Low Complexity Power Allocation in Multiple-antenna Relay Networks

Yi Zheng and Steven D. Blostein

Dept. of Electrical and Computer Engineering
Queen's University, Kingston, Ontario, K7L3N6, Canada
Email: yi.zheng@ece.queensu.ca, steven.blostein@queensu.ca

***Abstract*—In this paper we study a two-hop cooperative communication system to enhance system coverage and data rates for the uplink in a cellular network. The throughput maximization is formulated into a convex optimization problem. A suboptimal numerical algorithm to approach the maximum sum capacity based on the decode and forward scheme is proposed and the numerical algorithm has an iterative implementation with fast convergence. The performance is close to the min-cut max flow (MCMF) bound. The proposed scheme is compared with two other cooperative schemes and shows significant performance gain.**

I. Introduction

Cooperative communication is a growing area of research and enables efficient spectrum usage by resource sharing among multiple nodes in a network. Major contributions to cooperative communications can be found in [1]-[2]. As we can foresee that future generations of cellular network will migrate to higher carrier frequencies than today's 3G system, mobile handsets would be able to accommodate more antennas compared with those of today because of the decreased wavelength. However, at high frequency, path loss is increased significantly. Multi-hop communication, though a potential solution, does not itself yield diversity benefits, but may be used to combat severe signal attenuation in long-range communication.

In this paper, we consider a cooperative system with a source, a destination and multiple relays each equipped with multiple antennas based on the Decode and Forward (DF) protocol. The objective is to maximize the throughput for Gaussian MIMO relay channels. To the best knowledge of the authors, this fundamental problem has not been studied in the literature and poses the challenge of how to coordinate the multiple antenna relays to maximize throughput.

A four-node single antenna cooperative system is studied in [3], where the upper and lower bounds for capacity indicate that multiple antennas are necessary to achieve the multiplexing gain. In [4], the asymptotic upper and lower bound of the capacity of a MIMO relay system based on the amplify-and-forward (AF) scheme is derived and a scheme is proposed. But the scheme described in [4] is far from achieving the upper bound. In [5], the upper and lower bounds of the capacity of Gaussian MIMO relay channels are derived based on the min-cut max flow bound [6]. However, the number of relays is limited to one and the case when the signal cannot directly reach the destination is not considered. In [7], a MIMO relay system based on decode and forward (DF) is presented, but only the case of one relay is discussed and not studied in depth. In this paper, we study the throughput of two-hop multiple antenna relay systems and propose a low complexity suboptimal algorithm to approach the maximum throughput.

This paper is organized as follows. In Section II, we introduce the system model. Section III formulates the problem and presents an suboptimal iterative algorithm to approach the maximum throughput for the relay system. Numerical results and comparison with other cooperative systems are given in Section IV.

II. System model

We consider the uplink communication in a cellular network where the source subscriber's (SS) signals cannot reach the base station (BS) because of path loss but are in the range of other relay subscribers (RS) which can approach the base station. The uplink transmission from SS to BS which is divided into two hops. The hop from SS to RSs is a MIMO broadcast channel (MIMO-BC) [8], and the hop from RSs to BS is a MIMO multiple access channel (MIMO-MAC) [9]. The system is illustrated as Fig. 1. We assume in this case that the BS knows the channels from SS to RSs and the channels from RSs to itself. The base station makes all decisions and informs the relevant RSs and SS. The objective is the find a beamforming transmit matrix for the MIMO-BC link from SS to RSs and a set of transmit beamforming matrices for the MIMO-MAC link from RSs to BS such that the throughput of the cooperative communication system is maximized. The source subscriber has n_{SS} antennas, the base-station has n_{BS} antennas and each relay subscriber has n_{RS} antennas. We consider independent and identically (i.i.d) block fading channels between all nodes which are assumed to be frequency flat. The ith channel from SS to RS(i) is denoted by $H_{1,i} \in C^{n_{RS(i)} \times n_{SS}}$, $1 \leq i \leq K$ whereas the channel matrix from RS(i) to the BS is denoted by $H_{2,i} \in C^{n_{BS} \times n_{RS(i)}}$. We assume that RS$(i)$ can estimate the channel $H_{1,i}$ and forward this information to the BS by the data communication channel from SS to RS(i) and the BS can estimate the channel $H_{2,i}$. The transmit powers of SS and RS(i) are denoted by P_{SS} and $P_{RS}^{(i)}$, respectively. The received signal at RS(i) is

