

# Unequal Error Protection Rateless Coding Design for Multimedia Multicasting

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**Abstract**—We study the design and optimization of unequal error protection (UEP) rateless codes for scalable multimedia multicasting. We formulate two general problems of optimizing UEP rateless code for multimedia multicasting to heterogeneous users: one focusing on providing guaranteed quality of service (QoS) and the other focusing on providing best-effort QoS. A random interleaved rateless encoder design is proposed. Unlike previous designs, existing standardized raptor codes can be directly applied to this design without degrading performance. For each problem, optimal layer selection parameters are obtained either analytically or numerically. Numerical results demonstrate that the proposed optimized random interleaved UEP rateless code outperforms non-optimized rateless codes and recently proposed UEP rateless codes.

## I. INTRODUCTION

Fountain codes [1], such as Luby Transform (LT) codes [2] and raptor codes [3], are recently proposed efficient erasure error-control codes. Fountain codes are rateless in the sense that the transmitter can generate, as needed, a potentially infinite number of encoded symbols. Raptor codes have been used as application layer forward error correction (FEC) codes in the third generation partnership program (3GPP) Multimedia Broadcast/Multicast Services (MBMS) standard [4]. However, the original design of raptor codes, despite being highly efficient for broadcasting bulk data, has very poor progressive decoding performance. On the other hand, multimedia content often has a scalable structure in which some source layers have higher priorities than others. Therefore, an efficient fountain code designed for multimedia streaming applications should provide unequal error protection (UEP) of different source layers.

A few techniques have been proposed for UEP design of rateless codes in the literature. In [5], an UEP LT code is proposed by non-uniformly choosing message bits to encode each symbol. In [6] and [7], expanding window fountain (EWF) codes have been applied for multicasting data. The idea of EWF codes is to encode each symbol based on only source symbols inside a window. The windows are pre-designed in an overlapping and expanding manner such that any larger window contains all the symbols inside a smaller window. However, the designs in [5] and [6] use a large single code which alter the overall degree distribution and therefore change code behavior significantly. It is well known that both performance and complexity are sensitive to the choices of degree distribution. Therefore, without re-optimizing the degree distributions, the above designs may worsen code behavior.

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In [8], we propose a random interleaved UEP rateless code design which encodes and decodes different layers separately. The main advantage of the proposed UEP design compared to that in [5] and [6] is that it allows direct application of existing high performance raptor codes, such as that used in the 3GPP standard [4]. Some initial results have been provided in optimizing rateless code allocation parameters to minimize the required transmission overhead subject to providing users with QoS guarantees. In this paper, we address the following important issues that have not been considered in [8]: first, we provide a detailed proof of the convexity of the problem as well as how to use KKT optimality conditions to verify optimality of the solution. Second, we derive the frame error rate performance of a more general channel model where the number of received symbols are randomized. Third, we provide new comparisons of the proposed UEP scheme with EWF codes when different degree distributions are applied to different layers/windows to make a more detailed break down of the performance gain in terms of different code structures, degree distributions and decoders. Finally, we also investigate a different scenario where the objective is to optimize the proposed UEP rateless code to provide best-effort QoS for a given transmission overhead and provide numerical results.

The rest of the paper is organized as follows: Section II describes the system setup and the proposed UEP code design; Section III presents the problem formulation for the guaranteed QoS scenario and the solution of the problem; Section IV discusses the problem formulation for the best-effort QoS scenario; Section V provides analytical and numerical results and comparisons to other UEP schemes.

## II. SYSTEM SETUP AND PROPOSED DESIGN

In the system under consideration (Fig. 1), a multimedia server first compresses multimedia content using a pre-defined source coder and then encodes the source information using a UEP raptor code. The encoded symbols are multicast over a wireless lossy packet network. Let  $K$  represent the number of input symbols in a raptor code source block. The server transmits  $(1 + \varepsilon)K$  encoded symbols to end users in each block before moving to the next source block, where  $\varepsilon$  is the transmission overhead. Based on the importance of each source symbol, the raptor code source block is divided into  $L$  layers, where layer 1 contains the most important bits and layer  $L$  contains the least important bits. Let  $S_l$  represent the number of bits in layer  $l$  with  $\sum_{l=1}^L S_l = K$ . The users that subscribe to the multimedia streaming services are categorized into  $J$  different classes. For class  $j$  users, the channel can be modeled by a memoryless erasure channel with erasure

probability  $1 - \delta_j$ . The reception capability  $\delta_j$  is also the average proportion of symbols that the receiver can successfully receive compared to the number of transmitted symbols. Without loss of generality, we assume  $\delta_1 \leq \delta_2 \leq \dots \leq \delta_J$ .

