Reduced Complexity Multiband Multi-Sensor Spectrum Sensing

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***Abstract*—A spectrum sensing problem in which multiple sensors are used to detect an idle period in multiple channels is considered in this paper. By casting the problem using a partially observable Markov decision process (POMDP), a sequential detection scheme that minimizes the expected detection time and false alarm is described. Recent research shows that the POMDP formulation can be applied to spectrum sensing problems that sense multiple channels using only one sensor. This paper shows that this approach can be generalized to systems that incorporate an arbitrary number of sensors. Based on this more general procedure, this paper proposes two sequential detection schemes that exploit the additional sensors to reduce the detection time in the spectrum sensing system while maintaining its false alarm rate under a desired design constraint. The performances of the two detectors are investigated through Monte Carlo simulation.**

I. Introduction and Background

Spectrum sensing, which concerns the problem of characterizing the state of spectrum in terms of its availability, has recently become an important area of research with the promise to alleviate the strain caused by the scarcity of wireless spectrum. Spectrum sensing is naturally framed as a detection problem. Research on this problem is already abundant. Methods based on feature detection to detect different classes of signals are studied for various applications, such as for wireless microphones and analog TV signals [1], [2], or OFDM pilot signals [3]. There is also work on exploring alternative sensing methods such as energy detection [4].

The above publications all treat detector reliability as the most important requirement. Agility, which is determined by the time required to perform the detection, is also an important performance measure, since the time spent sensing a channel is an overhead that should be minimized.

The methods proposed by [1], [2], [3], [4] process a fixed block of observations, which means that their agility is fixed. In [5], agility is addressed through a sequential detection framework. By employing the sequential probability ratio test (SPRT) detector, decisions are produced after a sufficient number of observations is obtained, which is usually less than that of a fixed-sample-size detector for the same reliability of performance. The authors of [6][7] apply quickest detection to spectrum sensing, which is a sequential detection framework that detects for the changes in a signal. In [6][7], it is assumed that the change time, i.e., the time when the change event occurs, is unknown. By this assumption the potential statistical behaviour of the change time is ignored. Alternatively, [8] proposes a Bayesian design for quickest detection based on Shiryaev's problem, where it is assumed that there is a priori knowledge about the probability distribution of the change time.

While the classical change point detection framework studied in [5], [6], [7], [8] is based on detecting one single change event in one random process, [9] re-casts the problem under the framework of partially observable Markov decision processes (POMDP) and extends its solution to observing multiple processes. Through its POMDP formulation, the authors in [9] arrive at a very similar detector structure to that derived for Shiryaev's problem [8]. In the formulation [9], however, it is assumed that the detector may only observe one channel at a time. In practical CR systems, this is an unnecessary constraint as multiple spectrum sensors may be available, where each can be tuned to different spectrum band. The incorporation of more spectrum sensors should enable performance enhancement at moderate cost.

The contribution of this paper is to re-formulate the POMDP in [9] to accommodate multiple sensors, and propose two detector designs that incorporate an arbitrary number of sensors while maintaining the favourable properties of the detectors in [9]. The second detector design is motivated by the need to reduce complexity. Through evaluating the performance via Monte Carlo simulation, this paper studies system performance with varying numbers of sensors.

The paper is organized as follows: Section II describes the formulation and the proposed detectors for the multi-sensor case. Section III investigates the effects of introducing additional sensors, as well as provides a comparison between the two proposed detectors via simulation.

II. System and Methods

A. Problem Statement

Consider a discrete-time system with L channels that each may transition back and forth between occupied and unoccupied states. Among the L channels, a spectrum sensing system is tasked with finding an unoccupied channel. Suppose the spectrum sensing system is equipped with M sensors, so it can freely observe any M out of L channels. Each channel consists of time intervals of either occupied or unoccupied periods, termed busy and idle periods, respectively.

As in [9], it is assumed that the occupants emerge into and exit from any channel according to a Poisson arrival model, which implies that the length of each busy/idle period is modelled randomly using a geometric distribution. Given that a channel is in the busy state, the probability that the channel enters the idle state in the next time interval is represented by p_B; likewise, if a channel is in the idle state, there is a probability, which is denoted by p_I, that the channel switches to the busy state in the next time interval. The probability of a channel being in the idle state without knowledge of the channel's previous state is denoted as λ_o, which represents the fraction of idle time to total time as below,

$$\lambda_o = \frac{m_I}{m_I + m_B} \tag{1}$$

where $m_I \equiv 1/p_I$ and $m_B \equiv 1/p_B$ are the average busy and idle times.