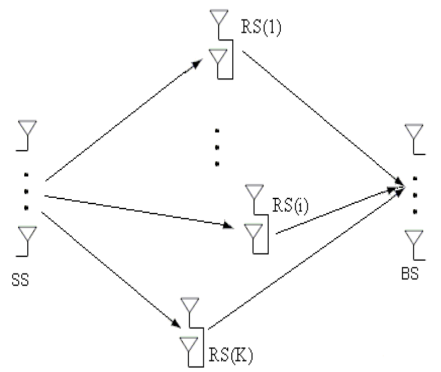

Fig. 1. Cooperative system Model

$$\boldsymbol{y}_{1,i} = H_{1,i}\boldsymbol{x} + \boldsymbol{n}_{1,i} \tag{1}$$

Let $S_1 = \mathbf{E}([\mathbf{x}\mathbf{x}^T])$. The power constraint is $tr(S_1) \leq P_{SS}$, and $\boldsymbol{n}_{1,i}$ is zero-mean circular symmetric complex Gaussian noise with covariance matrix $\sigma_{RS}^2 \mathbf{I}_{n_{RS(i)}}$.

The multiple access channel from RS to BS is

$$\boldsymbol{y}_2 = \sum_{j=1}^{K} H_{2,j}\boldsymbol{x}_{2,j} + \boldsymbol{n}_2 \tag{2}$$

where $\boldsymbol{x}_{2,j}$ is the input vector signal from RS(*j*), and $\boldsymbol{n}_2$ is the additive white Gaussian noise vector with covariance $\sigma_{BS}^2 \mathbf{I}_{n_{BS}}$. The input signals from RS are mutually independent with a joint distribution $\prod_{j=1}^{K} \mathbf{p}_j(\mathbf{x}_{2,j})$ that satisfies the power constraints $tr(\mathbf{E}([\mathbf{x}_{2,j}\mathbf{x}_{2,j}^T]) \leq P_j, 1 \leq j \leq K$. Letting $n_{RS} \times n_{RS}$ covariance matrix $S_{2,j} = \mathbf{E}([\mathbf{x}_{2,j}\mathbf{x}_{2,j}^T])$, the power constraints become $tr(S_{2,j}) \leq P_{RS}^{(j)}$. Also, we denote $\boldsymbol{S}_2$ as the $n_{RS} \times Kn_{RS}$ matrix $[S_{2,1}S_{2,2} \cdots S_{2,K}]$. Successive interference cancellation (SIC) is a commonly used decoding technique for MIMO-MAC and the choice of decoding order in SIC leads to different user data rates.

The objective is the find a transmit beamforming matrix for the MIMO-BC link from SS to RS and a set of transmit covariance matrices for the MIMO-MAC link from RS to BS so that the throughput for the cooperative communication system is maximized. From [10], we know that the throughput of the above system is min-cut max-flow (MCMF) bounded. More specifically, denote the maximum sum rate for the MIMO-MAC link as R_{MAC} [9], the maximum sum rate for the MIMO-BC link as R_{BC}. The maximum achievable throughput for the two hop cooperative system, R_{Coop}, based on the decode-and-forward scheme, is then

$$R_{Coop} \leq min\{R_{BC}, R_{MAC}\}. \tag{3}$$

From [8], the capacity regions for a K-user multiple access channel is the convex hull of the union of capacity pentagons defined in (2). From [8], it is proven that the capacity region of MIMO-BC is also a convex hull.