The proposed random interleaved UEP rateless coding works as follows: for each raptor encoded symbol  $y$ , a layer number  $l$  is chosen according to probability  $\rho_l$  where  $\rho_l, l = 1, 2, \dots, L$ , are a set of probabilities that satisfy  $\sum_{l=1}^L \rho_l = 1$ . Then, the encoded output symbol is generated by a raptor encoder with code dimension  $S_l$ , degree distribution  $\Omega_l(x)$ , precode  $C_l$  and input symbols limited to layer  $l$ . Therefore, the overall encoded data stream is an interleaved stream of raptor encoded symbols from each layer (Fig. 1).

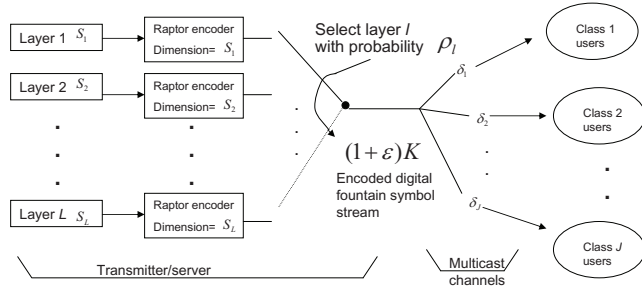


Fig. 1. The proposed random interleaved UEP raptor coder

### III. PROBLEM FORMULATION FOR MINIMIZING TRANSMISSION OVERHEAD

The objective of the UEP raptor code design is to provide different levels of QoS guarantees according to users' requirements while minimizing transmission overhead. The optimization of the proposed UEP raptor code design can be summarized as the following problem:

**Problem 1.0:**

$$\min_{\rho_1, \dots, \rho_L} \quad \varepsilon \quad (1)$$

$$\text{s.t. } \text{Prob}(PSNR_j \geq \gamma_j) \geq P_j, \quad j = 1, 2, \dots, L, \quad (2)$$

where  $PSNR_j$  represents the peak signal-to-noise ratio (PSNR) of the successfully recovered source data of a class  $j$  user,  $\gamma_j$  denotes a threshold and target outage probabilities  $1 - P_j$  ( $0 < P_j < 1, j = 1, 2, \dots, J$ ) are constants.

It has been shown in [8] that, given the  $PSNR$  (or an alternate fidelity measure) function of a scalable source, the above general problem can be transformed into an equivalent problem with a one to one mapping between the new  $L$  layers and new  $J$  classes with  $L = J$ . For users in class  $j$ , the probability that  $PSNR_j \geq \gamma_j$  is equal to the probability that the raptor decoder is able to decode layers 1 to  $j$ . Therefore, the QoS requirements are simplified to:

$$\prod_{l=1}^j (1 - P_e(l, j)) \geq P_j \quad j = 1, 2, \dots, L \quad (3)$$

where  $P_e(l, j)$  represents the frame error probability that the decoder of a class  $j$  user fails to fully decode layer  $l$ .

#### A. Frame error rate evaluation

Raptor codes used in the 3GPP standard, known as standardized raptor codes, can be directly applied to the raptor encoded blocks of our proposed layered design [4] (Annex B). When standardized raptor codes are used, for  $k > 200$ , the probability that the receiver fails to fully recover  $k$  source symbols after  $m$  symbols are successfully received can be well modeled by the empirically determined equation [10],

$$P_e^r(m, k) = \begin{cases} 1 & \text{if } m \leq k \\ a \times b^{m-k} & \text{if } m > k \end{cases} \quad (4)$$

where  $a$  and  $b$  are constants given by  $a = 0.85, b = 0.567$ .

