leq (M-1), |a| \leq (M-1), \\ a, b\text{odd} \end{array}\right\},$$
$$\mathcal{A}_{HEX} = \left\{a + \omega_3 b \middle| \begin{array}{l} |a| \leq (M-1), |a| \leq (M-1), \\ a, b\text{odd}, \omega_3 = e^{j2\pi/3} \end{array}\right\}.$$

A majority of high-rate algebraic designs of LDC are based on diversity criterion using layered or threaded structure, and the related research undergos three stages:

a) full diversity codes of symbol rate $R_{LDC}^{sym} = 1$, which do not satisfy NVD criteria: Damen et. al. proposed diagonal algebraic space-time (DAST) codes using the combination of rotated constellations and the Hadamard transform [21]. They also constructed a space-time block code of $R_{LDC}^{sym} = 1$ over $N_t = 2$ transmit antennas and $T = 2$ symbol periods using algebraic number theory in [22]. Xin et. al. employed algebraic number theoretic tools to design linear space-time constellation-rotating (ST-CR) block codes [23] or space-time linear constellation precoding [24] of $R_{LDC}^{sym} = 1$.

b) full diversity codes of symbol rate $1 < R_{LDC}^{sym} \leq N_t$, which do not satisfy NVD criteria: Ma and Giannakis designed a layered space-time (ST) scheme equipped with linear complex field (LCF) coding full-diversity full-rate (FDFR) codes of symbol rate of N_t for the data symbols carved from integer ring, which rely on the concatenation of two bandwidth efficient modules: the outer one implements LCF coding per layer and feeds the inner one which performs a circular form of layered ST multiplexing [25]. Gamal and Damen developed a framework based on the threaded layering concept, referred to as threaded algebraic space-time (TAST) coding of symbol rate up to N_t. Within the TAST framework, the authors recognized a special class of codes which use algebraic number-theoretic constellations as component codes for more general data symbols carved from either $\mathbb{Z}\left(i\right)$ or $\mathbb{Z}\left(e^{i2\pi/3}\right)$ [26]. In [27], Sethuraman et. al. presented some general techniques for constructing full-rank, minimal-delay code with symbol rate up to N_t over a variety of signal sets for arbitrary number of transmit antennas using commutative division algebras (field extensions) as well as using noncommutative division algebras of the rational field embedded in matrix rings.

c) full diversity codes of symbol rate $R_{LDC}^{sym} \leqslant N_t$, which satisfy NVD criteria: The research on NVD-related codes in the related literature can be categorized into three stages:

   a. codes of symbol rate 2 for $N_t = 2$ transmit antennas and $T = 2$ symbol periods: the first found NVD code, named Golden Code, is a space-time code for 2 transmit and 2 receive antennas for the coherent MIMO channel, and has been studied independently in [11], [12], and [13].

   b. codes of symbol rate $R_{LDC}^{sym} \leqslant N_t$ for certain number of transmit antennas: Wang et. al. have proposed a systematic and general structure of NVD multi-layer cyclotomic spacetime code design [14]. Several cases of optimality have been analyzed [14]:

      i. Optimal single-layer (diagonal) cyclotomic spacetime codes have been found for a certain number of N_t transmit antennas, where $N_t = \phi\left(3n\right)/\phi\left(3\right)$, where $\phi\left(3n\right)$ is the Euler number of n.

      ii. The optimal full rate cyclotomic spacetime codes for two and three transmit antennas have been obtained. Optimal two-layer cyclotomic spacetime codes have been obtained for three and four transmit antennas.

   In [15], based on cyclic division algebras, Kiran et. al. presented a systematic technique for constructing STBC-schemes with non-vanishing determinant for certain number of transmit antennas of the form $2^k, 3 \cdot 2^k, 2 \cdot 3^k, q^k(q-1)/2$, where q is any prime of the form $4s + 3$.

In parallel to the work mentioned above, Oggier et. al. introduced the notion of perfect spacetime block codes (STBCs), which are NVD codes for 2, 3, 4, and 6 antennas [16].

In [17], Liao and Xia presented a transformation technique to improve the normalized diversity product for a full rate algebraic spacetime block code (STBC) by balancing the signal mean powers at different transmit antennas. It was shown in [17] that the normalized diversity product of the transformed code with the multi-layer structure is better than that of the transformed code with the cyclic division algebra structure. A new full rate algebraic STBC with multi-layer NVD structure with a larger normalized diversity product for three transmit antennas has been presented [17].

c. for arbitrary number of transmit antennas: as an extension of $n \times n$ NVD perfect space-time codes, Elia has constructed NVD perfect codes for all channel dimensions, and extended the notion of a perfect code to the rectangular case [18].

1.6. Precoding Designs for LDC under Correlated MIMO Channels

1.6.1. Introduction

Under correlated space-time MIMO channels, LDC designed for i.i.d. MIMO channels may not work ideally. Due to simple structure, precoding for orthogonal space-time block codes [28, 29] has been investigated extensively and thoroughly [30–35]. Note that there exists a number of super non-orthogonal LDC designs for uncorrelated MIMO channels. However, due to difficulties intrduced by non-orthogonality, only very limited work have been done to exploit feedback resources for non-orthogonal LDC [36–38]. Sayeed et. al. provided capacity and pairwise error probability (PEP) analysis of LDC based on a unitarily equivalent eigen-domain representation of correlated MIMO fading channels [36]. In [36], LD are encoded via a family of structured code generator matrices, whose generator matrices is parameterized by three unitary matrices that determine the space-time structure of the codes and a diagonal power-shaping matrix. Hayes and Caffery also provided a two-stage DCC design procedure to utilize statistical channel knowledge to provide significantly improved capacity and bit error rate performance over LDC codes in the presence of channel correlations [37]. Vu and Paulraj proposed linear precoder designs exploiting channel mean and transmit antenna correlation in (MIMO) wireless system [38].

In the following, the work [38] by Vu and Paulraj is introduced in more details, since analytical precoding solutions for general non-orthogonal LDC are more thoroughly discussed in [38].

1.6.2. System Model

In [38],

a) the non-zero mean space-time MIMO channel model is defined as

$$\mathbf{H} = \mathbf{H}_m + \mathbf{H}_w \mathbf{R}_t^{1/2}, \tag{67}$$

where channel mean is $\mathbf{H}_m = \sqrt{K/(K+1)}\mathbf{H}_0$, transmit correlation matrix is $\mathbf{R}_t = (1/(K+1))\mathbf{R}_0$, $\mathbf{H}_w \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_{N_r \times N_t}\right)$, tr $\left((\mathbf{H}_0)^{\mathcal{H}}\mathbf{H}_0\right) = N_t N_r$, tr $(\mathbf{R}_0) = N_t$.

b) the system equation is

$$\mathbf{Y} = \mathbf{HW}\underline{\mathbf{C}} + \mathbf{V}, \tag{68}$$

where

a. precoding matrix $\mathbf{W}$ follows

$$\text{tr}\left(\mathbf{W}\mathbf{W}^{\mathcal{H}}\right) = 1, \tag{69}$$

b. LDC codeword can be equivalently written as $\underline{\mathbf{C}} = \sqrt{\frac{P}{N_t}}\left(\mathbf{S}_{LDC}\right)^{\mathcal{T}}$, where P is the sum transmit power,

c. additive complex Gaussian noise is $\mathbf{V} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_{N_r}\right)$.

c) Maximum-likelihood (ML) decoding is performed as

$$\widehat{\underline{\mathbf{C}}} = \arg\min_{\underline{\mathbf{C}} \in \underline{\mathcal{C}}} \left\| \mathbf{Y} - \sqrt{P}\mathbf{HW}\underline{\mathbf{C}} \right\|_F^2, \tag{70}$$

where $\mathcal{C}$ is the LDC codebook, and the subscript F denotes the Frobenius norm.

1.6.3. Precoding Design Criteria

The Chernoff bound of pairwise error probability (PEP) over codeword pair $\left\{\underline{\mathbf{C}}, \widehat{\underline{\mathbf{C}}}, \underline{\mathbf{C}} \neq \widehat{\underline{\mathbf{C}}}\right\}$ for the ML decoding (70) is [38]

$$P\left(\underline{\mathbf{C}} \to \widehat{\underline{\mathbf{C}}}\right) < \exp\left(-\frac{\left\|\mathbf{Y} - \mathbf{HW}\left(\underline{\mathbf{C}} - \widehat{\underline{\mathbf{C}}}\right)\right\|_F^2}{4\sigma^2}\right). \tag{71}$$

Denote $\underline{\mathbf{A}} = \frac{1}{P}\left(\underline{\mathbf{C}} - \widehat{\underline{\mathbf{C}}}\right)\left(\underline{\mathbf{C}} - \widehat{\underline{\mathbf{C}}}\right)^{\mathcal{H}}$, the Chernoff bound in (71) can then be written as

$$f(\mathbf{H}, \underline{\mathbf{A}}, \mathbf{W}) = \exp\left(-\frac{\rho}{4}\text{tr}\left(\mathbf{HW}\underline{\mathbf{A}}(\mathbf{W})^{\mathcal{H}}(\mathbf{H})^{\mathcal{H}}\right)\right).,\tag{72}$$

where SNR is $\rho = \frac{P}{\sigma^2}$. The Chernoff bound expression (71) is monotonic in codeword distance $\underline{\mathbf{A}}$. Two optimization criteria based on the codeword distance selection are considered in [38]:

a) minimum pairwise distance: when the minimum-distance codeword pairs is not small with high probability, these pairs will dominate the error performance, consequently, the minimum-distance design

$$\mathbf{F} = \arg\min_{\mathbf{F}}\left\{\max_{\underline{\mathbf{A}}} E_{\mathbf{H}}\left[f(\mathbf{H}, \underline{\mathbf{A}}, \mathbf{W})\right]\right\} \tag{73}$$

will lead to a reasonable overall performance gain. Note that criterion (73) was also given in [30].

b) average distance over all codeword pairs: when the minimum-distance codeword pairs is not small with low probability, average distance over all codeword pairs may become important. An average-distance measure was proposed in [38] as

$$\underline{\overline{\mathbf{A}}} = \frac{1}{PT} E\left[\left(\underline{\mathbf{C}} - \widehat{\underline{\mathbf{C}}}\right)\left(\underline{\mathbf{C}} - \widehat{\underline{\mathbf{C}}}\right)^{\mathcal{H}}\right] = \frac{1}{PT} \sum_{a \neq b} p_{a,b} \Delta_{a,b} \left(\Delta_{a,b}\right)^{\mathcal{H}}, \tag{74}$$

where $\Delta_{a,b} = \underline{\mathbf{C}}^{(a)} - \underline{\mathbf{C}}^{(b)}$, and $p_{a,b}$ is the probability of the pair $\left\{\underline{\mathbf{C}}^{(a)}, \underline{\mathbf{C}}^{(b)}\right\}$ among all pairs of distinct codewords.