For MIMO-BC, the optimization problem is non-convex and difficult to solve. Instead, we use the duality relationship between broadcast channel and multiple access channel [11][12] [13] to transform (1) into the following MIMO MAC model with sum power constraint:

$$\boldsymbol{y}_1 = \sum_{i=1}^{K} H_{1,i}^{\dagger}\boldsymbol{x}_{1,i} + \boldsymbol{n}_1 \tag{4}$$

where this multiple access model has sum power constraint $\sum tr(S_{1,i}) \leq P$ with $\mathbf{E}([\mathbf{x}_{1,i}\mathbf{x}_{1,i}^T])$ denoted as $S_{1,i}$ and $\boldsymbol{S}_1 = [S_{1,1}S_{1,2} \cdots S_{1,K}]$ and $E(\mathbf{n}_1\mathbf{n}_1^{\dagger}) = \sigma_{SS}^2 \mathbf{I}_{n_{SS}}$. Because of the duality relationship, once the set of covariance matrices $\boldsymbol{S}_1$ is obtained, we can easily derive the covariance matrix for MIMO-BC link through the one-to-one mapping relationship in [12]. Here we denote the transformed MIMO MAC through the duality relationship as T-MIMO MAC. We denote the MIMO MAC from RS to BS as RB-MIMO MAC and the T-MIMO MAC for the link from SS to RS as SR-T-MIMO MAC.

In this way, the original problem is then transformed into the problem of finding a point residing both in the capacity regions of RB-MIMO MAC and SR-T-MIMO MAC. As the capacity regions of RB-MIMO MAC and SR-T-MIMO MAC are both convex hulls, the intersection of two convex hulls is convex. The original problem is formulated into the following convex optimization problem. Let $r_i, 1 \leq i \leq K$ denote the achievable rates for each relay. We need to solve

$$\begin{aligned}
\max_{S_{1,1},S_{1,2},\cdots,S_{1,K},S_{2,1},S_{2,2},\cdots,S_{2,K}} & (r_1 + r_2 + \cdots + r_K) \\
s.t. \quad \sum_{i \in A} r_i \leq \text{logdet}(I + & \sum_{i \in A} H_{1,i}^{\dagger} S_{1,i} H_{1,i}), \\
& \forall A \subset \{1, \cdots, K\} \\
\sum_{j \in B} r_j \leq \text{logdet}(I + & \sum_{j \in B} H_{2,j} S_{2,j} H_{2,j}^{\dagger}), \\
& \forall B \subset \{1, \cdots, K\} \\
\text{tr}(S_{1,i}) \leq P_i & \quad \forall A \subset \{1, \cdots, K\} \\
\sum_{j \in B} \text{tr}(S_{2,j}) \leq P & \quad \forall B \subset \{1, \cdots, K\}
\end{aligned} \tag{5}$$

We remark that the number of constraints in (5) is equal to the number of possible subsets, $2^{(K+1)}$, which is exponentially large. As the objective function is an affine function and all the constraints are either concave or affine, this is a convex optimization problem. From [8], we know that the capacity region of a multiple access channel of K users with individual power constraints at each user admits a polymatroid structure with $K!$ vertices, while the capacity region of a multiple access channel with sum power constraint admits a polymatroid structure with a curved boundary in general, except at the maximum sum rate points [8]. The problem (5) requires a search for the points maximizing the throughput residing in the capacity regions of both RB-MIMO MAC and SR-T-MIMO MAC. There may exist multiple points maximizing the sum rate satisfying these constraints.

