# Low Complexity PN Code Acquisition with Tree Search in Wideband CDMA Multipath Channels

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*Abstract*— **In this paper, we propose a novel matched-filter-type parallel code acquisition scheme to detect uplink short-code wideband CDMA multipath signals. A tree-structured parallel adaptive network of RAKE fingers is created for all possible delay shifts and users. Employing a sequential detection scheme, only viable fingers are used to generate the output decision statistics. Monte Carlo computer simulation and numerical calculations have been carried out to study the performance of the algorithm in single-path and multipath channels corrupted with co-channel interference (CCI) and additive white Gaussian noise (AWGN). Both simulation and numerical calculations demonstrate that the proposed algorithm can, at the expense of signal combining loss, significantly reduce the computation complexity and latency of the receiver. Additional signal combining loss is also quantified when the channel estimation error from weighted multislot averaging (WMSA) is taken into account.**

## I. INTRODUCTION

In a Direct Sequence Code Division Multiple Access (DS-CDMA) systems, code synchronization is important because data demodulation is possible only after synchronization is performed. A feasible solution for PN synchronization in DS/CDMA systems might consist in the transmission of a series of pilot symbols (i.e, synchronization overhead), which contain the unmodulated PN code [1]. At the receiver side, the acquisition module is empolyed to find the correct delay by a code matched filtering process: the cyclically shifted replica encoded in code matched filter is matched to the incoming sequence contained in the pilot symbols. Code synchronization is typically achieved in two steps: acquisition for coarse alignment and tracking for fine alignment [2] [3], of which the former is addressed in this paper.

Extensive research on long PN code acquisition where the period of the PN code is much larger than the symbol duration, has been carried out on AWGN channels during the past two decades [2] [3] [4] [5]. Short PN codes are used to mitigate the multiple access interference (MAI) and incorporate advanced signal processing techniques such as multiuser detection (MuD) in wideband CDMA systems, where the PN code period is equal to the symbol duration [6]. The effective utilization of multipaths becomes more important in next generation CDMA communication systems, where wide bandwidths are allocated to support high data rates and multimedia services, resulting in an increase in the number of resolvable paths. In [7], a joint triple-cell code acquisition in frequency-selective Rayleigh fading channel is proposed. Unfortunately, if the number of multipaths that can be acquired is fixed, computation inefficiency and poor performance may arise in realistic time-varying wideband CDMA multipath channels.

In our paper, we proposed a matched filter-type short PN code parallel acquisition scheme with truncated sequential detection to prune a code tree. The number of acquired multipaths varies from symbol to symbol to achieve a desired signal detection performance, enabling receiver adaptation to the dynamic channel state. To assess receiver performance in the context of CDMA systems, the proposed code acquisition algorithm is combined with a WMSA RAKE combiner tap weight estimation algorithm.

## II. SYSTEM MODEL

We consider the basestation receiver for a synchronous uplink of a wideband CDMA system with binary phase shift keying (BPSK) modulation. It is assumed that a wideband CDMA signal is received without data modulation and coherent detection is employed with a bank of matched filters. It is also assumed that the spreading codes of all users are known at the receiver. Frequency selective Rayleigh fading is used to model the reverse channel from the mobile terminal to the basestation. The multipath radio channel for $i^{th}$ mobile may be then described as quasi-static with a wide band tapped delay line (TDL) model with statistically independent time-varying gains $\beta_{ij}$:

$$h_i(n) = \sum_{j=1}^{L} \beta_{i,j}\delta(n - \tau_{i,j}) \tag{1}$$

In (1), $L$ is the total number of multipaths. The index $j$ represents one of $L$ multipaths experienced by the $i^{th}$ user and $\tau_{i,j}$ is the relative integer chip time delay with respect to the first arriving component for the $j^{th}$ path of the $i^{th}$ user.

In a wideband CDMA time-dispersive channel, the multipath intensity profile (MIP) quantifies the relationship between temporal delay and amplitude attenuation of each multipath component. According to outdoor wideband CDMA channel sounding results in [8], the MIP in terms of delay can be modelled as:

$$E[\beta_{i,j}^2] = \begin{cases} \exp(-\frac{\tau_{i,j}^2}{\sigma_\tau^2}) & \tau_{i,j} \geq 0 \\ 0 & \tau_{i,j} < 0 \end{cases} \tag{2}$$

where $\sigma_\tau = \sqrt{E\{\tau^2\} - E\{\tau\}^2}$ is the rms delay spread.