The average distance criterion provided in [38] is

$$\mathbf{F} = \arg\min_{\mathbf{F}} \left\{ \max_{\underline{\mathbf{A}}} E_{\mathbf{H}} \left[f\left(\mathbf{H}, E\left[\underline{\mathbf{A}}\right], \mathbf{W}\right)\right] \right\}. \tag{75}$$

Since the average distance may lead to a smaller value of the Chernoff bound (74) compared to the minimum distance, the gain obtained using the average distance criterion may not be guaranteed to be the minimum precoding gain of the system. However, with the average distance approach for nonorthogonal STBCs or LDCs, it is more appropriate to approximate $\underline{\overline{\mathbf{A}}}$ as a scaled-identity matrix [38].

The probability density distribution of the non-zero mean channel is [38]

$$g\left(\mathbf{H}\right) = \frac{\exp\left(-\mathrm{tr}\left(\left(\mathbf{H} - \mathbf{H}_m\right)^{\mathcal{H}} \mathbf{R}_t^{-1} \left(\mathbf{H} - \mathbf{H}_m\right)\right)\right)}{\pi^{N_t N_r} \det\left(\mathbf{R}_t\right)^{N_r}}. \tag{76}$$

The average PEP of (72) over channel statistics is bounded by [38]

$$\overline{P}_e \leqslant \frac{\exp\left(-\mathrm{tr}\left(\mathbf{H}_m \left(\underline{\mathbf{W}}\right)^{-1} \left(\mathbf{H}_m\right)^{\mathcal{H}}\right)\right)}{\det\left(\mathbf{R}_t\right)^{N_r}} \exp\left(-\mathrm{tr}\left(\mathbf{H}_m \mathbf{R}_t^{-1} \left(\mathbf{H}_m\right)^{\mathcal{H}}\right)\right), \tag{77}$$

where

$$\underline{\mathbf{W}} = -\frac{\rho}{4} \mathrm{tr}\left(\mathbf{R}_t \mathbf{W} \underline{\mathbf{A}} \left(\mathbf{W}\right)^{\mathcal{H}} \mathbf{R}_t\right) + \mathbf{R}_t. \tag{78}$$

1.6.4. Problem Formulation and Analysis

Since minimizing the bound in (77) is equivalent to minimizing the logarithm of this bound, after ignoring the constant terms, the convex objective function in the matrix variable $\underline{\mathbf{W}}$ is obtained as [38]

$$J = \mathrm{tr}\left(\mathbf{H}_m \left(\underline{\mathbf{W}}\right)^{-1} \left(\mathbf{H}_m\right)^{\mathcal{H}}\right) - N_r \log\det\left(\underline{\mathbf{W}}\right). \tag{79}$$

Combining this objective function with the precoding power constraint (69), an optimization problem for designing can be formulated as [38]

$$\min_{\mathbf{F}} J = \mathrm{tr}\left(\mathbf{H}_m \left(\underline{\mathbf{W}}\right)^{-1} \left(\mathbf{H}_m\right)^{\mathcal{H}}\right) - N_r \log\det\left(\underline{\mathbf{W}}\right) \tag{80}$$

a) $\underline{\mathbf{W}} = -\frac{\rho}{4}\text{tr}\left(\mathbf{R}_t\mathbf{W}\underline{\mathbf{A}}\left(\mathbf{W}\right)^{\mathcal{H}}\mathbf{R}_t\right) + \mathbf{R}_t$

b) $\text{tr}\left(\mathbf{W}\mathbf{W}^{\mathcal{H}}\right) = 1$

This problem is not convex in $\mathbf{W}$ due to the nonlinear power constraint. To tackle this problem, we can add one more power constraint [38]

$$\text{tr}\left(\mathbf{W}\underline{\mathbf{A}}\mathbf{W}^{\mathcal{H}}\right) = \gamma, \tag{81}$$

where γ is a positive constant, and then apply different relaxations to obtain the precoder analytically.

Note that $\text{tr}\left(\mathbf{A}\mathbf{B}\right) \leqslant \sum_i \lambda_i\left(\mathbf{A}\right)\lambda_i\left(\mathbf{B}\right)$ [39]. Therefore

$$\gamma = \text{tr}\left(\mathbf{W}\underline{\mathbf{A}}\mathbf{W}^{\mathcal{H}}\right) = \text{tr}\left(\mathbf{W}^{\mathcal{H}}\mathbf{W}\underline{\mathbf{A}}\right) \leqslant \sum_i \lambda_i\left(\mathbf{W}^{\mathcal{H}}\mathbf{W}\right)\lambda_i\left(\underline{\mathbf{A}}\right), \tag{82}$$

where the equality occurs when the eigenvectors of $\mathbf{W}^{\mathcal{H}}\mathbf{W}$ are the same as those of $\underline{\mathbf{A}}$.

We perform

a) the singular value decomposition of $\mathbf{W}$ as $\mathbf{W} = \underline{\mathbf{U}}\underline{\mathbf{D}}\left(\underline{\mathbf{V}}\right)^{\mathcal{H}}$,

b) eigenvalue decomposition of $\underline{\mathbf{A}}$ as $\underline{\mathbf{A}} = \underline{\mathbf{U}_{\mathbf{A}}}\underline{\mathbf{\Lambda}_{\mathbf{A}}}\left(\underline{\mathbf{U}_{\mathbf{A}}}\right)^{\mathcal{H}}$.

This equality condition can be achieved if

$$\underline{\mathbf{V}} = \underline{\mathbf{V}_{\mathbf{A}}}. \tag{83}$$

The equality condition in (82) is fulfilled if $\lambda_i\left(\mathbf{W}\underline{\mathbf{A}}\mathbf{W}^{\mathcal{H}}\right) = \lambda_i\left(\mathbf{W}^{\mathcal{H}}\mathbf{W}\right)\lambda_i\left(\underline{\mathbf{A}}\right)$ [38]. In this case, the problem is then equivalent to the following problem [38]

$$\min_{\mathbf{B}} J = \text{tr}\left(\mathbf{H}_m\left(\underline{\mathbf{W}}\right)^{-1}\left(\mathbf{H}_m\right)^{\mathcal{H}}\right) - N_r \log\det\left(\mathbf{W}\right) \tag{84}$$

subject to

a)

$$\underline{\mathbf{W}} = -\frac{\rho}{4}\text{tr}\left(\mathbf{R}_t\mathbf{B}\mathbf{R}_t\right) + \mathbf{R}_t, \tag{85}$$

b)

$$\sum_i \xi_i\lambda_i\left(\mathbf{B}\right) = 1, \tag{86}$$

c)

$$\mathbf{B} \succeq 0, \tag{87}$$

where $\mathbf{B} = \mathbf{W}\underline{\mathbf{A}}\mathbf{W}^{\mathcal{H}}$, and $\xi_i = \left(\lambda_i\left(\underline{\mathbf{A}}\right)\right)^{-1}$ are the inverses of the nonzero eigenvalues of $\underline{\mathbf{A}}$.

However, the new formulation (85) is not convex in $\mathbf{B}$ due to the nonlinear equality constraint involving the eigenvalues of $\mathbf{B}$. The authors in [38] relaxed this constraint to obtain an analytical precoder solution which may be non-optimal. Two different relaxation methods presented in [38] are summarized as follows:

a) Minimum eigenvalue relaxation method: the problem is relaxed using $\sum_i \xi_i \lambda_i\left(\mathbf{B}\right) \leqslant \sum_i \xi_{\max} \text{tr}\left(\mathbf{B}\right)$, where $\xi_{\max} = \max\left\{\xi_i\right\}$. This approach is equivalent to approximating $\underline{\mathbf{A}}$ as an identity matrix, scaled by the minimum nonzero eigenvalue of $\underline{\mathbf{A}}$. This approximation effectively produces a smaller $\underline{\mathbf{W}}$ (in the positive semidefinite sense), hence loosening the upper bound on the PEP in (12). The problem formulation becomes the same form as that of orthogonal STBC, i.e.,

$$\min_{\mathbf{F}} J = \text{tr}\left(\mathbf{H}_m\left(\underline{\mathbf{W}}\right)^{-1}\left(\mathbf{H}_m\right)^{\mathcal{H}}\right) - N_r \log\det\left(\underline{\mathbf{W}}\right) \tag{88}$$

subject to

a.

$$\underline{\mathbf{W}} = -\frac{\mu_0 \rho}{4}\text{tr}\left(\mathbf{R}_t \mathbf{W}\left(\mathbf{W}\right)^{\mathcal{H}}\mathbf{R}_t\right) + \mathbf{R}_t, \tag{89}$$

b.

$$\text{tr}\left(\mathbf{W}\mathbf{W}^{\mathcal{H}}\right) = 1, \tag{90}$$

where the value for μ_0 is

a. for the minimum-distance design, $\mu_0 = \min_{\mathbf{B}}\left\{\lambda_{\min}\left(\Delta^{(a,b)}\left(\Delta^{(a,b)}\right)^{\mathcal{H}}\right)\right\}$,

b. for the average-distance design, $\mu_0 = \lambda_{\min}\left(\underline{\mathbf{A}}\right) \neq 0$.

The problem can be further formulated in terms of $\mathbf{W}$:

$$\min_{\underline{\mathbf{W}}} J = \text{tr}\left(\mathbf{H}_m\left(\underline{\mathbf{W}}\right)^{-1}\left(\mathbf{H}_m\right)^{\mathcal{H}}\right) - N_r \log\det\left(\underline{\mathbf{W}}\right) \tag{91}$$

subject to

a.

$$\underline{\text{tr}}\left(\left(\left(\mathbf{R}_t\right)^{-1}\underline{\mathbf{W}}\left(\mathbf{R}_t\right)^{-1} - \left(\mathbf{R}_t\right)^{-1}\right)\right) = \eta_0, \tag{92}$$

b.

$$\left(\mathbf{R}_t\right)^{-1}\underline{\mathbf{W}}\left(\mathbf{R}_t\right)^{-1} - \left(\mathbf{R}_t\right)^{-1} \geqslant 0, \tag{93}$$

where $\eta_0 = \frac{\mu_0 \rho}{4}$.