To calculate the frame error probability in the QoS constraints of Eq. (3), let  $n, m, k, 1-p$  represent the total number of transmitted symbols, the number of received symbols, the number of information symbols and the erasure rate for each code block, respectively. Then  $m$  is a Binomial random variable with probability density function (PDF)  $\text{Prob}(m = x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x}$ . Therefore, the probability of successfully decoding the whole code frame is equal to

$$\begin{aligned} 1 - P_e^f(n, k, p) &= \sum_{x=k}^n (1 - ab^{x-k}) P(m = x) \\ &= \sum_{x=k}^n (1 - ab^{x-k}) \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \end{aligned} \quad (5)$$

$$(6)$$

Since  $n$  is large, the above equation can be computationally complex. Let  $h = \min(k + 8, n)$ , we can approximate the above equation while significantly reducing computational complexity via

$$\begin{aligned} 1 - P_e^f(n, k, p) &\geq \sum_{x=k}^h (1 - ab^{x-k}) \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \\ &+ (1 - ab^8) \sum_{x=h}^n \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \\ &\approx \sum_{x=k}^h (1 - ab^{x-k}) \frac{1}{\sqrt{2\pi np(1-p)}} \exp \frac{-(x-np)^2}{2np(1-p)} \\ &+ (1 - ab^8) \left( Q\left(\frac{h-np}{\sqrt{np(1-p)}}\right) - Q\left(\frac{n-np}{\sqrt{np(1-p)}}\right) \right) \end{aligned} \quad (7)$$

where  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-x^2/2) dx$  is the Q-function. The difference between the two sides of the inequality in the first step is minimal because when  $x > k + 8$ , the outage probability  $ab^8 < 0.01$  is very small and can be ignored. The approximations made in the second step use the Normal Approximation (known also as DeMoivre-Laplace Theorem), which are also very accurate for large  $n$ . The probability that the decoder of a class  $j$  user fails to fully decode layer  $l$   $P_e(l, j)$  in (3) can be evaluated as  $P_e^f(n, k, p)$  of (7) with the number of transmitted symbols  $n = t_l = (1 + \varepsilon)K\rho_l$ , the code dimension  $k = S_l$  and  $p = \delta_j$ .

#### B. Simplified channel model and convex analysis

In the following, a simplified channel model is considered which assumes that the number of received symbols of class  $j$  users does not change and equals  $\delta_j$  times the number of transmitted symbols. Later we show numerically that the

difference created by this model is insignificant for Problem 1.0. We now show that by using this assumption, the problem can be transformed to a convex optimization problem which is much easier to solve. This simplified model is also the same channel model assumed by the EWF code in [7]. For the above reasons, this assumption is used for the rest of the paper unless otherwise specified. Based on this assumption, if standardized raptor codes are used, the frame error rate  $P_e(l, j)$  in (3) can be evaluated as  $P_e^r(m, k)$  of (4) with  $m = t_l \delta_j = (1 + \varepsilon) K \rho_l \delta_j$  and  $k = S_l$ . In [8], we have shown that Problem 1.0 can be transformed to the following problem.

**Problem 1.1:**

$$\min_{t_1, \dots, t_L} \sum_{i=1}^L t_i \quad (8)$$

such that

$$-\sum_{l=1}^j \log[1 - c_l \alpha_l^{t_l}] + \log P_j \leq 0, \quad j = 1, 2, \dots, L, \quad (9)$$

where  $c_l = a \times b^{-S_l}$  and  $\alpha_j = b^{\delta_j}$ .  $t_l = (1 + \varepsilon) K \rho_l$  is the number of transmitted symbols for layer  $l$  that satisfies  $\sum_{i=1}^L t_i = (1 + \varepsilon) K^1$ . The constraint that  $t_l$  is a non-negative integer is not explicitly shown here. However, the non-negativity is implicitly guaranteed by the constraints. To ensure an integer solution of  $t_l$ , we compute  $t_l$  as if it is a continuous variable, then round it to a nearest integer that is larger than the computed solution. We note that [8] does not give the proof of convexity and does not provide the optimality conditions for verifying the optimality of the solution, which will be addressed next.

