POWER OPTIMIZATION IN FADING GAUSSIAN MIMO BROADCAST CHANNELS

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ABSTRACT

In this paper, we consider the problem of minimizing long-term sum power under the constraint of an average sum rate in fading multiple input multiple output (MIMO) broadcast channels (BC) with additive Gaussian noise. This problem arises frequently in dynamic power allocation and power efficiency optimization for wireless communication systems. It is complementary to sum capacity maximization with a sum power constraint for a fading MIMO downlink. We first formulate the equivalent convex optimization problem using the duality between the MIMO multi-access channel (MAC) and the MIMO BC. Then we derive a simple and fast iterative water-filling algorithm based on the subgradient and bisection methods that computes the long-term sum power of the transmitter. Theoretical analysis and numerical simulations show that the proposed algorithm converges to the minimum sum power globally and efficiently.

1. INTRODUCTION

In cellular wireless MIMO communication systems, power allocation and power efficiency optimization have attracted considerable attention due to the fading characteristics of the wireless channel and the performance limitation caused by inter-cell or intra-cell interference. An iterative water-filling algorithm that maximizes sum capacity with individual power constraints for Gaussian vector multi-access channels was proposed in [1]. Jindal et al. [2] have considered the problem of maximizing sum rate with sum power constraint for a Gaussian MIMO BC based on the duality between the MIMO MAC and the MIMO BC [3]. The asymptotic convergence rate of the algorithm proposed in [2] becomes slow as the number of users increases. Yu introduced an algorithm to solve the same problem as in [2] by dual-composition [4], which is not influenced by the number of users. All of the above methods solve the capacity maximization problem with a power constraint. On the other hand, in [5], an iterative algorithm was proposed to solve the transmit power minimization problem in a Gaussian vector BC under rate constraints for individual users. In [6], a complete characterization of MIMO MAC and MIMO BC capacity regions and power regions were provided under various power and rate constraints. An efficient numerical algorithm was also proposed. Zhang et al. have studied the multi-user power region based on the channel distribution information and the rate demand of each individual user in [7]. These papers assume individual rate constraints for all users. More recently, Michel et al. [8] considered the minimization of sum power with a sum rate constraint for the MIMO BC. The algorithm assumes a short-term power constraint for MIMO broadcast channel and is only a minor modification of the algorithm proposed in [2]. Furthermore, the algorithm is only applicable to single block fading channels.

This paper considers the minimization of long-term sum power with an average sum rate constraint for the fading Gaussian MIMO BC. Power allocation under the long-term constraint provides more significant gain compared to the case under the short-term one [9]. Power can be allocated among different fading states and among different antennas. We consider power allocation and power efficiency optimization simultaneously. Firstly, we transform the non-convex downlink problem into a convex uplink problem by the duality between the MIMO MAC and the MIMO BC. Then an iterative water-filling algorithm is derived based on the subgradient and one-dimensional search method for computing transmitter sum power. In addition, the algorithm also determines the transmitter optimal input covariance matrices in different fading states.

Notation: In this paper, lower case bold letters denote vectors and upper case bold letters denote matrices. $|S|$ denotes the determinant, S^{-1} the inverse, S^H the Hermitian transpose and $Tr(S)$ the trace of a square matrix S . $E\{\}$ denotes statistical expectation. $\mathbb{C}^{M\times N}$ denotes the space of $M\times N$ matrices with complex entries. The distribution of a complex Gaussian vector with mean vector η and the covariance matrix Σ is denoted by $CN(\eta,\Sigma)$. $S\geq 0$ means that the matrix is positive semi-definite.

2. SYSTEM MODEL

We consider a discrete fading Gaussian MIMO BC with the base station equipped with r transmit antennas and K users each equipped with t receive antennas. (Note that our analysis below can be easily generalized to the case where users are equipped with different numbers of antennas). The base station communicates with all users simultaneously. A block fading channel model is assumed, i.e., the channel is assumed constant during each block, and possibly changing from one block to another. The fading process is assumed to be jointly stationary and ergodic. Thus, at each channel state n, the fading Gaussian MIMO BC is denoted as follows.