It is assumed that all L channels transit between the busy and idle states with the same p_B and p_I. Also, each busy/idle period is assumed to be statistically independent of the others.

Using observations from M sensors out of the L channels, the detector makes a decision between whether to continue to observe or to stop to declare that an idle channel is found. Furthermore, if the system decides to continue to observation process, it must also make an decision on which channels to observe.

Let T_{declare} be the random elapsed time from when the detection process begins to when the detection process declares a discovery of an idle time period. The false alarm rate of the detector, denoted as P_{FA}, is the probability of declaring a channel idle when the state the channel is busy.

The goal is to design a detection scheme that achieves the minimum expected detection time while maintaining its false alarm rate under a given constraint, α. That is,

$$\begin{aligned} &\min E\left[T_{\text{declare}}\right] \\ &\text{subject to } \text{P}_{FA} \leq \alpha. \end{aligned} \tag{2}$$

B. POMDP Formulation for Multi-Sensor Scenario

A POMDP is utilized to model the dynamics of the channels. In [9], it is pointed out that the POMDP approach differs from Shiryaev's problem [8] in that the POMDP models the dynamics of multiple change points, i.e., a channel may switch states following its change point, which is more realistic for spectrum sensing.

A POMDP formulation for the case of $M = 1$ is provided in [9], i.e., the case where a detector is only able to observe one channel at one time. In the following is a formulation that generalizes the POMDP in [9] to the case of multiple sensors, e.g., to $M > 1$.

1) State Space: The channels underlying the system each have independent states. These states are defined as $\{Z_1(t), ..., Z_L(t)\}$, where $Z_i(t) \in \{0, 1\}$ denotes the state of the i^{th} channel. State 0 and State 1 represent busy and idle states, respectively. In addition, there is an absorption state that is shared by all the channels that define the end of the detection process.

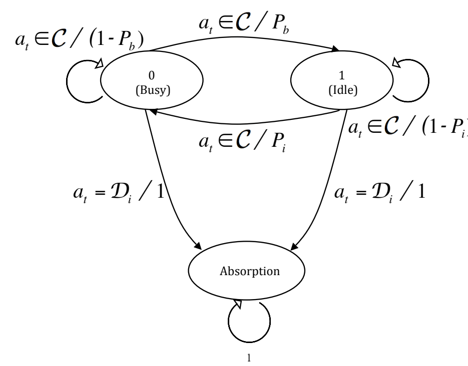

Fig. 1. The transition dynamics for the the i^{th} channel, $1 \leq i \leq M$. A similar diagram would apply to all other channels. When the action taken at t belongs to set $\mathcal{C}$, the channel will switch between the busy and idle states.

2) Action Space: Let a_t be the action taken at time t. The action space consists of all the possible actions for a_t, and is separated into two sets. The first set, denoted as $\mathcal{C}$, contains the action options to observe M out of L channels and therefore includes $\binom{L}{M}$ distinct options. An action within $\mathcal{C}$, denoted by $(C_{k_1}, C_{k_2}, ..., C_{k_M})$, represents continuing to observe channels with indices $k_1, ..., k_M$. The second set, denoted as $\mathcal{D}$,is related to actions of declaring a channel idle. Within $\mathcal{D}$, each action is denoted as (D_i) and represents declaring the i^{th} channel idle, $1 \leq i \leq L$.

3) State Transition: The transition dynamics are modelled in terms of p_B and p_I. Fig. 1 shows the state transition diagram for the i^{th} channel, and since the dynamics of all the channels are assumed to be independent and identically distributed, the same diagram applies for all channels.

4) Observation Model: If in the previous time instant an action belonging to set $\mathcal{C}$ is taken, M samples from M channels are observed from the process. When the total number of channels $L > M$, the detector observes a subset of the channels. The channels that fall within this observable subset are selected by the action taken at time $t - 1$. The pdf that the sample is drawn from depends on the state of the selected channel at time t. If the channel is busy, the observation is drawn from the pdf of f_0. On the other hand, if the channel is idle, the observation is drawn from pdf of f_1. When the action to declare a channel idle is taken in time t, the process enters the absorption state and stops generating further observations.