To solve Problem 1.1, we first prove that Problem 1.1 is a convex optimization problem. As the objective function is linear, we only need to prove that the constraint functions are convex functions with respect to the vector  $(t_1, t_2, \dots, t_L)$ . It can be shown that for  $l = 1, 2, \dots, L$ , the second derivative of  $-\log(1 - c_l \alpha_l^{t_l})$  with respect to  $t_l$  satisfies

$$\frac{\partial^2 [-\log(1 - c_l \alpha_l^{t_l})]}{\partial t_l^2} = \frac{c_l \alpha_l^{t_l} (\log \alpha_l)^2}{(1 - c_l \alpha_l^{t_l})^2} > 0. \quad j = 1, 2, \dots, L.$$

Therefore, according to the second order condition of convex functions [11],  $-\log[1 - c_l \alpha_l^{t_l}]$  is a convex function of  $t_l$ . Since  $-\log[1 - c_l \alpha_l^{t_l}]$  does not contain any other  $t_i$  term other than  $t_l$ , it is also a convex function of the vector  $(t_1, t_2, \dots, t_L)$ . In addition, because nonnegative weighted sums preserve convexity [11], the constraint functions, which are summations of  $-\log[1 - c_l \alpha_l^{t_l}]$  of  $t_l$ , are then convex functions of the vector  $(t_1, t_2, \dots, t_L)$ . Alternatively, one can also compute the Hessian matrix of the constraint functions in (9) with respect to vector  $(t_1, t_2, \dots, t_L)$ , which can be shown to be positive semi-definite and hence that the problem is a convex optimization problem. Therefore, the problem can be solved numerically by any available algorithms and tools for convex optimization [11].

However, in many practical cases, the above problem can be further significantly simplified if the optimal solution occurs when all the inequality constraints are active. This assumption is reasonable for many practical cases as users in subsequent

<sup>1</sup>Strictly speaking, for each implementation,  $t_l$  is a Binomial distributed random variable with mean  $(1 + \varepsilon) K \rho_l$ . However, the randomization of  $t_l$  has little effect on the problem of interest when averaged out over a large number of implementations. In addition, one can always monitor the selection of layers to make sure that  $t_l$  is proportional to  $\rho_l$  in every implementation.

classes with better receiving capabilities will likely be able to decode layer  $l$  with very high probability since the raptor code error rate curves are very steep [10]. Using the above simplification, the solution to Problem 1.1 can be obtained by finding  $t_1$  using the equality constraint for class 1 in Eq. (9) and substituting the solution of  $t_1$  into the next equality constraint, solving for  $t_2$  with the constraint for class 2 in Eq. (9), etc. That is, we can successively solve for all of the variables  $t_1, t_2, \dots, t_L$ .

To verify the optimality of the solution obtained from the above method, we can use the Karush-Kuhn-Tucker (KKT) optimality conditions. Let  $t = (t_1, t_2, \dots, t_L)$  and  $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_L)$  be the variable vector of the primal and dual problems of Problem 1.1, respectively. If  $t^* = (t_1^*, t_2^*, \dots, t_L^*)$  and  $\lambda^* = (\lambda_1^*, \lambda_2^*, \dots, \lambda_L^*)$  are a pair of primal and dual optimal points, they must satisfy the KKT optimality conditions below:

$$f_j(t^*) \leq 0, \quad j = 1, 2, \dots, L \quad (10)$$

$$\lambda_j^* \geq 0, \quad j = 1, 2, \dots, L \quad (11)$$

$$\lambda_j^* f_j(t^*) = 0, \quad j = 1, 2, \dots, L \quad (12)$$

$$\nabla f_0(t^*) + \sum_{j=1}^L \lambda_j^* \nabla f_j(t^*) = 0 \quad (13)$$

where  $f_0(t) = \sum_{i=1}^L t_i$  is the objective function and  $f_j(t) = -\sum_{l=1}^j \log[1 - c_l \alpha_l^{t_l}] + \log P_j$ ,  $j = 1, 2, \dots, L$  are the constraint functions. Since the original Problem 1.1 is a convex optimization problem and satisfies Slater's condition, the above KKT optimality conditions provide the necessary and sufficient conditions for optimality [11]. If all the inequality constraints are active, Eqs. (10) and (12) are automatically satisfied. Therefore, if we obtain a solution  $t^*$  of Problem 1.1 by solving  $f_j(t^*) = 0$ ,  $j = 1, 2, \dots, L$ , we can substitute the value of  $t^*$  into Eq. (13) and obtain  $\lambda^*$ . If  $\lambda^*$  satisfies Eq. (11), i.e.,  $\lambda_j^* \geq 0$ ,  $j = 1, 2, \dots, L$ , then we have proven that the value of  $t^*$  we obtained is indeed an optimal solution of Problem 1.1 and the assumption that all inequality constraints are active is correct.