This relaxation method is suitable for $\underline{\mathbf{A}}$ with reasonably small condition number.

b) Trace relaxation method: this relaxation method is to substitute linear constraint $\text{tr}\left(\left(\underline{\mathbf{\Lambda}_{\mathbf{A}}}\right)^{-1}\mathbf{B}\right) = 1$ for (86) in order to ensure the problem to be convex. However, $\sum_i \text{tr}\left(\mathbf{AB}\right) \geqslant \sum_i \lambda_{N-i+1}\left(\mathbf{A}\right)\lambda_i\left(\mathbf{B}\right)$, and thus $\text{tr}\left(\mathbf{W}\mathbf{W}^{\mathcal{H}}\right) \leqslant 1$, which may result in a precoder with the total transmit power less than the original constraint (69). This can be resolved using a scaling factor to reach the power constraint. The optimization problem for $\underline{\mathbf{W}}$ becomes

$$\min_{\underline{\mathbf{W}}} J = \text{tr}\left(\mathbf{H}_m\left(\underline{\mathbf{W}}\right)^{-1}\left(\mathbf{H}_m\right)^{\mathcal{H}}\right) - N_r \log\det\left(\underline{\mathbf{W}}\right) \tag{94}$$

subject to

a.

$$\underline{\text{tr}}\left(\left(\underline{\mathbf{\Lambda}_{\mathbf{A}}}\right)^{-1}\left(\left(\mathbf{R}_t\right)^{-1}\underline{\mathbf{W}}\left(\mathbf{R}_t\right)^{-1} - \left(\mathbf{R}_t\right)^{-1}\right)\right) = \frac{\rho}{4}, \tag{95}$$

b.

$$\left(\mathbf{R}_t\right)^{-1}\underline{\mathbf{W}}\left(\mathbf{R}_t\right)^{-1} - \left(\mathbf{R}_t\right)^{-1} \geqslant 0. \tag{96}$$

1.6.5. Remarks for Precoder Solutions

As in [38], the problem formulations {(91),(92),(93)} and {(94),(95),(96)} for minimum eigenvalue relaxation and trace relaxation methods can be solved analytically. Generally speaking, the precoder solutions should match the properties of both LDC and channel through matching singular vectors. We have the following remarks:

a) The precoder beam directions (the left singular vectors) depend only on the transmit channel side information (CSIT) [38].

b) The input shaping matrix (the right singular vectors) depends only on the precoder input signal - the LDC structure [38].

c) The power allocation, which can be allocated using dynamic water filling, depends on both LDC and channel [38].

1.7. Distributed LDC for Cooperative Communications

1.7.1. Introduction

Recently, relay based cooperative wireless communications have been attracting significant attention. One of promising cooperative techniques are relay based space-time coding. In [40], spatial diversity using relay based space-time code design are analyzed using capacity based outage bound. Jing and Hassibi in [41] used a two-stage relay based protocol, where in one stage the transmitter sends information and in the other, the relays encode their received signals into a distributed LDC, and then transmit the coded signals to the receive node. The diversity properties of distributed LDC were analyzed in [41]. In the following, distributed LDC approaches in [41] will be described.

1.7.2. System Model

The wireless network considered in [41] has $R+2$ nodes, including one transmit node, one receive node, and R relay nodes. Every node has a single antenna, which can be used for both transmission and reception in half-duplex mode. The transmitter sends a signal vector $\mathbf{s}$, where $\mathbf{s} = [s_1, ..., s_T]^{\mathcal{T}}$, of size $T \times 1$ over a period of T symbols. The receive vector $\mathbf{r}_i$ at i-th relay is

$$\mathbf{r}_i = \sqrt{P_1 T} f_i \mathbf{s} + \mathbf{v}_i, \tag{97}$$

where f_i is the channel from the transmitter to the i-th relay, $\mathbf{v}_i$ is the noise vector at the i-th relay, P_1 is transmit power of the transmit node. The receive node receives signal vectors from all R relay nodes over another period of T symbols, and the receive vector $\mathbf{r}_i$ at the receive node is [41]

$$\mathbf{x} = \sum_{i=1}^{R} g_i \mathbf{t}_i + \mathbf{w}, \tag{98}$$

where g_i is the channel from the i-th relay to the receive node, $\mathbf{w}$ is the noise vector at the receive node. We assume that f_i, g_i, $\mathbf{v}_i$, and $\mathbf{w}$ are independent complex Gaussian random variables with zero-mean and unit-variance. The receive signal vector $\mathbf{r}_i$ at i-th relay is encoded by

$$\mathbf{t}_i = \sqrt{\frac{P_2}{P_1+1}} \underline{\mathbf{A}}_i \mathbf{r}_i, \tag{99}$$

where P_2 is transmit power of each relay node. The receive vector $\mathbf{x}$ is written as [41]

$$\begin{aligned} \mathbf{x} &= \sum_{i=1}^{R} g_i \mathbf{t}_i + \mathbf{w}_i \\ &= \sqrt{\frac{P_1 P_2 T}{P_1+1}} \sum_{i=1}^{R} \left(\underline{\mathbf{A}}_i g_i f_i \mathbf{s}\right) + \sqrt{\frac{P_2}{P_1+1}} \sum_{i=1}^{R} \left(g_i \underline{\mathbf{A}}_i \mathbf{v}_i\right) + \mathbf{w}_i \\ &= \sqrt{\frac{P_1 P_2 T}{P_1+1}} \underline{\mathbf{S}} \widetilde{\mathbf{h}} + \widetilde{\mathbf{w}}, \end{aligned} \tag{100}$$

where $\underline{\mathbf{S}} = [\underline{\mathbf{A}}_1 \mathbf{s}, ..., \underline{\mathbf{A}}_R \mathbf{s}]$, $\widetilde{\mathbf{h}} = \begin{bmatrix} g_1 f_1 \\ \vdots \\ g_R f_R \end{bmatrix}$, and $\widetilde{\mathbf{w}} = \sqrt{\frac{P_2}{P_1+1}} \sum_{i=1}^{R} \left(g_i \underline{\mathbf{A}}_i \mathbf{v}_i\right) + \mathbf{w}$.

In [41], 2-D space-time dispersion matrices are not provided. $\{\underline{\mathbf{A}}_i, i = 1, ..., R\}$ in (99) and (100) are not space-time 2-D dispersion matrices. Rather, they are encoding matrices for 1-D dispersion codes over a T symbol period for the i-th relay. However, the corresponding 2-D space-time dispersion matrices can be constructed using

$$[\mathbf{A}_q]_{t,i} = [\mathbf{B}_q]_{t,i} = [\underline{\mathbf{A}}_i]_{t,q}, \tag{101}$$

where $q = 1, ..., T$, $t = 1, ..., T$, and $i = 1, ..., R$.

In [41], $\{\underline{\mathbf{A}}_i, i = 1, ..., R\}$ are assumed to be unitary matrices, thus $\widetilde{\mathbf{w}}$ is both spatially and temporally white since $\mathrm{var}\left(\widetilde{\mathbf{w}}\right) = \left(1 + \frac{P_2}{P_1+1} \sum_{i=1}^{R} \left(|g_i|^2\right)\right) \mathbf{I}_T$.

1.7.3. PEP, Power Allocation, and Diversity

The maximum-likelihood (ML) decoding rule of the system is [41]

$$\arg\min_{\mathbf{s}_k} P\left(\mathbf{x}|\mathbf{s}_k\right) = \arg\min_{\mathbf{s}_k} \left\| \mathbf{Y} - \sqrt{\frac{P_1 P_2 T}{P_1+1}} \underline{\mathbf{S}}_k \widetilde{\mathbf{h}} \right\|_F^2 . \tag{102}$$

Theorem 4 *With the ML decoding in (102), the PEP, averaged over the channel coefficients, of mistaking* $\mathbf{s}_k$ *by* $\mathbf{s}_l$ *has the following Chernoff bound [41]*

$$P\left(\mathbf{s}_k \to \mathbf{s}_l\right) \leqslant E_{\{g_i, f_i\}} \exp\left(-\frac{P_1 P_2 T \left(\widetilde{\mathbf{h}}\right)^{\mathcal{H}} \left(\underline{\mathbf{S}}_k - \underline{\mathbf{S}}_l\right)^{\mathcal{H}} \left(\underline{\mathbf{S}}_k - \underline{\mathbf{S}}_l\right) \widetilde{\mathbf{h}}}{4\left(1 + P_1 + P_2 \sum_{i=1}^{R} \left(|g_i|^2\right)\right)} \right). \tag{103}$$

Integrating over f_i*, the bound becomes [41]*

$$P\left(\mathbf{s}_k \to \mathbf{s}_l\right) \leqslant E_{\{g_i\}} \left[\det\left(\mathbf{I}_R + \frac{P_1 P_2 T \mathbf{M}\mathbf{G}}{4\left(1 + P_1 + P_2 \sum_{i=1}^{R} \left(|g_i|^2\right)\right)} \right) \right]^{-1}, \tag{104}$$

where $\mathbf{M} = \left(\underline{\mathbf{S}}_k - \underline{\mathbf{S}}_l\right)^{\mathcal{H}} \left(\underline{\mathbf{S}}_k - \underline{\mathbf{S}}_l\right)$ *and* $\mathbf{G} = diag\left\{|g_1|^2, ..., |g_R|^2\right\}$.

Since $g = \sum_{i=1}^{R} \left(|g_i|^2\right)$ has gamma distribution, (104) can be approximated as [41]

$$P\left(\mathbf{s}_k \to \mathbf{s}_l\right) \leqslant E_{\{g_i\}} \left[\det\left(\mathbf{I}_R + \frac{P_1 P_2 T \mathbf{M}\mathbf{G}}{4\left(1 + P_1 + P_2 R\right)} \right) \right]^{-1}. \tag{105}$$

Thus, the Chernoff bound is approaches the minimum value if

$$P_1 = R P_2 = \frac{P}{2}. \tag{106}$$

Denote the minimum nonzero singular value of M as $\sigma_{\min}^2$. After applying the power allocation (106), (104) is further approximated as [41]

$$P\left(\mathbf{s}_k \to \mathbf{s}_l\right) \leqslant \left(\frac{16R}{T\sigma_{\min}^2}\right)^{rank(\mathbf{M})} P^{-rank(\mathbf{M})\left(1 - \frac{\log\log P}{\log P}\right)}. \tag{107}$$

In the case of full rank $\mathbf{M}$, the correspoding diversity gain is $\min\{T, R\}\left(1 - \frac{\log\log P}{\log P}\right)$.

1.7.4. General Distributed LDC

In the previous introduction, $\{\mathbf{A}_q = \mathbf{B}_q, q = 1, ..., T\}$ have been considered. Now, we address a more general case in which $\mathbf{A}_q \neq \mathbf{B}_q$. The receive signal vector $\mathbf{r}_i$ at the i-th relay is encoded by [41]

$$\mathbf{t}_i = \sqrt{\frac{P_2}{P_1+1}} \left(\underline{\mathbf{A}}_i \mathbf{r}_i + \underline{\mathbf{B}}_i \left(\mathbf{r}_i\right)^*\right), i = 1, ..., R, \tag{108}$$

where $\underline{\mathbf{A}}_i$, $\underline{\mathbf{B}}_i$ are $T \times T$ matrices.