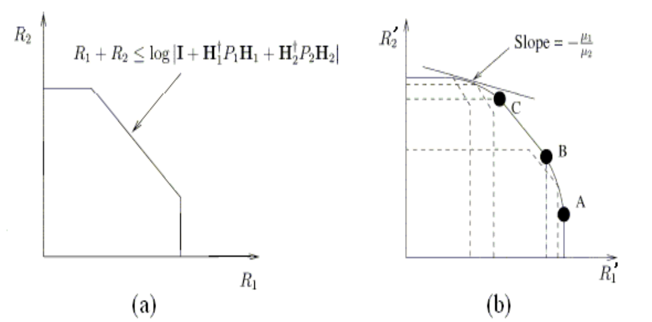

Fig. 2. Capacity Region of two-user case for (a) MIMO-MAC and (b) MIMO-BC

The problem (5) is a semidefinite programming problem with nonlinear constraints. There are the following difficulties associated with solving (5):

1) The number of constraints is $(K + 1 + 2^{K+1})$

2) For semidefinite programming, the dimension of the linear system is $\sum_{i=1}^{K} N_{RS(i)}^2$, so the computation needed to solve the problem is proportional to $(\sum_{i=1}^{K} N_{RS(i)}^2)^3$

3) The nonlinear constraints in the problem make it more involved, and developing numerically efficient and robust procedures to solve this problem are not straightforward to derive.

From [8], we know that the capacity regions of MIMO-MAC and MIMO-BC are convex hulls. The capacity region of a two-user MIMO-MAC and MIMO-BC is illustrated in Fig. 2. According to (2) and (4), the capacity regions of SR-T-MIMO MAC and RB-MIMO MAC are both convex hulls. However it is not easy to characterize the intersection of the capacity regions of SR-T-MIMO MAC and RB-MIMO MAC. Fortunately, it is obvious that the optimal point which gives the maximum throughput must be either on the surface of SR-T-MIMO MAC or on the surface of RB-MIMO MAC. From [8] and convex optimization theory, because of the convex hull geometry of SR-T-MIMO MAC capacity region, any point of the K-dimensional surface of SR-T-MIMO MAC is paretel optimal and can be described as follows:

$$\max_{S_{1,1},S_{1,2},\cdots,S_{1,K}} \mu_K \text{logdet}\left(\mathbf{I} + \sum_{l=1}^{K} \mathbf{H}_{1,l}^\dagger S_{1,l} \mathbf{H}_{1,l}\right) + \sum_{i=1}^{K-1} (\mu_i - \mu_{i+1}) \text{logdet}\left(\mathbf{I} + \sum_{l=1}^{i} \mathbf{H}_{1,l}^\dagger S_{1,l} \mathbf{H}_{1,l}\right) \quad (6)$$

where $\mu_i \in \mathbf{R}, 1 \le i \le K$. From [8], we know that any point on the surface of RB-MIMO MAC can be found by weighted maximum and time-sharing. The sum rate is determined by the weight vector $(\mu_1, \mu_2, \ldots, \mu_K)$ in (6). The decoding order is uniquely determined by $(\mu_1, \mu_2, \ldots, \mu_K)$ when for any $i \ne j$, $\mu_i \ne \mu_j$. For equal weight coefficients, different decoding orders can achieve the same sum rate. Once the decoding order and weight coefficients on RB-MIMO MAC and SR-T-MIMO MAC are determined, the capacity of each relay on RB-MIMO MAC and SR-T-MIMO MAC is determined, and hence the overall throughput. In (3), the throughput is MCMF bounded.

In the above discussion, the ordering strategy is the crucial point in solving this optimization problem. However, the number of possible orderings is $(K!)^2$ and finding the best possible ordering becomes a discrete optimization problem which is combinatorially prohibitive to solve as the number of relays becomes large. What is more, the possible combinations of weights for both RB-MIMO MAC and SR-T-MIMO MAC sides is infinite. This motivates a suboptimal algorithm which can provide close-to-optimal performance at significantly lower complexity.