$$\begin{bmatrix} y_1(n) \\ \vdots \\ y_K(n) \end{bmatrix} = \begin{bmatrix} H_1(n) \\ \vdots \\ H_K(n) \end{bmatrix} x(n) + \begin{bmatrix} z_1(n) \\ \vdots \\ z_K(n) \end{bmatrix} \quad (1)$$

where $x(n) \in \mathbb{C}^{r\times 1}$ denotes a transmitted signal vector. $y_k(n) \in \mathbb{C}^{t\times 1}$ and $H_k(n) \in \mathbb{C}^{t\times r}$ denote, respectively, the received signal vector and the complex channel matrix of user k, $k=1,\cdots,K$. The vector $z_k(n) \in \mathbb{C}^{t\times 1}$ denotes the additive Gaussian noise received by the kth user with $z_k(n) \sim CN(0,I)$. Now consider the following dual MIMO MAC:

$$\tilde{y}(n) = \left[H_1^H(n) \cdots H_K^H(n) \right] \begin{bmatrix} \tilde{x}_1(n) \\ \vdots \\ \tilde{x}_K(n) \end{bmatrix} + \tilde{z}(n). \quad (2)$$

Here $\tilde{y}(n) \in \mathbb{C}^{r\times 1}$ denotes the signal vector received by the base station; $\tilde{x}_k(n) \in \mathbb{C}^{t\times 1}$ denotes the signal vector transmitted by the kth user, and $\tilde{z}(n) \in \mathbb{C}^{r\times 1}$ denotes a complex additive Gaussian noise vector received by the base station with $\tilde{z}(n) \sim CN(0,I)$.

With perfect transmitter channel state information, Dirty Paper coding (DPC) can be employed at the transmitter. The Dirty Paper encoding order $\pi(K), \pi(K-1), \cdots, \pi(1)$ is assumed without loss of generality, where π is a permutation, and then the achievable rate of user $\pi(k)$ in fading state n can be denoted by:

$$\begin{aligned} R_{\pi(k)}^{BC}(n) &= \log\left| I + H_{\pi(k)}(n) \sum_{m=1}^{k} Q_{\pi(m)}(n) H_{\pi(k)}^H(n) \right| \\ &\quad - \log\left| I + H_{\pi(k)}(n) \sum_{m=1}^{k-1} Q_{\pi(m)}(n) H_{\pi(k)}^H(n) \right| \end{aligned} \quad (3)$$

where $Q_{\pi(k)}(n) = E\left[x_{\pi(k)}(n) x_{\pi(k)}^H(n) \right]$ denotes the signal covariance matrix transmitted to user $\pi(k)$ in fading state n, in which the expectation is taken over the code-book. Thus the sum capacity of MIMO BC can be written in the following form:

$$\begin{aligned} C_{BC}^{sum\ capacity} &= \log\left| I + H_{\pi(1)}(n) Q_{\pi(1)}(n) H_{\pi(1)}^H(n) \right| \\ &\quad + \log \frac{\left| I + H_{\pi(2)}(n)(Q_{\pi(1)}(n) + Q_{\pi(2)}(n)) H_{\pi(2)}^H(n) \right|}{\left| I + H_{\pi(2)}(n) Q_{\pi(1)}(n) H_{\pi(2)}^H(n) \right|} + \cdots \\ &\quad + \log \frac{\left| I + H_{\pi(K)}(n)(Q_{\pi(1)}(n) + \cdots + Q_{\pi(K)}(n)) H_{\pi(K)}^H(n) \right|}{\left| I + H_{\pi(K)}(n)(Q_{\pi(1)}(n) + \cdots + Q_{\pi(K-1)}(n)) H_{\pi(K)}^H(n) \right|} \end{aligned} \quad (4)$$

By the duality of the MIMO BC and MIMO MAC, the sum capacity of the MIMO BC is equal to the sum capacity of the dual MIMO MAC, and then (4) can be denoted by[2][3]:

$$C_{BC}^{\text{sum capacity}} = C_{MAC}^{\text{sum capacity}} = \log\left| I + \sum_{k=1}^{K} H_k^H(n) S_k(n) H_k(n) \right| \quad (5)$$

where $S_k(n) = E\{\tilde{x}_k(n)\tilde{x}_k^H(n)\}$ denotes the signal covariance matrix transmitted by the kth user for the MIMO MAC in fading state n.