Denote

$$\underline{\mathcal{D}}_i = \begin{bmatrix} \mathrm{Re}\left(\underline{\mathbf{A}}_i\right) + \mathrm{Re}\left(\underline{\mathbf{B}}_i\right) & -\mathrm{Im}\left(\underline{\mathbf{A}}_i\right) + \mathrm{Im}\left(\underline{\mathbf{B}}_i\right) \\ \mathrm{Im}\left(\underline{\mathbf{A}}_i\right) + \mathrm{Im}\left(\underline{\mathbf{B}}_i\right) & \mathrm{Re}\left(\underline{\mathbf{A}}_i\right) - \mathrm{Re}\left(\underline{\mathbf{B}}_i\right) \end{bmatrix},$$

$$\underline{\mathcal{G}}_i = \begin{bmatrix} \mathrm{Re}\left(g_i\right)\mathbf{I}_T & -\mathrm{Im}\left(g_i\right)\mathbf{I}_T \\ \mathrm{Im}\left(g_i\right)\mathbf{I}_T & \mathrm{Re}\left(g_i\right)\mathbf{I}_T \end{bmatrix},$$

$$\underline{\mathcal{F}}_i = \begin{bmatrix} \mathrm{Re}\left(f_i\right)\mathbf{I}_T & -\mathrm{Im}\left(f_i\right)\mathbf{I}_T \\ \mathrm{Im}\left(f_i\right)\mathbf{I}_T & \mathrm{Re}\left(f_i\right)\mathbf{I}_T \end{bmatrix},$$

and

$$\underline{\mathcal{R}}_k = \underline{\mathcal{G}}_k \underline{\mathcal{D}}_k \begin{bmatrix} \mathrm{Re}\left(\mathbf{s}_k - \mathbf{s}_l\right) & -\mathrm{Im}\left(\mathbf{s}_k - \mathbf{s}_l\right) \\ \mathrm{Im}\left(\mathbf{s}_k - \mathbf{s}_l\right) & \mathrm{Re}\left(\mathbf{s}_k - \mathbf{s}_l\right) \end{bmatrix}.$$

The system equation can now be written in real-valued form as [41]

$$\underline{\mathbf{x}} = \sqrt{\frac{P_1 P_2 T}{P_1+1}} \mathcal{H} \theta_{vec} + \mathcal{W}, \tag{109}$$

where $\mathcal{H} = \sum_{i=1}^{R} \left[\underline{\mathcal{G}}_i \underline{\mathcal{D}}_i \underline{\mathcal{F}}_i\right]$, $\mathcal{W} = \begin{bmatrix} \mathrm{Re}\left(\mathbf{w}\right) \\ \mathrm{Im}\left(\mathbf{w}\right) \end{bmatrix} + \sqrt{\frac{P_2}{P_1+1}} \sum_{i=1}^{R} \left[\underline{\mathcal{G}}_i \underline{\mathcal{D}}_i \begin{bmatrix} \mathrm{Re}\left(\mathbf{v}_i\right) \\ \mathrm{Im}\left(\mathbf{v}_i\right) \end{bmatrix}\right]$, $\theta_{vec} = \begin{bmatrix} \mathrm{Re}\left(\mathbf{s}\right) \\ \mathrm{Im}\left(\mathbf{s}\right) \end{bmatrix}$, and $\underline{\mathbf{x}} = \begin{bmatrix} \mathrm{Re}\left(\mathbf{x}\right) \\ \mathrm{Im}\left(\mathbf{x}\right) \end{bmatrix}$.

Theorem 5 *[41] Design the transmit signal at the i-th relay as in (108). The ML decoding is*

$$\arg\min_{\mathbf{s}_i} P\left(\mathbf{x}|\mathbf{s}_i\right) = \arg\min_{\mathbf{s}_i} \left\| \underline{\mathbf{x}} - \sqrt{\frac{P_1 P_2 T}{P_1+1}} \mathcal{H} \theta_{vec,i} \right\|_F^2.$$

Use the optimum power allocation given in (106). If $P \gg 1$, integrating over f_i, the PEP of mistaking $\mathbf{s}_k$ by $\mathbf{s}_l$ can be upper bounded by

$$P\left(\mathbf{s}_k \to \mathbf{s}_l\right) \leq E_{\{g_i\}} \left[\det\left(\mathbf{I}_{2R} + \frac{PT \sum_{i=1}^{R} \left(\underline{\mathcal{R}}_k \left(\underline{\mathcal{R}}_k\right)^{\mathcal{T}}\right)}{8\left(R + \sum_{i=1}^{R} \left(\left|g_i\right|^2\right)\right)}\right)\right]^{-1/2}. \tag{110}$$

1.8. LDC Designs for Single Stream Communications in Frequency Selective Channels

1.8.1. Introduction

LDC designs for space-time block fading channels have been discussed in the previous parts of this chapter. Here, we introduce LDC designs in frequency selective channels. In high data rate broadband communications, signals often experience frequency selective fading, introducing inter-symbol interference (ISI). There are two class of block based communications technologies (both using guard intervals) to tackle this problem

a) orthogonal frequency division multiplexing (OFDM) [42]: there are two types of OFDM, cyclic-prefix (CP) OFDM and zero-padded (ZP) OFDM. By serial-to-parallel (S/P) conversion, CP-OFDM, accepted as a key technique in multiple industrial standards for high-data-rate communications [43–47], transforms a single wideband multipath channel into multiple parallel narrowband flat fading channels, enabling simple equalization [48]. ZP-OFDM guarantees symbol recovery and assures FIR (even zero-forcing (ZF)) equalization of FIR channels regardless of the channel zero locations [48].

b) single-carrier block communications (SCBC): there two types of SCBC considered, cyclic-prefix single-carrier modulation (CP-SCM) and zero-padded single-carrier modulation (ZP-SCM). CP-SCM, accepted as an option in IEEE 802.16 standard [46], utilizes frequency-domain equalization (FDE) with similar lower complexity to CP-OFDM, due to its use of the computationally-efficient fast Fourier transform (FFT). Time-domain equalization may be applied in ZP-SCM systems [49].

Note that uncoded OFDM cannot provide the same order of diversity as uncoded single-carrier systems in severe frequency-selective fading environments, since the frequency responses of channel space branches differ from one another. One technique to mitigate this problem is the combination of interleaving and forward error correction across all subchannels at the price of reduced bandwidth efficiency, i.e., coded OFDM (COFDM) [50–55]. As an alternative to error control coding, linear precoding has been proposed to be combined with OFDM to exploit frequency diversity [56, 57]. To further improve performance, linear constellation precoding [24] was recently proposed to work in conjunction with OFDM, known as LCP-OFDM, to maximize not only frequency diversity gain but also coding gain [58]. However, LCP-OFDM is not able to exploit time diversity over different OFDM blocks in the channels.

Recently, LDC-OFDM including LDC-CP-OFDM and LDC-ZP-OFDM have been proposed to achieve full time and frequency diversity with high spectral efficiency [3, 59]. LDC were also proposed to be applied to SCBC as LDC-SCM, including LDC-CP-SCM and LDC-ZP-SCM, to exploit high diversity available in the channels [60, 61]. Note that both LDC-OFDM and LDC-SCM are designed for single antenna transmission systems.

1.8.2. Wideband Channel Model

Assume the communications channel experiences frequency-selective fading, and the channel for the k-th OFDM/SCM block is modeled as an Lth-order FIR filter with impulse response $\mathbf{h}^{(k)} = \left[h_0^{(k)}, ..., h_L^{(k)}\right]^{\mathcal{T}}$. Channel coefficients are constant within one OFDM/SCM block but vary statistically independently across different OFDM/SCM blocks. Each OFDM/SCM block is of size $P = N_C + N_g$, including a data symbol block of size N_C and a guard interval of size $N_g \geq L$ to avoid inter-block interference. Denote $H_p^{(k)}$ as the p-th frequency domain subcarrier channel gain during the k-th OFDM/SCM block, and $H_p^{(k)} = \sum_{l=o}^{L} h_l^{(k)} e^{-j(2\pi/N_c)l(p-1)}$, or $H_p^{(k)} = [\mathbf{w}_p]^{\mathcal{T}} \mathbf{h}^{(k)}$, where $\mathbf{w}_p = \left[1, \omega^{p-1}, \omega^{2(p-1)}, \cdots, \omega^{L(p-1)}\right]^{\mathcal{T}}$ and $\omega = e^{-j(2\pi/N_c)}$. The additive noise is circularly symmetric, zero-mean, complex Gaussian with variance N_0. Assume additive noise is statistically independent for different k, and ρ is the normalized signal to noise ratio (SNR).

a) OFDM case: denote $s_{OFDM(p)}^{(k)}, p = 1..., N_C$ as the LDC-encoded symbol transmitted on the p-th subcarrier during the k-th OFDM block. The receiver experiences additive complex Gaussian noise. Before transmission, a guard interval (e.g., cyclic prefix (CP)) is added to each OFDM block. After FFT processing, the received symbol is

$$x_p^{(k)} = \sqrt{\rho} H_p^{(k)} s_{OFDM(p)}^{(k)} + v_p^{(k)}, p = 1, ..., N_c. \tag{111}$$

The CP-OFDM system may also be written in block matrix form as

$$\mathbf{x}^{(k)} = \sqrt{\rho} \mathbf{D}_{\mathbf{H}}^{(k)} \mathbf{s}_{OFDM}^{(k)} + \mathbf{v}^{(k)}, \tag{112}$$

where $\mathbf{x}^{(k)}$ and $\mathbf{v}^{(k)}$ are the frequency domain received signal and noise vectors, respectively, $\mathbf{D}_{\mathbf{H}}^{(k)} = \mathbf{F}_{N_C} \mathbf{H}^{(k)} [\mathbf{F}_{N_C}]^{\mathcal{H}} = diag(H_1^{(k)}, ..., H_{N_C}^{(k)})$, where $\left[\mathbf{H}^{(k)}\right]_{m,n} = h_{((m-n) \bmod N_C}^{(k)}$.

When zero-padding (ZP) is used as the OFDM guard interval, orthogonality is destroyed, and the system model does not have a simple form as shown in (111). However, the ZP-OFDM system model can be expressed in block matrix form in the time domain as

$$\mathbf{x}_{ZP\_OFDM}^{(k)} = \sqrt{\rho} \mathbf{H}_0^{(k)} [\mathbf{F}_{N_C}]^{\mathcal{H}} \mathbf{s}_{OFDM}^{(k)} + \mathbf{v}_{ZP\_OFDM}^{(k)}, \tag{113}$$

with the k-th received ZP-OFDM block $\mathbf{x}_{ZP\_OFDM}^{(k)} \in C^{P\times 1}$, and the frequency selective channel matrix $\mathbf{H}_0^{(k)} \in C^{P\times N_C}$ corresponding to the k-th OFDM block. The Toeplitz channel matrix $\mathbf{H}_0^{(k)}$ is guaranteed to be invertible, regardless of the channel zero locations [48]. Zero-mean white additive complex Gaussian noise vector is represented by $\mathbf{v}_{ZP\_OFDM}^{(k)}$.

b) SCM case: denote $\mathbf{x}_{SC}^{(k)}$ as the channel data symbol vector transmitted during the k-th SCM block of size $N_C \times 1$, and $\mathbf{x}_{SC}^{(k)} = \left[x_{SC(1)}^{(k)}, ..., x_{SC(N_C)}^{(k)}\right]^{\mathcal{T}}$, where $x_{SC(p)}^{(k)}, p = 1, ..., N_C$ is the p-th data symbol of the k-th SCM block in sequence. Before transmission, a cyclic prefix (CP) guard interval is appended to each CP-SCM block. The CP is then removed at the receiver. The effective channel of the k-th SCM block is a circulant matrix $\mathbf{H}^{(k)}_{CP\_SC}$ with elements $\left[\mathbf{H}^{(k)}_{CP\_SC}\right]_{a,b} = h^{(k)}_{((a-b) \bmod N_C)}$. The CP-SCM block system can be modeled as

$$\mathbf{r}^{(k)}_{CP\_SC} = \sqrt{\rho}\mathbf{H}^{(k)}_{CP\_SC}\mathbf{x}^{(k)}_{SC} + \mathbf{v}^{(k)}_{CP\_SC}, \tag{114}$$

where $\mathbf{r}^{(k)}_{CP\_SC}$ is the received block after CP removal, and $\mathbf{v}^{(k)}_{CP\_SC}$ is the corresponding noise vector.