5) Cost Model: The cost model defines the cost incurred by each action taken. Let $R_{a_t}(t)$ be the cost incurred at time t under the action a_t. Similar to [9], if the action is to continue, i.e., $a_t \in \mathcal{C}$, unit cost is incurred, i.e., $R_{a_t}(t) = 1$. On the other hand, if the action is to stop and declare the i^{th} channel idle when the channel is in fact busy, a cost of $\gamma > 0$ is incurred to reflect the cost of the false alarm, i.e., $R_{a_t}(t) = \gamma, \gamma > 1$.

6) Sufficient Statistic: The detector does not directly observe the state of the channel. Instead through the POMDP model and the observations, sufficient statistics evolve to improve the observer's belief about the state of the process. The sufficient statistic for the process is a belief vector of size L: $\mathbf{\Lambda}(t) \equiv [\lambda_1(t), ..., \lambda_L(t)]$, where $\lambda_i(t)$ is the probability that the i^{th} channel is in an idle state at time t given all the past observations up to time t. In [9], an update function for the elements in a belief vector is derived for the case of $M = 1$. The following expression is the modified update function for $M > 1$:

When $a_{t-1} \in \mathcal{C}$,

$$\lambda_i(t) = \begin{cases} \widehat{\mathcal{T}}(\lambda_i(t-1), x) & C_i \in a_{t-1}, X_i(t) = x \\ \widetilde{\mathcal{T}}(\lambda_i(t-1)) & C_i \notin a_{t-1}. \end{cases} \quad (3)$$

The update equation for $\lambda_i(t)$ evolves recursively and differs depending on whether the i^{th} channel is being observed. From [9], the update function utilizing the observation x is

$$\widehat{\mathcal{T}}(\lambda, x) = \frac{(\lambda \overline{p}_I + \overline{\lambda} p_B) f_1(x)}{(\lambda \overline{p}_I + \overline{\lambda} p_B) f_1(x) + (\lambda p_I + \overline{\lambda}\, \overline{p}_B) f_0(x)} \quad (4)$$

where the operator $\overline{(\cdot)}$ is defined as $1 - (\cdot)$, and p_I and p_B are given by the prior geometric pdf.

When updating λ_i for which the i^{th} channel is not being observed, the update function on the transition dynamic evolves solely based on a priori information and is as follows [9]:

$$\widetilde{\mathcal{T}}(\lambda) = \lambda \overline{p}_I + \overline{\lambda} p_B. \quad (5)$$

In the context of the POMDP formulation, the detector is a function that maps the belief vector to a decision per time slot basis. Such a function, denoted as $\pi(\mathbf{\Lambda}(t)) = a_t$, is the policy obtained, which leads to the following the POMDP formulation that solves (2). For given M and L,

$$\pi^* = \arg\min_{\pi} E\left[\sum_{t=0}^{\infty} R_{\pi(\mathbf{\Lambda}(t))} | \mathbf{\Lambda}(0)\right] \quad (6)$$

where $R_{\pi(\mathbf{\Lambda}(t))} = R_{a_t}(t)|_{a_t = \pi(\mathbf{\Lambda}(t))}$ and $\mathbf{\Lambda}(0) = [\lambda_0, \lambda_0, ..., \lambda_0]$ is the a priori belief vector value.

C. Multiband Multi-Sensor Spectrum Sensing Detector

This section proposes a new multiband multi-sensor spectrum sensing detector (MMSSD) that has the following properties: (1) performs quickest detection over multiple channels (multiband), (2) simultaneously senses multiple channels (multi-sensor), and (3) tracks the complete belief vector. The last property is the result of the detector updating the belief values for all the channels.