#### IV. BEST-EFFORT QOS FORMULATIONS

The above formulation focuses on minimizing transmission overhead subject to satisfying users' guaranteed QoS. In the alternative criteria considered in this section, transmission overhead is upper bounded due to delay constraints or cost. For this scenario, given a maximum transmission overhead  $\varepsilon_0$ , the service provider attempts to provide users of different classes with the best possible QoS. This scenario is similar to the one considered in [7]. However, we have generalized the problem formulation by considering both constrained and unconstrained cases, allowing for potentially different weight factors to different user classes and using the proposed random interleaved UEP raptor codes.

The  $PSNR$  of users in class  $j$ , which serves as a measure of the best-effort QoS, can be evaluated as  $E(PSNR_j) = \sum_{l=1}^L p_{l,j} q_l$ , where  $q_l$  is the  $PSNR$  achieved when layer 1 up to layer  $l$  are successfully recovered. The quantity  $p_{l,j}$  represents the probability that a class  $j$  user successfully recovers layers 1 to  $l$  but fails to recover layer  $l + 1$ . The optimization of UEP fountain codes has to balance users of

different classes with different channel qualities. If we assign a weighting coefficient  $w_j$  for class  $j$  where the choice of  $w_j$  depends on both the importance of the user class as well as the number of users in each user class, then it is reasonable to consider the weighted average  $PSNR$  over all user classes as the objective function. When the proposed UEP fountain code is used, the above problem can be stated as:

**Problem 2.0:**

$$\max_{t_1, t_2, \dots, t_L} \sum_{j=1}^J w_j \cdot \left( \sum_{l=1}^L p_{l,j} \cdot q_l \right) \quad (14)$$

$$\text{such that} \quad \sum_{i=1}^L t_i \leq (1 + \varepsilon_0) \times K. \quad (15)$$

$$\text{where} \quad p_{l,j} = \begin{cases} \prod_{i=1}^l (1 - P_e(i, j)) & \times P_e(l+1, j) \\ & l = 1, 2, \dots, L-1 \\ \prod_{i=1}^L (1 - P_e(i, j)) & l = L, \end{cases} \quad (16)$$

and  $P_e(i, j)$  is the same frame error rate used in (3). In Problem 2.0, no guaranteed minimum QoS is provided. To address this concern, in a third scenario, for a given maximum transmission overhead, the service provider aims to provide best-effort QoS to multiple user classes, but under additional constraints that a minimum QoS guarantee for each user class is met. This problem (Problem 3.0) can be formulated by adding the following constraints to Problem 2.0:

$$\prod_{l=1}^{g_j} (1 - P_e(l, j)) \geq P_j \quad j = 1, 2, \dots, J \quad (17)$$

where  $g_j \in 1, 2, \dots, L$  ( $j = 1, 2, \dots, J$ ) is the minimum index that satisfies  $q_{g_j} \geq \gamma_j$ .

## V. NUMERICAL RESULTS

To demonstrate the advantage obtained by optimization, we first show in Fig. 2 the minimum overhead when different pre-designed layer allocation parameter  $\rho_1$  values are used. The proposed random interleaved UEP design employing standardized raptor codes is used. The dashed line represents the results of the original memoryless erasure channel model based on the frame error rate evaluated in Section III-A, while the solid line represents the results of simplified channel model that is analyzed in Section III-B. We also marked the operating point at  $\rho_1/\rho_2 = S_1/S_2$  on each curves with a star, which indicates the performance of the equal error protection (EEP) scheme. It can be seen that the minimum required transmission overhead is very sensitive to the choice of  $\rho_1$ : the minimum transmission overhead with the optimal choice of  $\rho_1$  performs significantly better than that of the EEP scheme and other arbitrary non-optimized allocation schemes. Also, the performance difference between the two curves are very small. And as shown in Fig. 5 and Fig. 6 of [8] and the later on in this paper, the use of an inferior raptor encoder/decoder has a much larger impact on performance. This validates the use of the simplified channel model which allows for a much simpler solution based on convex analysis.