At the receiver, the received block $\mathbf{r}^{(k)}_{CP\_SC}$ is first processed by an FFT to generate block $\mathbf{y}^{(k)}_{CP\_SC} = \mathbf{F}_{N_C}\mathbf{r}^{(k)}_{CP\_SC}$. Due to its circulant property, $\mathbf{H}^{(k)}_{CP\_SC}$ can be decomposed as

$$\mathbf{H}^{(k)}_{CP\_SC} = \left[\mathbf{F}_{N_C}\right]^{\mathcal{H}}\mathbf{D}^{(k)}_{CP\_SC}\mathbf{F}_{N_C},$$

where $\mathbf{D}^{(k)}_{CP\_SC}$ is diagonal with

$$\left[\mathbf{D}^{(k)}_{CP\_SC}\right]_{pp} = \sum_{l=0}^{L} h^{(k)}_l \exp\left(-j2\pi l(p-1)/N_C\right).$$

The frequency domain system equation can be expressed as

$$\mathbf{y}^{(k)}_{CP\_SC} = \sqrt{\rho}\mathbf{D}^{(k)}_{CP\_SC}\mathbf{F}_{N_C}\mathbf{x}^{(k)}_{SC} + \mathbf{F}_{N_C}\mathbf{v}^{(k)}_{CP\_SC}. \tag{115}$$

the ZP-SCM system model does not have a simple frequency domain format shown in (115). However, the ZP-SCM system model can be written in block matrix form in the time domain as,

$$\mathbf{r}^{(k)}_{ZP\_SC} = \sqrt{\rho}\mathbf{H}^{(k)}_{ZP\_SC}\mathbf{x}^{(k)}_{SC} + \mathbf{v}^{(k)}_{ZP\_SC}, \tag{116}$$

where $\mathbf{H}^{(k)}_{ZP\_SC}$ represents a Toeplitz convolution matrix with $\left[\mathbf{H}^{(k)}_{ZP\_SC}\right]_{a,b} = h^{(k)}_{(a-b)}$, where $\mathbf{r}^{(k)}_{ZP\_SC}$ is the received block of size $P \times 1$, and is the corresponding noise vector of size $\mathbf{v}^{(k)}_{ZP\_SC}$.

1.8.3. Coded Block Construction

a) One LDC-OFDM block consists of T adjacent OFDM blocks. An LDC-OFDM block includes D LDC codewords, each with LDC matrices occupying $N_{F(i)}$ subcarriers and T OFDM blocks $\in C^{T\times N_{F(i)}}, i = 1, ..., D$, with $\sum_{i=1}^{D} N_{F(i)} = N_C$. In OFDM systems, since the number of subcarriers is typically much larger than the number of antennas in space-time MIMO systems, the LDC-OFDM system has freedom to choose larger dispersion matrices and exploits low correlation across OFDM subcarriers.

One LDC-OFDM block is organized into the matrix $\mathbf{S}_{LDC-OFDM}$ of size $N_C \times T$, $\mathbf{S}_{LDC-OFDM} = \left[\mathbf{s}^{(1)}_{OFDM}, ..., \mathbf{s}^{(T)}_{OFDM}\right]$, where $\mathbf{s}^{(k)}_{OFDM}$ is the k-th OFDM block symbol vector of size $1 \times N_C$, and represents the transmitted complex symbol vector before inverse Fourier transformation (IFFT) in the transmitter for the k^{th} OFDM transmitted block. Elements $\mathbf{s}_{OFDM}^{(k)}$ consist of all the D row vectors $\mathbf{S}_{LDC(k,.)}^{(i)}, i = 1, ..., D$, where $\mathbf{S}_{LDC(k,.)}^{(i)} \in C^{1 \times N_{F(i)}}$ is the k-th row of the i-th LDC matrix codeword $\mathbf{S}_{LDC}^{(i)}$ in a single LDC-OFDM block. While $S_{LDC(k,.)}^{(i)}$ occupies $N_{F(i)}$ subcarriers, it is not necessary that these subcarriers be spectrally adjacent.

b) One LDC-CP-SCM block consists of T adjacent SCM blocks. In addition, one LDC-SCM block includes D LDC codewords, each of size $T \times N_{F_(i)}, i = 1, ..., D$, where $N_{F_(i)}$ is the number of channel symbols within one SCM block, which the i-th LDC codeword is across. Thus, the maximal size of one LDC-SCM block is $T \times N_C$.

The difference in allocation between a LDC-OFDM block and a LDC-CP-SCM block is that the LDC-OFDM block is located in frequency domain, while the LDC-CP-SCM block is located in time domain.

1.8.4. Two Step Estimation

Although LDC decoding was proposed using ML decoding or sphere decoding in earlier literature [2, 10], low complexity linear LDC decoding has been proposed and recommended in [3].

Note that it is desirable to maintain the existing receiver structure using modular components when introducing new system concepts, which may save investment cost in research and development. To this end, a two-step estimation (TSE) procedure is proposed for LDC-OFDM, permitting channel coefficients to change per OFDM block instead of per T OFDM blocks. This enables LDC decoding to be independent of the specific equalizers used, and in turn, enables wide applicability for enhancing different standards. One possible zero-forcing method to estimate the data symbol vector in (11) is via the Moore-Penrose pseudo-inverse of LDC encoding matrix $\mathbf{G}_{LDC}$, which is calculated and stored offline.

To remove dependence of LDC decoding on symbol estimation, LDC designs need to meet the following criterion:

Correlation criterion: denote the correlation matrix of $vec\left([\mathbf{S}_{LDC}]^{\mathcal{T}}\right)$ as $\mathbf{R}_{vec\left([\mathbf{S}_{LDC}]^{\mathcal{T}}\right)}$. In the case that LDC-encoded symbols per channel use or per row of $\mathbf{S}_{LDC}$ are block-wise estimated, $\mathbf{S}_{LDC}$ needs to be row-wise uncorrelated. In other words, $\mathbf{R}_{vec\left([\mathbf{S}_{LDC}]^{\mathcal{T}}\right)}$ needs to have the block diagonal form

$$\mathbf{R}_{vec\left([\mathbf{S}_{LDC}]^{\mathcal{T}}\right)} = \begin{bmatrix} \mathbf{R}_{\mathbf{S}_{LDC(1,.)}} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{R}_{\mathbf{S}_{LDC(T,.)}} \end{bmatrix} \tag{117}$$

where $\mathbf{R}_{\mathbf{S}_{LDC(k,.)}} \in C^{M \times M}, k = 1..., T$ is the correlation matrix of the k-th row vector of $\mathbf{S}_{LDC}$, and $\mathbf{0}$s are $M \times M$ zero matrices. In the case that LD-coded symbols are estimated per element of $\mathbf{S}_{LDC}$, $\mathbf{S}_{LDC}$ needs to be element-wise uncorrelated. In other words, $\mathbf{R}_{vec\left([\mathbf{S}_{LDC}]^{\mathcal{T}}\right)}$ needs to be diagonal, and more restrictive constraints are applied. The two steps are:

a) *Signal estimation per channel use:*

Signals in each of T channel uses are estimated. No immediate signal detection is performed. (In different channel uses, channel matrices may be different);

b) *Data symbol estimation and detection per LDC block:*

The data symbols corresponding to one LDC codeword are estimated. (In this step, channel knowledge is not required). Bit detection is then performed.

Unlike other two-stage estimation methods, such as Kalman filter, the same core matrix-vector TSE processing may operate on different signal dimensions with symbol blocks of different sizes. The per-data-symbol complexity of encoding and decoding is constant and proportional to the LDC data symbol coding rate.

Similar to LDC-OFDM, LDC-SCM may advantageously utilize two-step-estimation (TSE) procedure in receivers.

1.8.5. Diversity Properties

It is more tractable to analyze the diversity properties of CP-based LDC systems rather than ZP-based LDC systems, since CP-based systems satisfy frequency domain orthogonality. In the following, the diversity for LDC-CP-OFDM is first introduced, and then the approach of diversity analysis is extended to LDC-CP-SCM.

Without loss of generality, we consider a single time-frequency (TF) block, i.e., a single $T \times N_{F(i)}$ block $\mathbf{C}^{(i)}, i = 1, ..., D$ within a LDC-OFDM block [59]. The block $\mathbf{C}^{(i)}$ is created after encoding the i-th LDC codeword within a LDC-OFDM codeword. Denote subcarrier indices chosen for TF block $\mathbf{C}^{(i)}, i = 1, ..., D$ as $\{p^{(k)}_{n_{F(i)}}, n_{F(i)} = 1, ..., N_{F(i)}, i = 1, ..., D, k = 1, ..., T\}$, and the block components

$$\mathbf{C}^{(i)} = \begin{bmatrix} c^{(1)}_{p_{1_{(i)}}} & c^{(1)}_{p_{2_{(i)}}} & \cdots & c^{(1)}_{p_{N_{F(i)}}} \\ c^{(2)}_{p_{1_{(i)}}} & c^{(2)}_{p_{2_{(i)}}} & \cdots & c^{(2)}_{p_{N_{F(i)}}} \\ \vdots & \vdots & \ddots & \vdots \\ c^{(T)}_{p_{1_{(i)}}} & c^{(T)}_{p_{2_{(i)}}} & \cdots & c^{(T)}_{p_{N_{F(i)}}} \end{bmatrix}.$$

The transmission of a general LDC codeword $\mathbf{C}^{(i)}$ is expressed as

$$\mathbf{r}^{(i)} = \sqrt{\rho}\mathbf{M}^{(i)}\mathbf{H}^{(i)} + \mathbf{v}^{(i)}, \tag{118}$$

where received signal vector $\mathbf{r}^{(i)}$ and noise vector $\mathbf{v}^{(i)}$ are of size $N_{F(i)}T \times 1$, the i-th LDC symbol matrix [59]

$$\mathbf{M}^{(i)} = diag(c^{(1)}_{p_{1_{(i)}}}, ..., c^{(1)}_{p_{N_{F(i)}}}, ..., c^{(T)}_{p_{1_{(i)}}}, ..., c^{(T)}_{p_{N_{F(i)}}}), \tag{119}$$

is of size $N_{F(i)}T \times N_{F(i)}T$, $c^{(k)}_{p_{n_{F(i)}}}$ is the channel symbol of the k-th OFDM block, $p_{n_{F(i)}}$-th subcarrier, and i-th LDC codeword, $n_{F(i)} = 1, ..., N_{F(i)}$, and $i = 1, ..., D$. The channel matrix

$$\mathbf{H}^{(i)} = \left[\begin{array}{l} H_{p_{1(i)}}^{(1)}, H_{p_{2(i)}}^{(1)}, ..., H_{p_{N_{F(i)}}}^{(1)}, \\ ..., H_{p_{1(i)}}^{(T)}, H_{p_{2(i)}}^{(T)}, ..., H_{p_{N_{F(i)}}}^{(T)} \end{array} \right]^{\mathcal{T}} \tag{120}$$

is of size $N_{F(i)}T \times 1$, and each element

$$H_{p_{n_{F(i)}}}^{(k)} = \left[\mathbf{w}_{p_{n_{F(i)}}} \right]^{\mathcal{T}} \mathbf{h}^{(k)} \tag{121}$$

is the path gain of the k-th OFDM block and $p_{n_{F(i)}}$-th subcarrier for block $\mathbf{C}^{(i)}$.