The MMSSD has the following structure for the case of general M and L:

$$\pi_{\text{MMSSD}}(\mathbf{\Lambda}(t)) = \begin{cases} D_i, & \text{if } i = \arg\max\limits_{1 \le j \le L}(\mathbf{\Lambda}(t)) \\ & \text{and } \lambda_i \ge \eta_d(\mathbf{\Lambda}^-(t)) \\ (C_{k_1}, ..., C_{k_M}), & \text{Otherwise} \end{cases} \quad (7)$$

where $k_1, ..., k_M$ denote the indices of the L channels that have the M largest belief values, $\mathbf{\Lambda}(t)$ is updated using (3) for $t = 1, 2, ...$, and $\mathbf{\Lambda}^-(t)$ denotes the set of belief values for the $L - M$ unobserved channels at time t.

In the policy (7), the detector chooses the M channels with the highest belief values to observe the next time slot. The detector continues to observe and update the belief values until one belief value crosses the threshold, $\eta_d(\mathbf{\Lambda}^-(t))$.

To develop a detector structure that minimizes (6) for arbitrary M and L, where $M \le L$, a value function is defined to represent the optimal expected cost as

$$V(\lambda_1, ..., \lambda_L) = \min\{V_{\overline{C}_1}(\lambda_1, ..., \lambda_L), V_{\overline{C}_2}(\lambda_1, ..., \lambda_L), ..., V_{\overline{C}_{|\mathcal{C}|}}(\lambda_1, ..., \lambda_L), V_{D_1}(\lambda_1, ..., \lambda_L), V_{D_2}(\lambda_1, ..., \lambda_L), ..., V_{D_L}(\lambda_1, \lambda_2, \lambda_3)\}, \quad (8)$$

where $V_{\overline{C}_j}$, $1 \le j \le |\mathcal{C}|$, denotes the value function for the action $(C_{k_1}, C_{k_2}, ..., C_{k_M})$. There exists a total of $|\mathcal{C}|$ $V_{\overline{C}_j}$ functions since all possible combinations of $C_{k_1}, ..., C_{k_M}$ need to be included. V_{D_j}, $1 \le j \le L$, are the value functions of declaring the j^{th} channel as idle. The action-specific value functions are defined as follows:

$$V_{\overline{C}_j}(\lambda_1, ..., \lambda_L) = 1 + \int ... \int P(x_{k_1}, ..., x_{k_M}; \lambda_{k_1}, ..., \lambda_{k_M}) V(\lambda_1^{'}, ..., \lambda_L^{'}) dx_{k_1} ... dx_{k_M}, \quad (9)$$

$$V_{D_i}(\lambda_1, \lambda_2, \lambda_3) = (1 - \lambda_i)\gamma \quad (10)$$

where $P(x_{k_1}, ..., x_{k_M}; \lambda_{k_1}, ..., \lambda_{k_M})$ is the joint pdf of observations of the channels $k_1, ...k_M$, given the channels' belief values, $\lambda_{k_1} ... \lambda_{k_M}$. The arguments of the $V(\cdot)$ function, $\lambda_j^{'}$, $1 \le j \le L$, are the transformed belief values defined as

$$\lambda_j^{'} = \begin{cases} \widehat{\mathcal{T}}(\lambda_j, x_j) & j \in \{k_1, ..., k_M\} \\ \widetilde{\mathcal{T}}(\lambda_j) & j \notin \{k_1, ..., k_M\} \end{cases}, \quad (11)$$

where the functions $\widehat{\mathcal{T}}(\cdot)$ and $\widetilde{\mathcal{T}}(\cdot)$ are defined in (4) and (5), respectively.

To describe the properties of the defined value functions, this paper proposes Lemma 1 that extends Lemma 2 from [9] to general values M and L:

Lemma 1:

L1.1: When $p_B + p_I \le 1$ and $P(x_{k_1}, ..., x_{k_M}; \lambda_{k_1}, ..., \lambda_{k_M}) = P(x_{k_1}; \lambda_{k_1}) P(x_{k_2}; \lambda_{k_2}) ... P(x_{k_M}; \lambda_{k_M})$, the value functions $V_{A_j}(\lambda_1, ...\lambda_L)$ are concave and monotonically decreasing with λ_j, for $1 \le j \le L$.

L1.2: $V_{A_j}(\lambda_1, ...\lambda_L)$ are symmetric with respect to the planes $\{\lambda_i = \lambda_j; 1 \le i \le L, 1 \le j \le L, i \ne j\}$.