In [8] (Fig. 5 and Fig. 6), we have provided some initial results which show the minimum transmission overhead performance of proposed UEP design, Rahnavard's UEP rateless

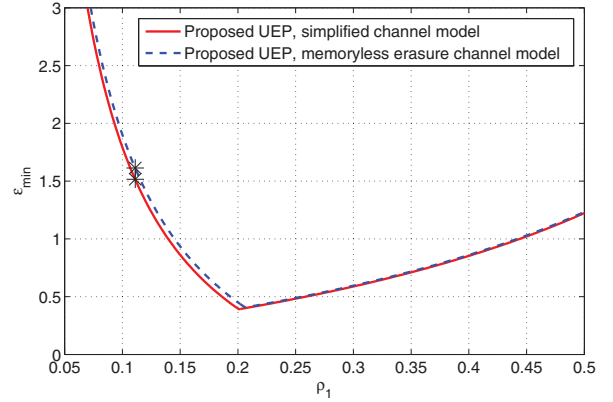


Fig. 2. The effect of layer allocation probability  $\rho_1$ . (Standardized raptor codes,  $L = 2$ ;  $K = 9000$ ;  $S = [1000, 8000]$ ;  $\delta = [0.4, 0.8]$ ;  $P = [0.95, 0.8]$ ).

code [5], the EWF code [7] versus their respective configuration parameters. To make fair comparisons, we use LT codes with an iterative decoder applied to all three schemes because Rahnavard's UEP rateless code and the EWF code cannot deploy existing standardized raptor codes directly. The frame error rate  $P_e(l, j)$  is evaluated by  $P_e(l, j) = 1 - (1 - e_{l,j})^{S_l}$ , where  $e_{l,j}$  is bit error probability of a class  $j$  user successfully decoding layer  $l$  and  $e_{l,j}$  is evaluated by the "and-or" tree technique used also in [5]. For the proposed random interleaved UEP scheme,  $e_{l,j} = \lim_{n \rightarrow \infty} e_{l,j}^n$ , where  $e_{l,j}^n$  is evaluated iteratively as<sup>2</sup>

$$e_{l,j}^n = \exp(-(1 + \varepsilon)K\rho_l\delta_j\Omega'_l(1 - e_{l,j}^{n-1})/S_l), \quad n \geq 1 \quad (18)$$

with an initial value  $e_{l,j}^0 = 1$ , where  $\Omega'_l(x)$  denotes the derivative of  $\Omega_l(x)$ .

As pointed out in [7], one of the advantages of EWF codes over Rahnavard's UEP scheme is the flexibility of using different degree distributions applied to different windows. Therefore, in order to break down the performance gain obtained by different code structure, degree distributions and decoder efficiency, we use different degree distributions applied to different windows of EWF codes as well as to different layers of the proposed UEP scheme. Fig. 3 shows the performance of the three different UEP schemes using LT codes after optimization over their respective configuration parameters. The performance curves show minimum transmission overhead for different parameter sets by varying the number of symbols in the first layer. Apart from using the degree distribution  $\Omega_r(x)$  described by Eq. (12) of [8] or Eq. (2) of [7], we also show the performance when a truncated robust soliton distribution (RSD)  $\Omega_{r,s}(k_{r,s}, \delta, c)$  with maximum degree bounded by  $k_{r,s}$  applied to the more important bits (MIB) for EWF codes and the proposed random interleaved UEP scheme. The truncated RSD is a stronger degree distribution compared to  $\Omega_r(x)$  at the cost of higher decoding complexity. It can be seen from Fig. 3 that using the stronger truncated RSD for the MIB provides a significant performance boost for both the EWF codes and the proposed UEP codes. For the same degree distribution, the minimum required transmission overhead is still lower for the proposed UEP scheme. A more important advantage

<sup>2</sup>We note that this expression corrects a typo in Eq. (8) in [8]



of the proposed scheme is easy adoption of standardized raptor codes. As shown in Fig. 2, the minimum overhead for  $S_1 = 1000$  using a standardized raptor code is only around 0.4, which is significantly lower than any curve shown in Fig. 3. This difference comes from the superior code design used by standardized raptor codes, which includes the use of a high performance pre-code as well as a much more efficient maximum likelihood (ML) decoder as opposed to the iterative decoders used for Fig. 3.