Considering a pair of matrices $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ corresponding to two different time-frequency (TF) blocks $C^{(i)}$ and $\tilde{C}^{(i)}$, the upper bound pairwise error probability (PEP) [62] between $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ is [59]

$$P\left(\mathbf{M}^{(i)} \rightarrow \tilde{\mathbf{M}}^{(i)}\right) \leq \left(\begin{array}{c} 2r-1 \\ r \end{array} \right) \left(\prod_{a=1}^{r} \gamma_a \right)^{-1} (\rho)^{-r} \ , \tag{122}$$

where r is the rank of

$$\mathbf{\Lambda}^{(i)} \triangleq \left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right) \mathbf{R}_{\mathbf{H}^{(i)}} \left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right)^{\mathcal{H}},$$

and $\mathbf{R}_{H^{(i)}} = E\left\{\mathbf{H}^{(i)} \left[\mathbf{H}^{(i)}\right]^{\mathcal{H}}\right\}$ is the correlation matrix of vector $\mathbf{H}^{(i)}$, $\gamma_a, a = 1, ..., r$ are the non-zero eigenvalues of $\mathbf{\Lambda}^{(i)}$.

The corresponding rank and product criteria are

a) Rank criterion: the minimum rank of $\mathbf{\Lambda}^{(i)}$ over all pairs of different matrices $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ should be as large as possible.

b) Product criterion: the minimum value of the product $\prod_{a=1}^{r} \gamma_a$ over all pairs of different $\mathbf{M}^{(i)}$ and $\tilde{\mathbf{M}}^{(i)}$ should be maximized.

It has been proved that the rank of $\Lambda^{(i)}$ satisfies [59]

$$rank\left(\mathbf{\Lambda}^{(i)}\right) \leq \min\left\{rank\left(\mathbf{M}^{(i)} - \tilde{\mathbf{M}}^{(i)}\right), rank\left(\mathbf{R}_{\mathbf{H}^{(i)}}\right)\right\} \ , \tag{123}$$

where

$$\mathbf{R}_{\mathbf{H}^{(i)}} == \left[\mathbf{I}_T \otimes \mathbf{W}^{(i)}\right] \Phi \left[\mathbf{I}_T \otimes \left[\mathbf{W}^{(i)}\right]^H\right], \tag{124}$$

where

a)

$$\mathbf{W}^{(i)} = \left[\mathbf{w}_{p_{1(i)}}, \cdots, \mathbf{w}_{p_{N_{F(i)}}}\right]^{\mathcal{T}}, \tag{125}$$

b)

$$\mathbf{h} = \left[\left[\mathbf{h}^{(1)} \right]^{\mathcal{T}}, \cdots, \left[\mathbf{h}^{(T)} \right]^{\mathcal{T}} \right], \tag{126}$$

c)

$$\mathbf{H}^{(i)} = \left(\mathbf{I}_T \otimes \mathbf{W}^{(i)} \right) \mathbf{h}, \tag{127}$$

d)

$$\mathbf{\Phi} = E \left\{ \mathbf{h} \left[\mathbf{h} \right]^{\mathcal{H}} \right\}. \tag{128}$$

Clearly, the maximum of rank of $\mathbf{\Phi}$ is $T(L+1)$. To maximize the rank of $\mathbf{R}_{H^{(i)}}$ [59], it is necessary to maximize the rank of matrix $\mathbf{W}^{(i)}$ of size $N_{F(i)} \times (L+1)$ [59]. This may be achieved by selecting a special subcarrier set to ensure full rank $\mathbf{W}^{(i)}$, where $N_{F(i)} \geq L+1$. For full diversity, the channels need to be full rank jointly in frequency and time domains. For a description on how to choose subcarrier sets to achieve full rank $\mathbf{W}^{(i)}$, refer to [58], [63], and [59]. The full diversity conditions for LDC-CP-OFDM are summarized as follows.

Theorem 6 *a) If the correlation matrix $\mathbf{R}_{\mathbf{H}^{(i)}}$ of channel vector $\mathbf{H}^{(i)}$ has full rank $T(L+1)$, the necessary condition that the frequency-time (FT) block $\mathbf{C}^{(i)}$ of LDC-OFDM achieves full joint frequency-time diversity order, i.e. $rank(\mathbf{\Lambda}_{(i)}) = T(L{+}1)$, is that the frequency dimension size of the FT block $\mathbf{C}^{(i)}$ satisfies $N_{F(i)} \geq L+1$.*

b) The sufficient condition that the frequency-time (FT) block $\mathbf{C}^{(i)}$ of LDC-OFDM achieves available joint frequency-time diversity order, $rank(\mathbf{R}_{\mathbf{H}^{(i)}})$, is that any two elements $c^{(k)}_{p_{n_{F(i)}}}$ and $\widetilde{c^{(k)}_{p_{n_{F(i)}}}}$, of any two different blocks, $\mathbf{C}^{(i)}$ and $\widetilde{\mathbf{C}^{(i)}}$ are different. Mathematically, the sufficient condition is

$$c^{(k)}_{p_{n_{F(i)}}} - \widetilde{c^{(k)}_{p_{n_{F(i)}}}} \neq 0, \tag{129}$$

where $n_{F(i)} = 1, ..., N_{F(i)}, k = 1, ..., T$;

c) If both $N_{F(i)} = L+1$ and $rank(\mathbf{R}_{\mathbf{H}^{(i)}}) = T(L+1)$ are satisfied, the condition (129) is the sufficient and necessary condition that the frequency-time (FT) block $\mathbf{C}^{(i)}$ of LDC-OFDM achieves joint full frequency-time diversity order, $rank(\mathbf{\Lambda}_{(i)}) = T(L+1)$;

d) The related product criterion of design is that the minimum of products

$$\prod_{k=1}^{\mathcal{T}} \prod_{a=1}^{N_{F(i)}} \left| c^{(k)}_{p_{a(i)}} - \widetilde{c^{(k)}_{p_{a(i)}}} \right|^2$$

over distinct FT block $\mathbf{C}^{(i)}$ and $\widetilde{\mathbf{C}^{(i)}}$ must be maximized.

Now it is time to discuss the diversity properties of LDC-CP-SCM. Unlike LDC-CP-OFDM, it is necessary to choose the frequency domain size N_C for LDC-CP-SCM to conduct diversity analysis instead of $N_{F(i)}$, since the whole time domain based LDC-CP-SCM block is transformed into frequency domain only if all subcarriers are considered. Denoting $\mathbf{z}_{CP-SC}^{(k)} = \mathbf{F}_{Nc}\mathbf{x}_{SC}^{(k)}, k = 1, ..., T$, the whole LDC-CP-SCM block with FFT outer processing in each SCM block can expressed as

$$\mathbf{C} = \begin{bmatrix} c_1^{(1)} & c_2^{(1)} & \cdots & c_{N_C}^{(1)} \\ c_2^{(2)} & c_2^{(2)} & \cdots & c_{N_C}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{(T)} & c_2^{(T)} & \cdots & c_{N_C}^{(T)} \end{bmatrix},$$

where $c_p^{(k)} = \left[\mathbf{z}_{CP-SC}^{(k)}\right]_{p,1}, p = 1, ..., N_C, k = 1, ...T.$

After this frequency domain transformation, the rest of analysis of LDC-CP-SCM follows the similar strategy in diversity analysis of LDC-CP-OFDM. However, the results are different, and a sufficient condition for LDC-CP-SCM to achieve full available joint frequency and time diversity in the channels, is summarized in the following theorem.

Theorem 7 *a) The necessary and sufficient condition to ensure* $rank\left(\mathbf{M} - \tilde{\mathbf{M}}\right) = N_C T$ *is*

$$\left[\mathbf{F}_{Nc}\left(\mathbf{x}_{SC}^{(k)} - \widetilde{\mathbf{x}}_{SC}^{(k)}\right)\right]_{p,1} \neq 0, k = 1, ..., T, p = 1, ..., N_C$$

b) In a LDC-CP-SCM system, the rank of $\left(\mathbf{M} - \tilde{\mathbf{M}}\right)$ *satisfies*

$$rank\left(\mathbf{M} - \tilde{\mathbf{M}}\right) = N_C T.$$

a. The LDC-CP-SCM system achieves full available diversity order in the frequency selective channels, i.e. $rank\left(\mathbf{\Lambda}\right) = rank\left(\mathbf{R_H}\right)$

b. The corresponding product design criterion for LDC-CP-SCM block is that the minimum of the product

$$\Delta = \prod_{k=1}^{T}\prod_{p=1}^{N_C}\left|\left[\mathbf{F}_{Nc}\mathbf{x}_{SC}^{(k)}\right]_{p,1} - \left[\mathbf{F}_{Nc}\widetilde{\mathbf{x}}_{SC}^{(k)}\right]_{p,1}\right|^2 \tag{130}$$

taken over all pairs of distinct frequency domain symbol matrices $\mathbf{M}$ *and* $\tilde{\mathbf{M}}$ *must be maximized.*

c) Assume that the frequency selective channel order L *is constant over time. A condition for LDC-SCM to achieve the available full joint frequency and time diversity order* $r_d = rank(\mathbf{R_H})$ *is that there always exist (L+1) indices,* $1 \leqslant p^{(k)} = p_1^{(k)}, ..., p_{L+1}^{(k)} \leqslant N_C$*, for each* $k = 1, ..., T$ *such that*

$$\left[\mathbf{F}_{Nc}\left(\mathbf{x}_{SC}^{(k)} - \widetilde{\mathbf{x}}_{SC}^{(k)}\right)\right]_{p^{(k)},1} \neq 0.$$

Note that this condition is a sufficient and necessary condition for frequency diversity and a sufficient condition for time diversity.

Note that single-carrier systems inherently achieve some level of frequency diversity. However, full frequency diversity cannot be guaranteed in conventional uncoded single-carrier communications systems, especially in uncoded CP-SCM systems, and the frequency coding gain may be further improved through careful signal design [64, 65].

A LDC-SCM block is across multiple SCM blocks in block time varying channels, and the LDC-SCM system has potential to achieve joint frequency-time diversity order up to $T(L+1)$. Although the design strategy of LDC-SCM systems to support a certain order of frequency diversity is different from that of LDC-OFDM, from Theorem 6 and 7, we can come up with the following Corollary on the relation between full joint frequency and time diversity LDC-CP-SCM and LDC-CP-OFDM.