L1.3: $V_{D_i}(\lambda_1, ...\lambda_L)$ is linearly decreasing with λ_i, for $1 \le i \le L$.

Due to space limitations, a detailed proof for Lemma 1 is omitted. However, using the assumption

$$P(x_i, x_j; \lambda_i, \lambda_j) = P(x_i; \lambda_i) P(x_j; \lambda_j), \ \ \forall i, j; i \ne j \quad (12)$$

and the value functions defined in (8)-(10), Lemma 1 can be proved straightforwardly using the procedures provided in Appendix B of [9].

The condition (12) arises if channels are mutually independent so that the joint pdf of observations from different channels is separable into products of marginal pdf's of individual observations.

Having shown Lemma 1, Theorem 2 from [9] then follows as such:

Theorem 2 [9]: The optimal structure $\pi^*(\mathbf{\Lambda}(t))$ under the belief vector $\mathbf{\Lambda}(t)$ is the following:

$$\pi^*(\mathbf{\Lambda}(t)) = \begin{cases} D_i, & \text{if } i = \underset{1\le j\le L}{\arg\max}(\mathbf{\Lambda}(t)) \\ & \text{and } \lambda_i \ge \eta_d(\mathbf{\Lambda}^-(t)) \\ \mathcal{C}, & \text{otherwise} \end{cases} \tag{13}$$

where $\mathcal{C}$ is the set of actions of continuing to observe channels, as defined in Section II-B2, and $\eta_d(\mathbf{\Lambda}^-(t))$ is the function that maps $\mathbf{\Lambda}^-(t)$ to a threshold value in $[0, 1]$.

Identifying how the values of $\lambda_1, ..., \lambda_L$ optimally map to the action $(C_{k_1}, ..., C_{k_M})$ requires the computation of the value functions (9)(10), which is a challenging task due to the nested integrations involved. Due to the complexity, the optimal policy (13) is difficult to implement. Therefore, MMSSD (7), which selects the action $(C_{k_1}, ..., C_{k_M})$ by the ranking of $\lambda_1, ..., \lambda_L$, is proposed as a suboptimal low-complexity version of the policy (13).

The threshold design for (7) is also an challenging task because the function $\eta_d(\mathbf{\Lambda}^-(t))$ is difficult to compute. In addition, the detector must satisfied $P_{FA} \le \alpha$. An alternate threshold design method is proposed here for (7), where the threshold is assumed to be a constant, η_d, rather than a function.

Let $T_{\text{stop}}(\eta_d)$ denote the random detection time of the MMSSD detector employing the threshold η_d, i.e.,

$$T_{\text{stop}}(\eta_d) \equiv \inf\{n \ge 1 : \lambda_{\max}(n) \ge \eta_d\} \tag{14}$$

where $\lambda_{\max}(n) \equiv \max(\lambda_1(n), ..., \lambda_L(n))$. By definition, $1 - \lambda_{\max}(T_{\text{stop}})$ is the a posteriori probability that the channel declared to be idle is in a busy state, given the observation. Therefore, the false alarm rate of the MMSSD detector is

$$P_{FA} = E_{\lambda_o}[1 - \lambda_{\max}(T_{\text{stop}})] \tag{15}$$

where the expectation operator $E_{\lambda_o}[\cdot]$ is the expected value over all the possible random observation sequences with initial belief value equal to λ_o. From (14) and (15) , an upper bound on P_{FA} can be derived using $\lambda_{\max}(T_{stop}) \ge \eta_d$ to obtain $P_{FA} = 1 - E_{\lambda_o}[\lambda_{\max}(T_{stop})] \le 1 - \eta_d$. The inequality suggests that a detector threshold that satisfied the false alarm constraint α is $\eta_d = 1 - \alpha$. This threshold design may be conservative if $\lambda_{\max}(T_{stop})$ overshoots the threshold η_d significantly in the detection process. However, the upper bound on P_{FA} becomes tighter as α tends to zero since

$$0 \le P_{FA} \le 1 - \eta_d = \alpha. \tag{16}$$

D. Reduced Complexity Multiband Multi-Sensor Spectrum Sensing Detector

The detector in (7) carries a high complexity cost, i.e., the cost of memory to store the belief vector that is of length L and the cost of computation to rank all the belief values in order to find the top M elements. To address this concern, we seek a way to reduce MMSSD's complexity.