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After RF analog downconversion, chip matched filtering and chip-rate sampling, the received baseband discrete-time signal in one symbol period is:

$$r(n) = \sum_{i=1}^{K} \sum_{j=1}^{L} \beta_{i,j} d_i c_i(n - \tau_{i,j}) + z(n) \tag{3}$$

where $c_i(n-\tau_{i,j})$ is the PN code of user $i$ after a $\tau_{i,j}$ circular shift, $z(n)$ is zero-mean additive white Gaussian noise (AWGN) that is bandlimited at the receiver and $K$ is the total number of users. Since code acquisition is aided by pilot symbols, which are not data modulated, $d_i = 1$.

## III. CODE ACQUISITION BY TRUNCATED SEQUENTIAL DETECTION

Code acquisition is the process of determining the time delay of each path of each user. Strictly speaking, determining the exact sampling time is an estimation problem. However, to reduce the problem to one of finite dimensionality, the delay is assumed to be a multiple of $T_c$. Therefore, we can formulate code acquisition into a framework of multiple hypothesis testing [9].

The received signal is matched filtered by the PN sequence for the $i^{th}$ user with integer time delay $\tau_{i,j}$ of the $j^{th}$ path, and the code matched filter output is denoted as $Z_{i,j}$,

$$\begin{aligned} Z_{i,j} &= \sum_{n=1}^{G} r(n) c_i(n - \tau_{i,j}) \\ &= \beta_{i,j} G + IN_{i,j} \end{aligned} \tag{4}$$

where (3) is used and where,

$$IN_{i,j} = \sum_{n=1}^{G} \sum_{k=1}^{K} \sum_{\substack{l=1 \\ k,l \neq i,j}}^{L} \left(\beta_{k,l} d_k c_k(n - \tau_{k,l}) + z(n)\right) c_i(n - \tau_{i,j})$$

The signal from the $i^{th}$ mobile with shift $\tau_{i,j}$ will have a processing gain of $G$ chips per symbol, and interference from other users can be modeled as AWGN [3]. The correlation statistic $Z_{i,j}$ can be used to test for the presence of a signal delayed by $jT_c$ seconds from the $i^{th}$ mobile. Define $H_{i,j}$ as the hypothesis that there exists a strong path at the $j^{th}$ delay for the $i^{th}$ mobile. The test statistic $Z_{i,j}$ can belong to one of two mutually exclusive subsets, $\Gamma_0$ and $\Gamma_1$ of the observation space. Let

$$Z_{i,j} \in \Gamma_0 \quad \text{if no signal is present} \tag{5}$$

$$Z_{i,j} \in \Gamma_1 \quad \text{if } H_{i,j} \text{ is true} \tag{6}$$

Define the individual multiplicative term in the correlation metric at stage $n$ of the test for signal presence in $Z_{i,j}$ as $\chi_{i,j}(n)$:

$$\chi_{i,j}(n) = r(n) c_i(n - \tau_{i,j})$$

where $r(n)$ is the received signal. Using a Gaussian approximation,

$$\chi_{i,j}(k) \sim \begin{cases} \mathcal{N}(0, \sigma_{i,j}^2) & \text{if a signal path does not exist} \\ \mathcal{N}(\beta_{i,j}, \sigma_{i,j}^2) & \text{if a signal path exists} \end{cases}$$

where $\sigma_{i,j}^2$ contains both interference and additive thermal noise experienced by the $j^{th}$ path of the $i^{th}$ user. Since the additive thermal noise (with variance $\sigma_t^2$) and interference are uncorrelated, we have that

$$\sigma_{i,j}^2 = Var\{IN_{i,j}\} + \sigma_t^2 \tag{7}$$

The decision at stage $n$ $(1 \leq n < G)$ of the test can be written as:

$$\sum_{k=1}^{n} \chi_{i,j}(k) \begin{cases} \leq B_n & \Rightarrow \text{choose } H_0 \text{ and terminate test} \\ \text{otherwise} & \Rightarrow \text{continue to stage n+1} \end{cases}$$

At stage $G$, the test is terminated, i.e., a final decision is reached,

$$\sum_{k=1}^{G} \chi_{i,j}(k) \begin{cases} \leq B_G & \Rightarrow \text{choose } H_0 \text{ and terminate test} \\ > B_G & \Rightarrow \text{choose } H_1 \text{ and terminate test} \end{cases}$$