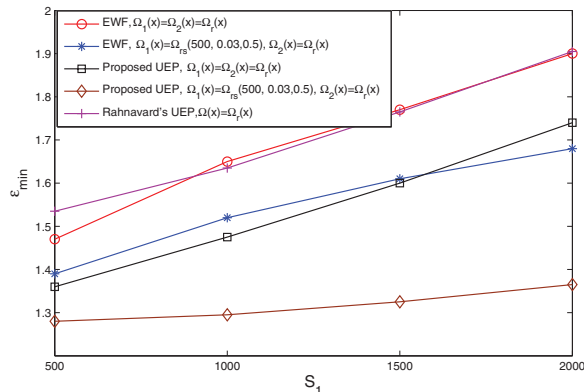


Fig. 3. Performance comparisons using LT codes with different degree distributions.  $L = 2$ ;  $K = 9000$ ;  $\delta = [0.4, 0.8]$ ;  $P = [0.95, 0.8]$

Fig. 4 shows the PSNR performance of the proposed random interleaved UEP scheme and the EWF scheme for the best-effort QoS formulation described by Problem 2.0. We assume transmission of the H.264 SVC coded CIF *stefan* video sequence, where we consider the case of two layers, with the base layer as the first layer, containing  $S_1 = 400$  symbols and all enhancement layers as the second layer with  $S_2 = 3400$  symbols. Successfully decoding the first layer provides a PSNR of 25.79 dB while decoding both first and second layers provides a PSNR of 40.28 dB. The performance is shown by the maximum average PSNR versus the selection probability  $\rho_1$  for the proposed random interleaved UEP scheme and the first window selection probability  $\Gamma_1$  of the EWF code. Due to space limitations, we have shown all performance curves in one figure, however, the selection probabilities for the two different schemes have different meanings and therefore are not comparable. For the first two performance curves, we have used the LT code with an iterative decoder and degree distribution  $\Omega_r(x)$  applied to all windows and layers. For these parameters, the proposed random interleaved UEP scheme provides a maximum average PSNR of 32.420 dB when optimized while the EWF scheme provides 32.355 dB when optimized. If constraints from Eq. (17) are added, the results remain the same since both the optimal operating points of the proposed UEP scheme and the EWF code are inside the feasible regions. Therefore, the performance difference between the two code design is much lower compared to Problem 1.0 with a slight edge towards the proposed UEP scheme. However, as shown in the third performance curve, when standardized raptor codes are employed for the proposed random interleaved UEP scheme, a maximum average PSNR of 40.28 dB can be achieved when  $0.11 \leq \rho_1 \leq 0.18$ , which is significantly higher than other two curves employing inferior raptor coders. We can also observe from Fig. 4 that different

choices of  $\rho_1$  can also result in a large difference in average PSNR, showing that the optimization process is necessary.

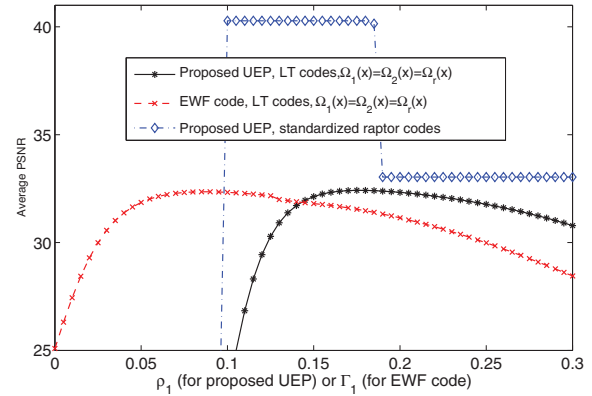


Fig. 4. Average PSNR performance of two UEP schemes.  $L = 2$ ;  $S = [400, 3400]$ ;  $\delta = [0.55, 1]$ ;  $P = [0.95, 0.8]$ ;  $\epsilon_0 = 1$ ;  $w = [0.5, 0.5]$ .

## VI. CONCLUSIONS

Two general problems are formulated for optimizing UEP rateless codes for scalable multimedia multicasting systems with heterogeneous users. The design objective is to either minimize transmission overhead for guaranteed QoS or provide best-effort QoS for a given transmission overhead. A random interleaved UEP raptor code design is proposed that can take advantage of the high performance of existing standardized raptor codes. The formulated problem is converted into a convex optimization problem which can be solved analytically. Numerical results demonstrate that the optimized proposed UEP raptor codes perform better than existing UEP raptor code designs when the same degree distribution and iterative decoding is applied. Large additional gain for the proposed UEP scheme can be obtained by using the superior existing standardized raptor codes which use an efficient ML decoder.

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