III. A SUBOPTIMAL ALGORITHM

The high complexity discussed above and implementation difficulties motivate the development of a simpler suboptimal algorithm. Rather than employing nonlinear optimization, we instead propose an iterative greedy method that achieves good performance with significantly lower complexity. In [9], Yu finds the optimal solution for the maximum sum rate of the MIMO-MAC channel using an individually power constrained iterative waterfilling algorithm (IP-IWA). In [14], Goldsmith utilizes a similar iterative algorithm to find the maximum sum rate for MIMO-BC and the associated transmit beamforming matrices, which is termed a sum power constrained iterative waterfilling algorithm (SP-IWA). However, we notice that in both RB-MIMO MAC and SR-T-MIMO BC described in the previous section, inequality constraints will be active for the optimal solution. However, this is not true for problem (5) in general. Moreover, in both the RB-MIMO MAC and SR-T-MIMO MAC, the optimal solution is not affected by the decoding order. However, in problem (5), the decoding order plays a crucial role. In [15], Yu illustrates that the encoding order plays an important role to achieve a target rate for MIMO-BC with minimum power consumption. In problem (5), the situation is similar to that in [15] but more complicated. The difference between RB-MIMO MAC and SR-T-MIMO MAC are the power constraints. More specifically, RB-MIMO MAC is individually power constrained while SR-T-MIMO MAC is sum-power constrained.

A. ALGORITHM DESCRIPTION

Inequality (3) provides a hint that the optimal solutions for the RB-MIMO MAC and SR-T-MIMO MAC channels may be good candidates as a starting point, since the objective is to maximize the throughput bounded by the minimum of the maximum rate of RB-MIMO MAC and SR-T-MIMO MAC. From this starting point, we can adjust the decoding orders and power allocations for both RB-MIMO MAC and SR-T-MIMO MAC iteratively to satisfy the rate vector constraints on both sides in a greedy way. The IP-IWA [9] and SP-IWA [14] are used as the building blocks for the proposed suboptimal algorithm. As the problem size decreases with each iteration, the complexity reduces correspondingly. The last decoded relay for MIMO MAC is processed first, and its signal is treated as interference for the remaining RSs. This process is iterated for the remaining relays. We denote this algorithm as greedy search algorithm (GS alg).

Algorithm Description:

Step 1. Apply IP-IWA for RB-MIMO MAC and SP-IWA for SR-T-MIMO MAC

Step 2. Calculate $r_{1,i}^{(k)}$ and $r_{2,i}^{(k)}$ for the ith relay at kth iteration [9] [14].

Step 3. Calculate $r_{i,diff}^{(k)} = r_{2,i}^{(k)} - r_{1,i}^{(k)}$ and decide the decoding order for RB-MIMO MAC.

Step 4. Choose the jth relay which satisfies $r_{rb-mac,j}^{(k)} \leq r_{sr-mac,j}^{(k)}$ as the last decoded relay. If no such relay exists, then stop and calculate the sum capacity of the remaining relays on SR-T-MIMO side plus the sum capacity of the relays on RB-MIMO MAC side in previous iterations as the overall throughput.

Step 5. Calculate $S_{1,i}$ and $S_{2,i}$. On both RB-MIMO MAC side and SR-T-MIMO MAC side the ith relay's signal is regarded as interference for later iterations . Update interference and power on both sides [9] [14].

Step 6. Take out the ith relay and go to step 1 for the next iteration or if $k = K$ then stop the algorithm and calculate the sum capacity of the relays on RB-MIMO MAC side in previous iterations as the overall throughput.

In step 2, $r_{1,i}$ and $r_{2,i}$ denote the capacity of the ith relay on the RB-MIMO MAC side and SR-T-MIMO MAC side with the other relays' signals as interference respectively. In step 3, the decoding order on RB-MIMO MAC side is inverse to the ascending order of sorted $\mathbf{r}_{diff}$, for example, if $r_{m,diff}$ is the smallest then the mth relay's signal is the last decoded on RB-MIMO side. In step 4, $r_{rb-mac,j}^{(k)}$ denotes the maximum achievable capacity of jth relay at kth iteration based on IP-IWA and $r_{sr-mac,j}^{(k)}$ denotes the minimum achievable capacity of jth relay based on SP-IWA.