The above threshold parameters to detect multipath signal, namely $B_n (1 \leq n \leq G)$, are designed by modifying the well-known truncated sequential test design procedure to create a one-sided test that will choose $H_1$ only at the final stage. In [12] it is shown that for given nominal probability of missed detection $P_M$, truncation stage $G$ (equal to the processing gain) and $\sigma_{i,j}^2$ (given in (7)), the test thresholds can be designed as:

$$B_n = \frac{\ln[(1-c)P_M]\sigma_{i,j}^2}{E\{\beta_{i,1}\}} + \frac{nE\{\beta_{i,1}\}}{2} \tag{8}$$

$$B_G = \sigma \sqrt{G} \Phi^{-1}((1-c)P_M) + GE\{\beta_{i,1}\} \tag{9}$$

where $E\{\beta_{i,1}\}$ is the average signal strength of the dominant multipath component.

## IV. TREE SEARCH ALGORITHM

Instead of testing all possible circular shifts of all users' PN codes independently by a bank of code matched filters, we propose to test them jointly and utilize common testing statistics. Due to the fact that some shifts of PN codes have initial chips in common, simultaneous hypothesis testing is possible by organizing all the users' PN codes into a code tree. We achieve this firstly by enumerating all possible shifts of PN codes of all users in a code book, where each codeword denotes a shift of some user's PN code. Then we proceed through the codebook, codeword by codeword. Each codeword is processed chip by chip: each chip value is encoded in a code tree node.

In wideband CDMA systems, there are $K$ users transmitting the data symbols simultaneously in the same channel. We assume that the multipath delay spreads over $L_d$ chips in time. Let code book $C$ denote a matrix with $K{*}L_d$ rows. Each row corresponds to a shift of a PN code of a user in the system with length $G$ chips. To identify each node at a particular depth level of the tree search, we define the following rule:

Tree Node Labelling Rule

Node (i,j ) denotes the $j^{th}$ tree node at tree depth level $i$
Node (i,j)'s left child is labelled as node (i+1,2j-1).
and the right child is labelled as (i+1,2j).
$i = 1, \ldots, G$, $1 \leq j \leq 2^i$, where $G$ is the PN code length

The procedure of constructing the code tree works recursively. The chip values of a code table are labelled row by row according to the tree node labelling rule. For row $i$, chip $j$:

If $C_{i,j} = -1$,
then generate a left child
If $C_{i,j} = 1$,
then generate a right child

The last column of the code book consists of $K * L_d$ leaf nodes of the tree, i.e., no children are generated at the end of each row. The depth of the tree is, therefore, $G$ levels.

In our search procedure, we use the modified truncated sequential detection thresholds in Section III to prune the code tree. Because multiple hypotheses are tested jointly, some paths that are deemed to contain noise-only components will be eliminated simultaneously. The detection thresholds $B_i$ are given by (8), (9). The following information is stored within each tree node:

1. The current test stage $i$, i.e., the tree depth level of node $j$.
2. PN code value of tree node $j$.
3. The test statistic of the parent node $ZF_{i,j}$, where $i$ is the tree depth level and $j$ is the tree node identifier.
4. A linked list that contains all paths intersecting this node.

The CDMA receiver code acquisition is a joint procedure of tree search, code correlation and sequential detection. We denote, $Z_{i,j}$ as the truncated SPRT test statistic of tree node (i,j), $r(i)$ as the $i^{th}$ signal chip sample to be processed and $ZF_{i,j}$ as the input test statistic of the parent node of tree node (i,j). $C_{i,j}$ is the PN code value encoded in tree node (i,j). The array *df* with size of $1{*}KL_d$ is used to record the detected paths in the code acquisition procedure. The algorithm is executed as follows:

1. Initialize test statistic value at the tree root node (tree depth level 0) as: $Z_{0,1} = 0$ and input this value to tree nodes (1,1) , (1,2), i.e., $ZF_{1,1} = ZF_{1,2} = 0$.
2. Recursively execute the test in depth level $i$ by breadth-first tree search. All the tree nodes at tree depth level $i$ are tested. The test statistic value at tree node (i,j) is computed by:

$$Z_{i,j} = C_{i,j} * r(i) + ZF_{i,j} \tag{10}$$

If $Z_{i,j} \leq B_i$, the subtree from this tree node is discarded from further processing.
Otherwise, output $Z_{i,j}$ to its children nodes at stage $i+1$ and proceed to the next tree depth level, i.e., set $i = i+1$ and repeat.
3. If the final stage is reached, i.e., $i = G$, and if all the tree nodes in this stage are tested, then terminate the test. The test statistics of detected paths generate channel estimates.