3. PROBLEM FORMULATION

In this paper, we study the long-term sum power optimization (spatial and temporal power optimization) under the constraint of an average sum rate in fading Gaussian MIMO BC. The problem can be formulated as follows:

$$\begin{aligned} &\min\ E\{\sum_{k=1}^{K} Tr(Q_k(n))\} \\ &\text{s.t. } E\{\log\left| I + H_{\pi(1)}(n) Q_{\pi(1)}(n) H_{\pi(1)}^H(n) \right| \\ &\quad + \log \frac{\left| I + H_{\pi(2)}(n)(Q_{\pi(1)}(n) + Q_{\pi(2)}(n)) H_{\pi(2)}^H(n) \right|}{\left| I + H_{\pi(2)}(n) Q_{\pi(1)}(n) H_{\pi(2)}^H(n) \right|} + \cdots \\ &\quad + \log \frac{\left| I + H_{\pi(K)}(n)(Q_{\pi(1)}(n) + \cdots + Q_{\pi(K)}(n)) H_{\pi(K)}^H(n) \right|}{\left| I + H_{\pi(K)}(n)(Q_{\pi(1)}(n) + \cdots + Q_{\pi(K-1)}(n)) H_{\pi(K)}^H(n) \right|} \} \geq R^* \end{aligned} \quad (6)$$

where the expectation is with respect to the channel fading distribution. Obviously, this is a non-convex function of $Q_{\pi(1)}(n), Q_{\pi(2)}(n), \cdots, Q_{\pi(K)}(n)$. However, by combining (4-6), we can translate (6) into an uplink problem denoted by

$$\begin{aligned} &\text{minimize } \mathrm{E}\{\sum_{k=1}^{K} Tr(S_k(n))\} \\ &s.t.\ \ \mathrm{E}\{\log\left| I + \sum_{k=1}^{K} H_k^H(n) S_k(n) H_k(n) \right|\} \geq R^* \\ &\qquad S_k(n) \geq 0, \quad \forall k, n \end{aligned} \quad (7)$$

Here, we can remove the expectation and assume instead that n is a random variable taking on the values of fading states, $n = 1, 2, \cdots, N$ each state having the same probability distribution $f(n)$ [6] as the channel is stationary. Thus

$$\min \sum_{k=1}^{K}\sum_{n=1}^{N} Tr(S_k(n))f(n)$$
$$s.t. \quad \sum_{n=1}^{N}\log\left|I+\sum_{k=1}^{K}H_k^H(n)S_k(n)H_k(n)\right|f(n)\geq R^* \tag{8}$$
$$S_k(n)\geq 0, \quad \forall k,n$$

The objective function is linear, and the constraints are concave in the space of positive semidefinite matrices. Therefore (8) is a convex optimization problem. In general, a convex optimization problem can be solved using the interior point method [10]. Here, we will exploit the structure of the optimization problem and derive a simple and efficient iterative water-filling algorithm.

4. ITERATIVE WATER-FILLING ALGORITHM

For ease of computation, assume uniform state distribution for all n, $f(n)=1/N$. The Lagrangian associated with the optimization problem (8) is

$$L(\{S_k(n)\},\mu)=\frac{1}{\mathrm{N}}\sum_{k=1}^{K}\sum_{n=1}^{N}Tr(S_k(n))-\mu\left(\frac{1}{\mathrm{N}}\sum_{n=1}^{N}\log\left|I+\sum_{k=1}^{K}H_k^H(n)S_k(n)H_k(n)\right|-R^*\right), \tag{9}$$

where μ is the dual variable (Lagrange multiplier).

The Lagrange dual function is defined as [10, 11]

$$g(\mu)=\min_{S_k(n)\geq 0} L(\{S_k(n)\},\mu). \tag{10}$$

For a convex problem, if Slater's condition holds, the optimal duality gap is zero. Note that by using enough power, $\frac{1}{\mathrm{N}}\sum_{n=1}^{N}\log\left|I+\sum_{k=1}^{K}H_k^H(n)S_k(n)H_k(n)\right|$ can be made large enough to contain the specified rate as an interior point. Thus for expression (8), Slater's condition holds. The primal problem (8) can be solved by solving the following Lagrange dual problem.

$$\begin{aligned}&\max \quad g(\mu)\\ &\text{s.t.} \quad \mu>0.\end{aligned} \tag{11}$$

Here, as the Lagrange dual function $g(\mu)$ is concave [10] and μ is a single variable, a standard one-dimension search method can be utilized to obtain the result. However, it is observed from the structure of $g(\mu)$ that it is not easy to take its gradient. Thus, we find a subgradient [12] instead of the gradient here.