IV. NUMERICAL RESULTS AND DISCUSSSSION

In this section, numerical results are employed to provide insight into the behavior of GS alg. The focus is an comparison of the achievable system throughput using GS alg with the MCMF bound. GS alg is also compared with three other cooperative schemes to determine the effectiveness of it is. The simulation scenario is as follows: the system consists of SS, four RS and BS each with two antennas, and $\mathbf{n}_1, \mathbf{n}_2$ are independent additive Gaussian noise vectors with unit variance on each vector component. The first cooperative scheme is for the MIMO-BC link to allocate equal power for each relay and employing IP-IWA on both SR-T-MIMO MAC and RB-MIMO MAC. Equal power allocation for relays is the best strategy when the information of the link from RSs to BS is unavailable. This scheme is denoted as UNI alg. The second ad-hoc algorithm is to use SP-IWA on SR-T-MIMO MAC and IP-IWA on RB-MIMO MAC link only once without adjusting the power allocation. The second ad-hoc algorithm is denoted as ITR alg. For both these two schemes, the encoding order on the MIMO-BC link from SS to RS and the decoding order on the MIMO MAC link from RS to BS are randomly chosen. The third scheme is based on UNI alg with the additional feature that on the MIMO-BC link all the possible encoding orders are enumerated and the best encoding order is chosen. This scheme is denoted as enumerated UNI alg.

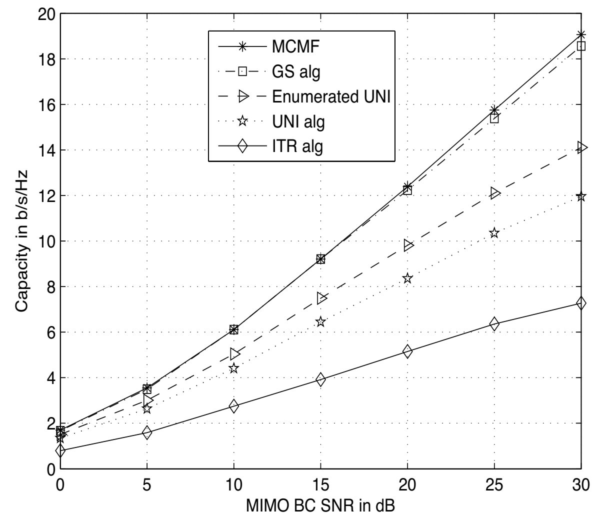

Fig. 3. Comparison of achievable system throughput using GS alg with MCMF bound, enumarated UNI alg, UNI alg and ITR alg for MIMO-BC link with power at SS twice the sum of the powers at relays.

In Fig. 3, a comparison of the system throughput achieved using GS alg with the MCMF bound is illustrated. The plot indicates that the achievable throughput of GS alg is very close to that of the MCMF bound. This means in this case, with a very high probability (close to 1), the rate vector with maximum sum rate on the RB-MIMO MAC side resides in the capacity region of SR-T-MIMO MAC.

The optimal system throughput is in between GS alg and the MCMF bound. Since the gap between the achievable system throughput using GS alg is no larger than the gap between the achievable system throughput using GS alg and the MCMF bound, GS alg can achieve close to optimal performance and hence very effective.

From Fig. 3, it is obvious that GS alg is significantly better than all the other schemes. The performance of these three cooperative schemes, however, are far from the MCMF bound. We also notice that there is a performance gap between UNI-alg and ITR-alg, which is as expected since when channel state information is not utilized, uniform power allocation is the best strategy [16]. When enumerating all possible encoding orders and choosing the best, the performance can be significantly improved, which demonstrate the importance of ordering. However, the enumeration is very costly, for a K relay network, the complexity increase to $K!$ times.