Corollary 1 *Assume that a LDC-CP-OFDM block $\mathbf{C}_{LDC\_OFDM}$ with N_C subcarriers and T OFDM blocks achieves full joint frequency and time diversity order. Before IFFT, the k-th OFDM block within the LDC-CP-OFDM block $\mathbf{C}_{LDC\_OFDM}$ is expressed as $\mathbf{x}_{OFDM}^{(k)}$, where $k = 1, ..., T$ and $\mathbf{x}_{OFDM}^{(k)} = \left[x_{OFDM(1)}^{(k)}, ..., x_{OFDM(N_C)}^{(k)}\right]^{\mathcal{T}}$. Then the k-th SCM block $\mathbf{C}_{LDC\_SCM}$ within a LDC-CP-SCM can be designed as*

$$\mathbf{x}_{SC}^{(k)} = [\mathbf{F}_{Nc}]^{\mathcal{H}} \mathbf{x}_{OFDM}^{(k)}, \tag{131}$$

where $k = 1, ..., T$ and $\mathbf{x}_{SC}^{(k)} = \left[x_{SC}^{(k)}, ..., x_{SC}^{(k)}\right]^{\mathcal{T}}$. The resulting LDC-CP-SCM achieves full joint frequency and time diversity order in the time varying frequency selective channel.

Corollary actually 1 provides a method to construct full joint frequency and time diversity LDC-CP-SCM. However, since the IFFT is involved, this LDC-CP-SCM construction is the same as LDC-CP-OFDM with IFFT processing, one might be concerned with the related problems, such as high PAPR.

1.9. Other Topics in LDC Designs and Applications

This subsection briefly introduces several other topics in LDC designs and applications.

1.9.1. Trace Based Design Criteria

Recall that in (4), one LDC codeword can be written as

$$\mathbf{S}_{LDC} = \sum_{q=1}^{2Q} \tau_q \mathbf{K}_q.$$

Let $\underline{\mathbf{C}}_k = \tau_k \mathbf{K}_k$, one LDC codeword can be written as

$$\mathbf{S}_{LDC} = \sum_{k=1}^{2Q} \underline{\mathbf{C}}_k. \tag{132}$$

Its Hermitian square has the form

$$(\mathbf{S}_{LDC})^{\mathcal{H}} \mathbf{S}_{LDC} = \sum_{k=1}^{K} \underline{\mathbf{C}}_k^{\mathcal{H}} \underline{\mathbf{C}}_k + \sum_{i<k} \left(\underline{\mathbf{C}}_i^{\mathcal{H}} \underline{\mathbf{C}}_k + \underline{\mathbf{C}}_k^{\mathcal{H}} \underline{\mathbf{C}}_i \right). \tag{133}$$

Denote the difference of a pair of LDC codewords as $\underline{\mathbf{D}}^{(a,b)} = \mathbf{S}_{LDC}^{(a)} - \mathbf{S}_{LDC}^{(b)} = \sum_{k=1}^{2Q} \underline{\mathbf{D}}_k$, where $\underline{\mathbf{D}}_k = \left(\tau_k^{(a)} - \tau_k^{(b)}\right) \mathbf{K}_k$.

Then, the following relation holds [66, 67]

$$\left[\mathbf{D}^{(a,b)}\right]^{\mathcal{H}} \mathbf{D}^{(a,b)} = \mathcal{D} + \mathcal{N}, \tag{134}$$

where

$$\mathcal{D} = \sum_{k=1}^{K} \left(\underline{\mathbf{D}}_k^{\mathcal{H}} \underline{\mathbf{D}}_k\right)$$

and

$$\mathcal{N} = \sum_{i<k} \left(\underline{\mathbf{D}}_i^{\mathcal{H}} \underline{\mathbf{D}}_k + \underline{\mathbf{D}}_k^{\mathcal{H}} \underline{\mathbf{D}}_i\right).$$

The concept of maximal symbol-wise diversity (MSD) is introduced in [66, 67] in a non-orthogonal case: the individual code matrices $\underline{\mathbf{C}}_k$ should be scaled unitary matrices with $\left[\underline{\mathbf{C}}_k\right]^{\mathcal{H}} \underline{\mathbf{C}}_k = |\tau_k|^2 \mathbf{I}$. For a maximal symbolwise diversity code, the distance matrix is $\left[\underline{\mathbf{D}}^{(a,b)}\right]^{\mathcal{H}} \underline{\mathbf{D}}^{(a,b)} = \sum_{k=1}^{K} \left(\left| \tau_k^{(a)} - \tau_k^{(b)} \right|^2 \mathbf{I} \right) + \mathcal{N}$. Note that maximal symbol-wise diversity can be defined more generally than that in [66, 67] such that for $\forall k, rank(\mathbf{C}_k) = \min\{T, N_T\}$.

Design Criterion 3 *Traceless non-orthogonality [10]: if maximal symbol-wise diversity is satisfied, $\mathbf{C}_k$ should be designed so that the non-orthogonality matrix $\mathcal{N}$ is traceless [66, 67], i.e.,*

$$Tr\left(\mathcal{N}\right) = 0. \tag{135}$$

As illustrated in the following theorem, another trace related design criterion is Frobenius orthogonality, also called traceless self-interference [68].

Theorem 8 *[68] For a linear matrix modulation with a Frobenius orthogonal basis, i.e.,*

$$Tr\left(\underline{\mathbf{C}}_i^{\mathcal{H}} \underline{\mathbf{C}}_k + \underline{\mathbf{C}}_k^{\mathcal{H}} \underline{\mathbf{C}}_i\right) = 0, \tag{136}$$

the union bound on the pairwise error probabilities increases with increased self-interference at any SNR.

In [68], it is shown that Frobenius orthogonality supports minimizing the union bound.

In [69], Zhang et. al. employed both trace and diversity criteria to design full diversity cyclotomic codes, which generally allow LDC dispersion matrices with $\{\mathbf{A}_q \neq \mathbf{B}_q\}$. The trace related criteria in [69] are given as follows.

Definition 1 *Let $T \geq N_t$. A sequence of matrices $\mathbf{C}_q$ and $\mathbf{D}_q$, $q = 1, ..., Q$, and is said to constitute a trace-orthonormal LD code if the following conditions are satisfied:*

a)

$$\left[\mathbf{C}_q\right]^{\mathcal{H}} \mathbf{C}_q + \left[\mathbf{D}_q\right]^{\mathcal{H}} \mathbf{D}_q = \frac{T}{Q} \mathbf{I}_M, \tag{137}$$

b)

$$[\mathbf{D}_q]^{\mathcal{H}}\,\mathbf{C}_q + [\mathbf{C}_q]^{\mathcal{H}}\,\mathbf{D}_q = \mathbf{0}, \tag{138}$$

c)

$$\textit{Tr}\left(\mathbf{C}_q\,[\mathbf{C}_p]^{\mathcal{H}} + \mathbf{D}_p\,[\mathbf{D}_q]^{\mathcal{H}}\right) = \frac{MT}{Q}\delta(p-q), \tag{139}$$

d)

$$\textit{Tr}\left(\mathbf{D}_q\,[\mathbf{C}_p]^{\mathcal{H}} + \mathbf{D}_p\,[\mathbf{C}_q]^{\mathcal{H}}\right) = 0, \tag{140}$$

where $p = 1, ..., Q$ and $q = 1, ..., Q$, $\mathbf{C}_p$ (or $\mathbf{C}_q$) and $\mathbf{D}_p$ (or $\mathbf{D}_q$) are dispersion matrices defined in (3).

1.9.2. Space-Time-Frequency Codes

To further exploit space diversity, LDC concepts may extend to 3-D space-time-frequency channels as STF codes [70–72].

General MIMO-OFDM channel model has been provided in [70]. Consider a MIMO-OFDM system with N_t transmit antennas, N_r receive antennas and a OFDM block of N_c subcarriers per antenna. The channel between the m-th transmit antenna and n-th receive antenna in the k-th OFDM block experiences frequency-selective, temporally non-selective Rayleigh fading with channel coefficients $\mathbf{h}_{m,n}^{(k)} = \left[h_{m,n(0)}^{(k)}, ..., h_{m,n(L)}^{(k)}\right]^{\mathcal{T}}, m = 1, ..., N_T, n = 1, ..., N_R$, where

$$L = \max\{L_{m,n}, m = 1, ..., N_T, n = 1, ..., N_R\},$$

and $L_{m,n}$ is the frequency-selective channel order of the path between the m-th transmit antenna and n-th receive antenna. Note that the above model is based on the fact that frequency selective channels are different from one pair of transmitter and receiver antennas to another, since different transmitter-receiver channel often experience different physical environments, especially for outdoor communications. Using the approach in [62], Wu and Blostein in [70] provided a general diversity order bound of one STF block as $\min\left\{N_{freq(i)}N_RT, T\sum_{m=1}^{N_T}\sum_{n=1}^{N_R}(L_{m,n}+1)\right\}$, where $N_{freq(i)}$ is the given frequency domain size of the STF block. Note that, in [73], the diversity order bound of STFC was given provided that $L_{m,n} = L$ holds for all $m = 1, ..., N_t$ and $n = 1, ..., N_r$.

Using a simpler MIMO OFDM channel model as in [73], Zhang et. al. constructed a high rate STFC in [72], achieving a full diversity order $N_tN_rT(L+1)$. For $N_t = N_r = T = 2$ and $L = 1$, the design example is shown as follows,

$$\mathbf{X} = \begin{pmatrix} X_1(1) & \phi X_2(1) & X_1(5) & \phi X_2(5) \\ \phi X_2(2) & X_1(2) & \phi X_2(6) & X_1(6) \\ X_1(3) & \phi X_2(3) & X_1(7) & \phi X_2(7) \\ \phi X_2(4) & X_1(4) & \phi X_2(8) & X_1(8) \end{pmatrix}, \tag{141}$$

where $[X_1(1), ..., X_n(8)]^{\mathcal{T}} = \Theta \left[s_{8(n-1)+1}, ..., s_{8n}\right]$, $n = 1, 2$, ϕ and Θ is defined in [72].

Note that $\mathbf{X}$ of size $N_t(L+1) \times N_t T$ also use layered structure, and the number of layer is N_t. The designs in [72] are extensions of the layered structure of the ST codes in [25–27]. Even the frequency diversity order is $L+1$, the minimal frequency domain size is set to $N_t(L+1) > (L+1)$ to achieve full diversity.

1.9.3. LDC in Multiuser Communications

Applications of O-STBC to CDMA systems have been studied, e.g., in [74–76]. However, very limited efforts have been made in investigating the application of LDCs to CDMA systems. In order to support future high data rate CDMA systems, the use of high-rate space-time block codes, e.g, LDCs, may be desirable. In [77], a LDC decoder combined with a blind subspace-based multi-user detector is studied for the downlink of a DS-CDMA system, and a subspace-based sphere decoding algorithm is proposed to further improve the performance. The iterative decoding of LDC codes in a frequency-selective channel is considered in [78], where only a single-user approach is studied, and multi-user scenarios were not investigated.