To obtain the subgradient of $g(\mu)$, we need solve the optimization problem (10). Firstly, the Lagrangian (9) can be denoted by the following expression through simple manipulations:

$$L(\{S_k(n)\},\mu)=\mu R^*+\frac{1}{\mathrm{N}}\sum_{n=1}^{N}\left(\sum_{k=1}^{K}Tr(S_k(n))-\mu\log\left|I+\sum_{k=1}^{K}H_k^H(n)S_k(n)H_k(n)\right|\right). \tag{12}$$

Using the well-known dual-decomposition method [11], the minimization of the expression (10) can be decomposed into N independent problems

$$\tilde{g}_n(\mu)=\min_{S_k(n)}\sum_{k=1}^{K}Tr(S_k(n))-\mu\log\left|I+\sum_{k=1}^{K}H_k^H(n)S_k(n)H_k(n)\right|. \tag{13}$$

Then, the Lagrange dual function $g(\mu)$ can be denoted by

$$g(\mu)=\frac{1}{N}\sum_{n=1}^{N}\tilde{g}_n(\mu)+\mu R^*. \tag{14}$$

Each optimization problem introduced in (13) can be written as

$$\tilde{g}_n(\mu)=\max_{S_k(n)} \log\left|I+\sum_{k=1}^{K}H_k^H(n)S_k(n)H_k(n)\right|-\frac{1}{\mu}\sum_{k=1}^{K}Tr(S_k(n)) \tag{15}$$

Obviously, (15) belongs to a class of convex optimization problem. Here we employ a simple iterative water-filling algorithm to compute $S_k(n)$ in (15). The Lagrangian associated with (15) is denoted as

$$\tilde{L}(S_k(n),\varphi_k)=\log\left|I+\sum_{k=1}^{K}H_k^H(n)S_k(n)H_k(n)\right|-\frac{1}{\mu}\sum_{k=1}^{K}Tr(S_k(n))+Tr(\varphi_k S_k(n)) \tag{16}$$

where φ_k are the matrix dual variables associated with the positive definiteness constraints. The Karush-Kuhn-Tucker (KKT) condition can be obtained by setting $\partial\tilde{L}/\partial S_k(n)=0$:

$$H_k(n)\left(I+\sum_{k=1}^{K}H_k^H(n)S_k(n)H_k(n)\right)^{-1}H_k^H(n)=\frac{1}{\mu}I-\varphi_k. \tag{17}$$

This is the water-filling condition for a Gaussian vector channel. In the problem given by (15), μ is given and thus the water-filling level is fixed. The procedure of water-filling can be described as follows [4]

Algorithm 1 iterative Multiuser water-filling with fixed water level

Given a fixed water level μ

While the algorithm is not convergent **do**
  For $k=1$ to K **do**
    Perform multiuser water-filling over { S_k } while fixing $\{S_i\}_{i=1,i\neq k}^{K}$
  end for
end while

The above table presents an algorithm to solve (15) for a given μ. To solve the problem given by (11), we still need to find the subgradient of $g(\mu)$. To the end, substituting (13) into (14) results in:

$$g(\mu)=\min_{S_k(n)\geq 0} \mu R^*+\frac{1}{\mathrm{N}}\sum_{n=1}^{N}\left(\sum_{k=1}^{K}Tr(S_k(n))-\mu\log\left|I+\sum_{k=1}^{K}H_k^H(n)S_k(n)H_k(n)\right|\right). \tag{18}$$

Let $S_k^*(n), k = 1,\cdots,K, n = 1,\cdots,N$ be the optimizing solutions of (18). Then for $\delta \ge 0$, the following expression can be obtained:

$$\begin{aligned} g(\mu) &= \mu R^* + \frac{1}{\mathrm{N}}\sum_{n=1}^{N}(\sum_{k=1}^{K} Tr(S_k^*(n)) \\ &\quad - \log\left|I + \sum_{k=1}^{K} H_k^H(n) S_k^*(n) H_k(n)\right|) \\ &\ge g(\delta) + (\mu - \delta)(R^* - \frac{1}{\mathrm{N}}\sum_{n=1}^{N}\log\left|I + \sum_{k=1}^{K} H_k^H(n) S_k^*(n) H_k(n)\right|) \end{aligned} \quad (19)$$

Thus, $h = (R^* - \frac{1}{N}\sum_{n=1}^{N}\log\left|I + \sum_{k=1}^{K} H_k^H(n) S_k^*(n) H_k(n)\right|)$ is a subgradient of $g(\mu)$. By the bisection linear search method, if $R^* > \frac{1}{N}\sum_{n=1}^{N}\log\left|I + \sum_{k=1}^{K} H_k^H(n) S_k^*(n) H_k(n)\right|$, then increase water-filling level μ. Otherwise decrease μ until the stopping criteria for the bisection method is met.