Inspired by the detector proposed by [9] for the infinite regime problem, we consider the case where L is infinitely larger than M. Let $j, 1 \le j \le L - M$, be the index of an unobserved channel; T^j_{unobs} be the random elapsed time during which j^{th} channel is not observed; and the $r_j(t)$ is the ranking of jth channel at the time t, in which a lower ranking means having a higher belief value. Then

$$T^j_{\text{unobs}} = \sum_{t=1}^{\infty} \{1\}_{(r_j(t)>M)} \tag{17}$$

where $\{1\}_{(r_j(t)>M)}$ is an indicator function for the event $r_j(t) > M$. As $L - M$ approaches infinity, Prob$(r_j(t) > M) \to 1$ because the probability that the belief value for jth channel becomes the top ranked M elements becomes vanishingly small. As a result, T^j_{unobs} approaches infinity.

While a channel is not being observed, the corresponding belief value is updated recursively via (5). After N time slots pass, the resulting belief value is expressed as a function of an initial belief value as

$$\widetilde{\mathcal{T}}^N(\lambda) = p_B(1 + \overline{p}_{I,B} + \overline{p}^2_{I,B} + ... + \overline{p}^N_{I,B}\lambda), \tag{18}$$

where $\overline{p}_{I,B} \equiv (1 - p_I - p_B)$. As $T^j_{\text{unobs}} \to \infty$,

$$\lim_{N\to\infty} \widetilde{\mathcal{T}}^N(\lambda) = \frac{m_I}{m_B + m_I} = \lambda_o, \tag{19}$$

with the condition that $p_I + p_B \ge 0$.

The above results suggest that if the set of unobserved channels is infinitely large, the belief values of all the channels approach λ_o. Therefore, the procedure to find the top ranked M belief values, as required by MMSSD, is reduced to comparing the belief value of the observed channels to λ_o: If a belief value of a channel is greater than λ_o, the detector continues to observe the channel. Otherwise, the detector stops observing the channel and starts sensing another channel from the unobserved set instead. Based on this idea, this paper proposes the following variant of MMSSD with reduced complexity, called RC-MMSSD:

The detector maintains a belief vector of size M, $[\lambda_1(t), ..., \lambda_M(t)]$, where $\lambda_j(t), 1 \le j \le M$, is the belief value for the l^{th}_j channel, and $l_j \in (1, 2, ..., L)$. With every iteration, the detector updates each belief value using the observation from its corresponding observed channels via (4). Based on the updated belief vector, the detector decides on the action at time t based on the following policy:

Let Ω be the set of channels that are not under observation at time t.

for $j = 1$ to M **do**
  if $\lambda_j \ge \eta_d$ **then**

{Declare channel l_j as idle}
else if $\lambda_j < \lambda_o$ **then**
{Switch out of channel l_j}
$l_j \leftarrow$ index of a new channel selected from Ω
$\lambda_j(t)$ to λ_o {Resetting $\lambda_j(t)$}
Update Ω accordingly
else
{Continue to observe channel l_j}
end if
end for

RC-MMSSD has removed the need to store and update the belief values for all L channels. In addition, the decision to choose which channels to observe depends only on M belief values, rather than on a ranking of L belief values in the case of (7), which is costly to compute.

In the asymptotic case where $L - M \to \infty$, MMSSD and RC-MMSSD behave the same because all the belief values in the unobserved pool converge to λ_o. In practice, (18) shows that the convergence to λ_o can occur rather quickly as $p_I + p_B \to 1$ and hence RC-MMSSD can be applied to problems with a moderately-sized unobserved channel pool.

1) Problem with False Alarm Rate for Small $L - M$: To demonstrate such problem, consider a scenario where $L - M \to 0$ and RC-MMSSD is used. An event occurs in the following sequence: an jth channel has been swapped out due to $\lambda_j(t) < \lambda_o$; after a short T^j_{unobs} has lapsed, the jth channel is called upon to be sensed again with its belief value assumed to be λ_o; and eventually the jth channel is declared idle by RC-MMSSD.