In [79], Xiao et. al. have proposed a joint multi-user detection, space-time LDC decoding, and Q-ary demodulation algorithm for DS-CDMA sytems, and the turbo processing principle is applied to improve the system performance, while maintaining a reasonable computational complexity. Results show that in comparison to the spatial multiplexing (SM) system with the same transmission rate, the LDC coded systems have superior performance and faster convergence. Furthermore, by exploiting time diversity, LDCs also provide us a powerful means to combat impairment caused by fast fading channels. However, it has been observed in [79] that the advantages of applying STBC become smaller when a strong channel code is used and/or when the receive diversity increases. Considering the fact that strong channel codes are usually employed in practical communication systems, and a high receive diversity order can be readily implemented at the base station. The simple SM scheme using turbo MIMO approaches would work properly for MIMO CDMA systems in slow fading channels, whereas LDCs are more helpful for rapid fading channels.

1.10. Performance Examples

Although we will not provide a thorough investigation of system performance, this subsection provides several performance examples of LDC based systems to give readers some visionary feelings on advantages of LDC.

1.10.1. ST-LDC

In the following comparisons, ST MIMO flat fading channels are assumed. Perfect channel knowledge (amplitude and phase) is assumed at the receiver but not at the transmitter. Each LDC codeword is of size $T \times N_t$. The symbol coding rates of all tested codes are one. Data symbols use 4-QAM modulation in all simulations. Maximum likelihood decoding is performed at the receiver. Average SNR per receive antenna is used in all figures. The matrix channel is assumed to be constant over one ST-LDC codeword. In the case of $N_t = N_r = 2$, R^{sym}_{LDC} = 2, the following ST-LDCs are compared in Fig. 1:

a) HH: size 2×2, proposed by Hassibi and Hochwald, Eq. (31) of [2].

b) TAST: size 2×2, Eqs. (13) and (15) in [26]. This code achieves full diversity over constellations carved from $\mathbb{Z}[j]$.

c) GD: size 2×2, proposed by Gohary and Davidson [80],

d) TON(Zhang): size 2×2, given in Example 3 on p. 626 of [69].

Note that the performance of TON 2×2 (Zhang) is quite close to Golden codes. As mentioned in [69], TON 2×2 (Zhang) is also a NVD ST-LDC.

In the case of $N_t = N_r = 3$, R^{sym}_{LDC} = 3, the following ST-LDCs are compared in Fig. 2:

a) TAST of size 3×3, from Eq. (18) of [26]. It achieves full diversity over constellations carved from $\mathbb{Z}[j]$.

b) HP of size 3×3, as proposed by Heath and Paulraj, from (30) and (31) of [10],

c) FDFR: size 3×3, as proposed by Ma and Giannakis, a full-diversity full-rate (FDFR) code corresponding to Design A in [25],

d) HH of size 3×3, proposed by Hassibi and Hochwald, from (31) of [2].

All the curves except the one for TAST are very close to each other, thus the investigated ST-LDCs have similar diversity properties in the shown SNR range in the case of $N_t = N_r = 3$.

1.10.2. LDC-OFDM

Each of the D LDC demodulators decodes $T \times N_{F(i)}$ LDC matrices. In particular, we set $N_{F(i)} = N_F = T, i = 1, ..., D$, and $N_C = 16$ OFDM subcarriers are chosen. An evenly and maximally spaced subcarrier mapping with respect to the subcarrier indices is used within LDC codewords. Data symbols use 4-QAM modulation. The frequency-selective Rayleigh fading channel has 4 paths with uniform power delay profile. The channel is assumed to be constant over an integer number of OFDM blocks, independently and identically-distributed between blocks. Denote this interval of OFDM blocks as the channel change interval (CCI) for LDC-OFDM. Linear constellation precoded CP-OFDM (LCP-CP-OFDM) with subcarrier grouping has been proposed as a non-redundancy approach to improve BER performance [58]. Although LCP-CP-OFDM achieves both maximum frequency selective diversity gain and coding gain, it cannot exploit time diversity over OFDM blocks. Using MLD, we investigate the performance limitations of LDC-CP-OFDM. For a fair comparison, all parameters of LCP-CP-OFDM are chosen to be the same as those of LDC-CP-OFDM. Thus the available diversity in the channels is the same for both systems. In Figure 3, it is observed that LDC-CP-OFDM, which achieves full joint frequency and time diversity, significantly outperforms LCP-CP-OFDM with the frequency-domain MLD in rapid fading channels ($CCI = 1$).

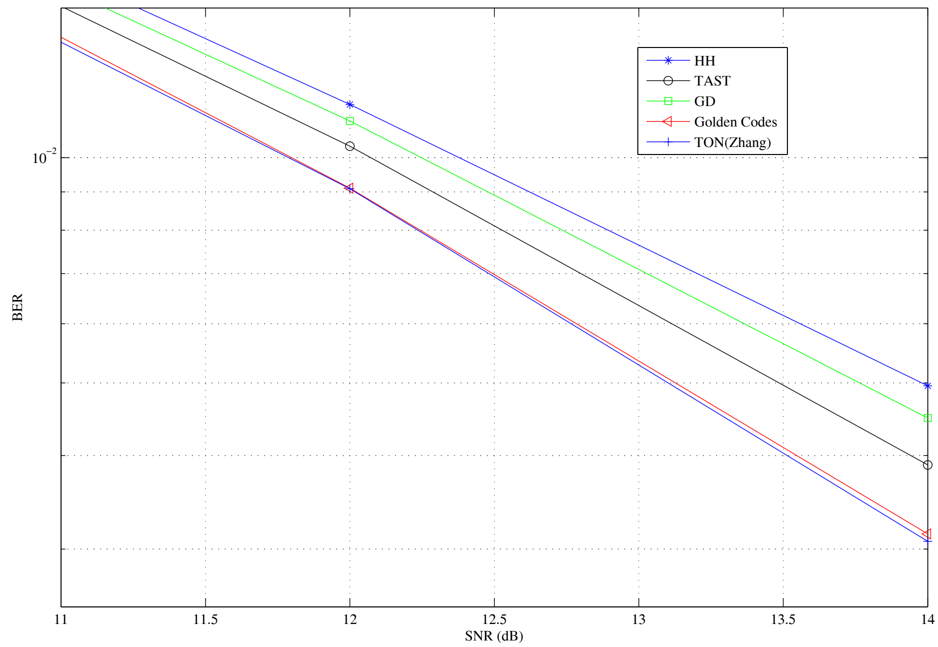

Figure 1. ST-LDC performance $N_t = N_r = 2$, R^{sym}_{LDC} = 2.

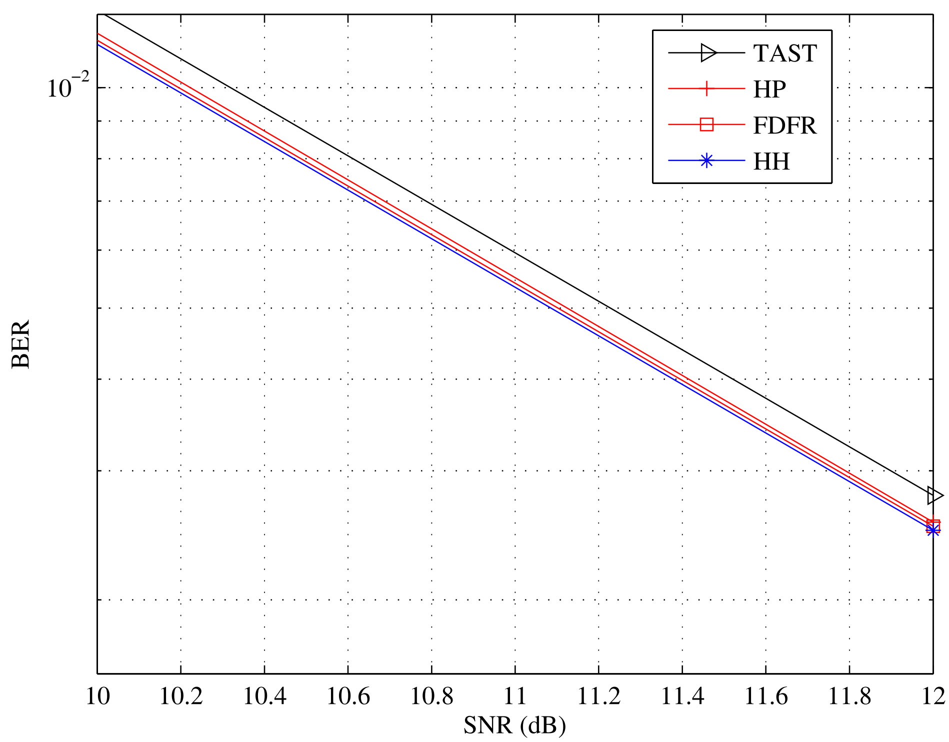

Figure 2. ST-LDC performance $N_t = N_r = 3$, R^{sym}_{LDC} = 3.

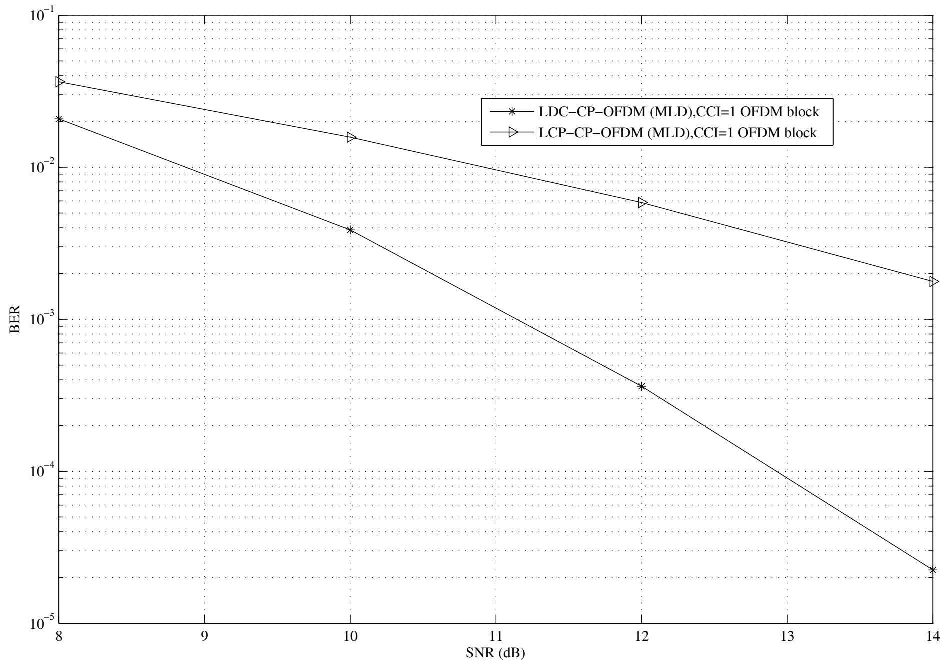

Figure 3. Performance comparison between LDC-OFDM and LCP-OFDM under MLD.

1.11. Summary

This chapter have given a systematic survey of LDC designs and applications for wireless communications. LDC have been and will further be considered as a general class of block coding techniques in improving quality of wireless information transmission.

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