The detected RAKE fingers provide multipath delay information to the RAKE combining stage. The final test statistics correspond to matched filter output amplitudes.

## V. STEADY-STATE PERFORMANCE ANALYSIS

We quantify the steady-state performance of the proposed algorithm by both computation complexity savings and signal combining loss. Steady-state complexity savings $\eta$ is measured in terms of the number of multiplications and additions (MADs) performed by the proposed MHT-MRC RAKE receiver, $MAD_{MHT-MRC}$, compared to that of a Full-Search-MRC RAKE receiver, $MAD_{FS-MRC}$, and is given by:

$$\eta = 1 - \frac{MAD_{MHT-MRC}}{MAD_{FS-MRC}} \tag{11}$$

The steady-state computation complexity of MHT-MRC RAKE receiver in multipath channels is given by (details appear in [12])

$$MAD_{multipath} = \sum_{l=1}^{P} \sum_{m=(v(l) mod G)}^{G} r(l,m)\rho(l,m) \tag{12}$$

where $v(l)$ is the stage at which the first $H_0$ label appears in path type $l$, $r(l,m)$ is the probability of continuing of path type $l$ at stage $m$ and $\rho(l,m)$ is the number of tree nodes at stage $m$ in subtree $T_l$.

The complexity of the Full-Search-MRC RAKE receiver is measured by complexity needed for testing $K{*}G$ possible PN code shifts, using code matched filtering, each of which introduces $G$ multiplications and additions (MADs) over one symbol period.

$$MAD_{FS-MRC} = K * L_d * G \tag{13}$$

Signal combining loss $\Gamma_{MHT-MRC}$ is measured by the received signal power after maximal ratio combining (MRC) of the MHT-MRC RAKE receiver compared to that of the Full-Search-MRC:

$$\Gamma_{MHT-MRC} = SNR_{FS-MRC} - SNR_{MHT-MRC} \tag{14}$$

### A. Perfect Channel Knowledge

In the Full-Search-MRC, the combined SNR is obtained by optimal MRC combining. The SNR after RAKE combining is averaged over all $K$ active users in the system via:

$$SNR_{FS-MRC} = \frac{\sum_{i=1}^{K} \sum_{j=1}^{L} SNR_{i,j}}{K} \tag{15}$$

where $SNR_{i,j}$ is the SNR of path $j$ of user $i$.

In the MHT-MRC RAKE receiver, the SNR of each multipath component is weighted by its corresponding probability of detection $P_{d_{i,j}}$. The signal combining loss stems from missed detection of multipath components.

$$SNR_{MHT-MRC} = \frac{\sum_{i=1}^{K} \sum_{j=1}^{L} SNR_{i,j} * P_{d_{i,j}}}{K} \tag{16}$$

In (16), $P_{d_{i,j}}$ is calculated numerically using the truncated SPRT analysis in [10].

### *B. WMSA Channel Estimator*

To determine the performance of a RAKE receiver without multipath channel knowledge, channel tap weights may be estimated by averaging the soft-decision amplitude statistics of the received signal after despreading over several symbol periods:

$$\tilde{\beta}_{i,j} = \frac{1}{N_w} \sum_{q=1}^{N_w} \frac{1}{G} \sum_{n=1}^{G} r(q * G + n) c_i(n - \tau_{i,j}) \tag{17}$$

where $N_w$ is the number of pilot symbols. The SNR at the output of the WMSA Full-Search MRC RAKE receiver is:

$$SNR_{W-FS-MRC} = \frac{1}{K} \sum_{i=1}^{K} \sum_{j=1}^{L} \widetilde{SNR}_{i,j} \tag{18}$$

where

$$\widetilde{SNR}_{i,j} = \frac{E(\tilde{\beta}_{i,j})}{\sigma^2} \tag{19}$$

The channel estimation error adds to the signal combining loss,

$$\Gamma_{W-FS-MRC} = SNR_{FS-MRC} - SNR_{W-FS-MRC} \tag{20}$$

Similarly, in the WMSA MHT-MRC RAKE receiver:

$$SNR_{W-MHT-MRC} = \frac{1}{K} \sum_{i=1}^{K} \sum_{j=1}^{L} \widetilde{SNR}_{i,j} * P_{d_{i,j}} \tag{21}$$