Let $\delta\lambda(t)$ be the error at time t between the assumed belief value and the true belief value

$$\delta\lambda(t) = \lambda_o - \lambda_j(t). \tag{20}$$

Based on (18), a small T^j_{unobs} implies that $\delta\lambda$ is a finite positive quantity. The false alarm rate, using (20), can be re-written as

$$1 - E_{\lambda_o}[\lambda_{\max}^{T_{\text{stop}}}] = P_{FA,\lambda_j(t)} - \Delta P_{FA} \tag{21}$$

where $P_{FA,\lambda_j(t)}$ is the false alarm based on the true belief value of the jth channel and is therefore the true false alarm rate of the detector. ΔP_{FA} is false alarm caused by the error $\delta\lambda(t)$. Note that ΔP_{FA} is a positive value and approaches 0 as $\delta\lambda(t) \to 0$.

Using the same threshold design method as MMSSD, by setting $\eta_d = 1 - \alpha$, the inequality below then follows:

$$P_{FA,\lambda_j(t)} \leq 1 - \alpha + \Delta P_{FA}. \tag{22}$$

As is evident in (22), the design threshold $1-\alpha$ is no longer a sufficient upper bound for the false alarm rate of the system.

III. Simulation Results

The performance of the proposed detectors MMSSD and RC-MMSSD are evaluated using Monte Carlo simulation. The marginal distributions for both observation states, f_0 and f_1, are both zero-mean Gaussian distributions with variances σ_0^2 and σ_1^2, respectively. In a physical sense, the variance σ_0^2 in the busy state is the transmission power of the channel occupant, and the variance σ_1^2 in the idle state is the power of the noise of an unoccupied channel. The SNR is termed as the amount of the shift and is determined by $10\log(\sigma_0^2 - \sigma_1^2)$ (in dB). To specify the busy/idle transitions of the channels statistically, the average busy time, m_B and the fraction of the idle period, λ_o are defined from which the other related parameters can be derived (See (1)). The value α is chosen to be 0.1 for all the scenarios described.

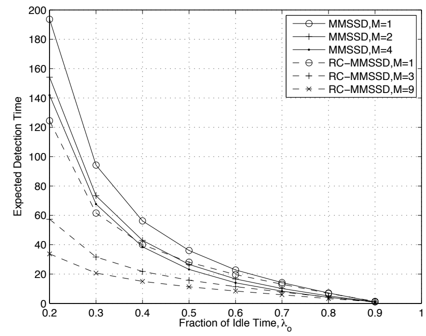

Fig. 2. The expected detection times of MMSSD and RC-MMSSD with different M's as the fraction of idle time varies. ($m_B = 300, SNR = 0dB, \eta_d = 0.9$, L=4 (MMSSD), and L=100 (RC-MMSSD)). We have simulated the case of M=3 for MMSSD, which has very similar result as that of M=4, and the cases of M=5 and 7 for RC-MMSSD, which performs similarly to M=9. These cases are not displayed in order to make the plot more legible.

A. Effects of Adding Sensors

Fig. 2 compares the expected detection time of MMSSD and RC-MMMSD as the the prior probability of an idle channel, λ_o, varies. The gain in performance is more significant when the idle periods are more rare. As idle periods become more probable, the benefit of adding extra sensors vanishes. By adding one sensor from $M = 1$ to $M = 2$ in the MMSSD system, the expected detection time converges quickly to that of $M = 4$. With $L = 4$, the MMSSD with $M = 4$ is effectively a full sensing detector. Therefore, it is fair to claim that the performance of a full-sensing detector, in terms of expected detection time, can be in large part achieved by deploying fewer than L sensors. A similar effect is observed in the RC-MMSSD system, where the most significant reduction in detection time comes from the addition of the first few sensors. Fig. 3 shows the comparison of the false alarm rates for both detectors. It is clear the false alarm constraint α has been satisfied. No single detector is able to outperform others in false alarm rates across all values of λ_o and for lower λ_o values, the false alarms rates are very similar and quite invariant to λ_o.