The proposed algorithm is summarized as follows:

Algorithm 2

1) Initialize $\mu_{\min}, \mu_{\max}$;

2) Set $\mu = (\mu_{\min} + \mu_{\max})/2$;

3) Solve the optimization problem (12) with the water-filling level μ by Algorithm 1;

4) Compute the subgradient h. If $h > 0$, then $\mu_{\min} = \mu$, otherwise $\mu_{\max} = \mu$;

5) If $|\mu_{\max} - \mu_{\min}|$ is less than the given tolerance, stop. Otherwise, goto 2).

As the proposed algorithm in this paper exploits the specific structure of a convex optimization problem and uses the iterative water-filling method, the complexity of the algorithm is linear with K, whereas the complexity of the standard interior point convex optimization algorithm is cubic with K using Newton's method. Using the bisection search method, our algorithm convergents globally. This is true even when we choose arbitrary feasible initial points for μ.

5. NUMERICAL SIMULATION AND ANALYSIS

In the following simulations, show three examples where an overall spectral efficiency of $R^* = 40$nats/Hz is achieved. The base station is equipped with $r = 4$ antennas and each user has $t = 4$ antennas. A block fading channel model is assumed. It is sufficient to choose the number of channel state $N = 50$ [6]. The channel realizations are independently drawn from the matrices with independent zero-mean and unit-variance circularly symmetric complex Gaussian elements.

Fig.1 illustrates the convergence behavior of Algorithm 1 when the water-filling level μ is fixed and $K = 10$. It is observed that the algorithm of multi-user water-filling with fixed μ converges, and the rate of convergence is fast.

Fig.2 illustrates the convergence behavior of Algorithm 2 for a fading Gaussian MIMO BC with 10 users. It is seen that the algorithm converges at about the 10th iteration. Our algorithm thus provides a considerably fast convergent rate.

Fig. 3 illustrates the convergence behavior of Algorithm 2 for a fading Gaussian MIMO BC with 50 users. It is also shown that the algorithm converges at about the 10th iteration. Thus the convergence rate of the proposed algorithm does not appear to be influenced by the number of users. We have also tried to vary the number of users and have observed convergence rate that seems to be independent of the number of users.

6. CONCLUSION

In this paper, an iterative water-filling algorithm is proposed to solve the problem of minimizing of long-term sum power for fading Gaussian MIMO BC with a system throughput constraint. We transform the non-convex downlink problem into a convex uplink problem by the duality between the MIMO MAC and the MIMO BC. Then by the equivalence of the primal problem and Lagrange dual problem, we solve the Lagrange dual problem based on the subgradient and one-dimensional search method, which allows us to solve the primal problem. Theoretical analysis and numerical simulations show that the proposed algorithm provides low complexity and fast convergence rate. Furthermore, the convergence rate is not influenced by the number of users.

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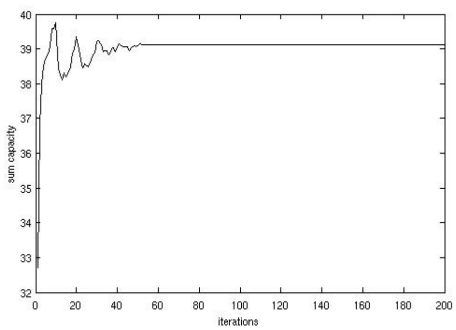

Fig.1 the convergent behavior of Algorithm 1 when μ is fixed

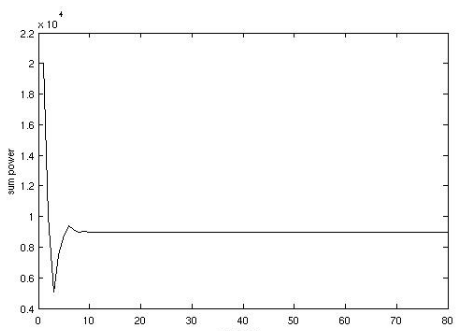

Fig.2 the convergent behavior of Algorithm 2 with 10 users

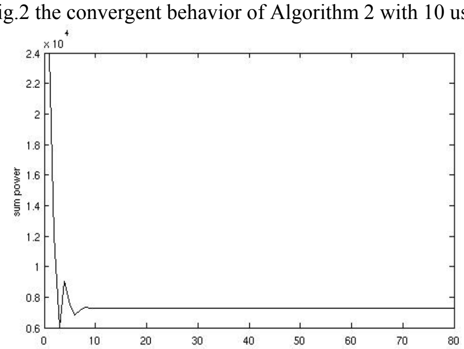

Fig.3 the convergent behavior of Algorithm 2 with 50 users