V. CONCLUSION

In this paper we studied the throughput of the two-hop DF multi-antenna cooperative system, and proposed a relay strategy for the uplink two-hop MIMO transmission in a cellular network based on a decode-and-forward protocol. We have shown that the computational complexity of the optimal solution grows exponentially in the number of relays. We then proposed a low-complexity suboptimal algorithm which has performance very close to the min-cut max-flow bound, and in some cases, the min-cut max-flow bound can be achieved.

REFERENCES

[1] N.J. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. on Info. Theory*, vol. 49, no. 3, pp. 2415-2425, Oct 2003.

[2] A. Sendonaris and E. Erkip and B. Aazhang, "Cooperation diversity-Part I: System description," *IEEE Trans. on Info. Theory*, vol. 50, no. 3, pp. 3062-3080, Nov 2003.

[3] Anders Host-Madsen, "Capacity Bounds for Cooperative Diversity," *IEEE Trans. on Info. Theory*, vol. 52, no. 4, pp.1522-1544, Apr 2006.

[4] H. Bolcskei and R. U. Nabar and O. Oyman and A. J. Paulraj, "Capacity Scaling Laws in MIMO Relay Networks," *IEEE Trans. on Communications*, vol. 5, pp.1433-1444, June 2006.

[5] Bo Wang and Junshan Zhang and Anders Host-Madson, "On the Capacity of MIMO relay Channels," *IEEE Trans. on Info. Theory*, vol. 51, pp.29-43, Jan 2005.

[6] T. M. Cover and El Gamal, "Capacity theorems for the relay Channel," *IEEE Trans. on Info. Theory*, vol. 5, pp.572-584, Sep 1979.

[7] Y. Fan and J. Thompson, "MIMO Configurations for Relay Channels: Theory and Practice," *IEEE Trans. on Wireless Communication*, vol. 6, pp.1774-1786, May 2007.

[8] A. Goldsmith and S. A. Jafar and N. Jindal and S. Vishwanath, "Capacity Limits of MIMO Channels," *IEEE Journal on Selected Areas in Communications*, vol. 21, pp.684-702, June 2003.

[9] W. Yu and W. Rhee and S. Boyed and J. M. Cioffi, "Iterative Water-Filling for Gaussian Vector Multiple-Access Channels," *IEEE Trans. on Info. Theory*, vol. 50, pp.145-152, Jan 2004.

[10] T. M. Cover and J. A. Thomas, "Elements of Information Theory," *John Wiley and Sons Inc*, 1991.

[11] W. Yu and J. M. Cioffi, "Sum Capacity of Gaussian Vector Broadcast Channels," *IEEE Trans. on Info. Theory*, vol. 50, pp.1875-1892, Sept 2004.

[12] S. Vishwanath and N. Jindal and A. Goldsmith, "Duality, Achievable Rates, and Sum-Rate Capacity of Gaussian MIMO Broadcast Channels," *IEEE Trans. on Info. Theory*, vol. 49, pp.2658-2668, Sept 2003.

[13] P. Viswanath and D. C. TSE, "Sum Capacity of the Vector Gaussian Broadcast Channel and Uplink Downlink Duality," *IEEE Trans. on Info. Theory*, vol. 49, pp.1912-1921, Aug 2003.

[14] N. Jindal and W. Rhee and S. Vishwananth and A. Goldsmith, "Sum power iterative water-filling for multi-antenna Gaussian broadcast channels," *IEEE Trans. on Info. Theory*, vol. 51, pp.1570-1580, Apr 2005.

[15] C. H. F. Fung and W. Yu and T. J. Lim, "Precoding for the Multi-Antenna Downlink: Multi-user Gap Approximation and Optimal User Ordering," *IEEE Trans. on Communications*, vol. 55, pp.1570-1580, Jan 2007.

[16] Daniel P. Palomar, John M. Cioffi, and Miguel Angel Lagunas, "Uniform Power Allocation in MIMO Channels: A Game-Theoretic Approach," *IEEE Trans. on Info. Theory*, vol. 49, pp.1707-1727, July 2003.