B. MMSSD versus RC-MMSSD

The system performances of both proposed detectors are compared in Figs. 4 and 5 over different values of L, at SNR levels of 10dB and 0dB. Fig. 5 shows the impact on reliability when $L - M$ is small. For a fixed M and $L \leq 30$, the false alarm rates of RC-MMSSD at both high and low SNRs

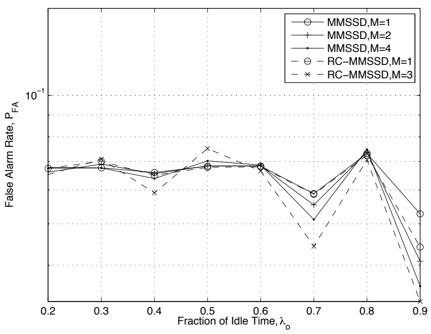

Fig. 3. The false alarm rate of MMSSD and and RC-MMSSD with different M's as the fraction of idle time varies. ($m_B = 300, SNR = 0dB, \eta_d = 0.9$, L=4 (MMSSD), and L=100 (RC-MMSSD)). Again, the cases of M=3 for MMSSD and M=5 and 7 for RC-MMSSD are not displayed in order to make the plot more legible. These cases are simulated and the performance of MMSSD for M=3 is similar to that of M=4, and the performance of RC-MMSSD for M=5 and 7 are similar to that of M=9.

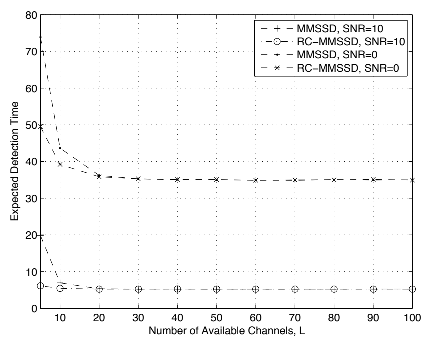

Fig. 4. The expected detection times of MMSSD and RC-MMSSD with respect to the number of available channels, for both SNR=0dB and SNR=10dB ($\lambda_o = 0.25, m_B = 300, \eta_d = 0.9, M = 4$).

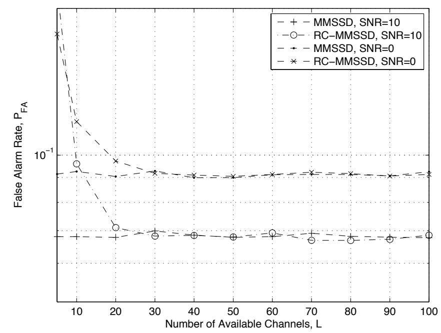

Fig. 5. The false alarm rate of MMSSD and RC-MMSSD with respect to the number of available channels, for SNR=0dB and SNR=10dB. The false alarm constraint is 0.1. ($\lambda_o = 0.25, m_B = 300, \eta_d = 0.9, M = 4$).

violate the false alarm constraint α. This observation validates the earlier analysis on the impact of having a small pool of unobserved channels on the false alarm rate. For $L > 30$, the false alarm rates of the two detectors are similar. Over this range, Fig. 4 shows the expected detection times between MMSSD and RC-MMSSD are identical. This result indicates that when the number of available channels greatly outnumbers the number of sensors, MMSSD and RC-MMSSD have similar performance despite MMSSD's extra complexity, This result is expected because it has been shown in Section II-D that MMSSD behaves similarly to RC-MMSSD as the pool of unobserved channels grows large.

IV. Conclusion

In this paper, it is shown that the POMDP formulation and the optimal multiple process detection structure proposed in [9] can be generalized to incorporate an arbitrary number of sensors. Based on this result, a new detector, MMSSD, is proposed as the low-complexity and suboptimal form of the optimal detector. Our simulation results show that the the detector derives most of its utility from the first few added sensors and that the performance of a detector with a large bank of sensors can be achieved with a relatively small number of sensors. This paper also addresses the high complexity of implementing MMSSD when the number of available channels grows large. To address this issue, a second detector, RC-MMSSD, is proposed. RC-MMSSD reduces both the storage requirement and the computational complexity in MMSSD. Through simulation, it is shown that the system performance of RC-MMSSD is comparable to that of MMSSD. However, a caveat with RC-MMSSD is that the number of channels must greatly outnumber the number of sensors. If such a condition is not met, MMSSD has to be used. In spectrum sensing, where there is likely to be few sensors available, one can make suitable choice between MMSSD and RC-MMSSD, depending on the number of available channels, to achieve both system agility and reliability.

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