The signal combining loss of the WMSA MHT-MRC RAKE receiver, $\Gamma_{W-MHT-MRC}$, is obtained by:

$$\Gamma_{W-MHT-MRC} = SNR_{FS-MRC} - SNR_{W-MHT-MRC} \tag{22}$$

## VI. Numerical and Simulation Results

Monte Carlo simulation is performed to compare the proposed receiver and the optimal full-search receiver in multipath channels. Extended Gold code sequences [13] of length $G = 256$ are used as short spreading codes. The CDMA signals emitted from each mobile terminal propagate through $L = 4$ paths to reach the basestation receiver. We assume that the delay of each multipath component is uniformly distributed over $[0, 37]$ chips [14]. The multipath delay profile in Table I is used. The test design parameter $P_M = 0.001$ (See test design procedure in Section III). To achieve a 90% confidence interval of less than 10% errors over the range of nominal missed detection probabilities $10^{-4}$, we use $10^5$ Monte Carlo trials.

| Path | Amplitude | Delay spread in chips |
|---|---|---|
| 1st | 0.0 dB | 0 |
| 2nd | -5.0 dB | 14 |
| 3rd | -6.0 dB | 19 |
| 4th | -7.0 dB | 25 |

TABLE I

Multipath delay profile obtained from wideband CDMA channel sounding [8]

Figures 1 and 2 illustrate complexity savings and signal combining loss of the proposed receiver as a function of the SNR of the strongest path in a WCDMA system with $K = 10$ users. It can be seen that significant complexity reduction can be achieved with only modest signal combining loss. Specifically, complexity savings of 90% or more is obtained with no more than 2.0 dB signal combining loss at an SNR of 5 dB or higher. Due to the joint code acquisition and channel estimation in the proposed code tree search algorithm, the complexity savings due to MHT in both MHT-MRC and W-MHT-MRC receivers are the same. As shown in Figure 2, less than 0.6 dB additional signal combining loss is incurred due to channel estimation error .

Next, we examine the effect of the number of users on receiver performance in terms of complexity savings and SNR at the RAKE receiver output when the SNR of the strongest path is fixed at 15 dB. A wide range of WCDMA systems loaded with 1 to 50 users are compared. From Figure 3, we observe that the complexity savings increases dramatically when the number of users increases from 1 to 10 and saturates at more than 10 users. The relationship between RAKE combiner output SNR and the number of users is illustrated in Figure 4. As expected, the SNR decreases proportionally as the number of users increases due to the increasing multiple access interference. Also, the performance gap due to channel estimation also increases with user loading.

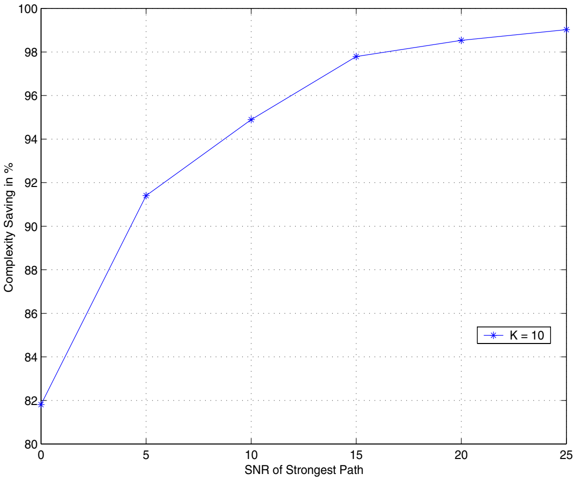

Fig. 1. Complexity savings (multipath channel) as a function of SNR of the strongest path: $K = 10$ users, $L = 4$ paths per user and nominal missed detection probability $P_M = 0.001$

## VII. conclusions

Our study has shown that tree structured search of circularly shifted PN codes via multistage hypothesis testing achieves about one order of magnitude complexity savings (90%) while incurring 2.0 dB signal combining loss at an SNR of 5 dB in WCDMA systems with 10 users. When

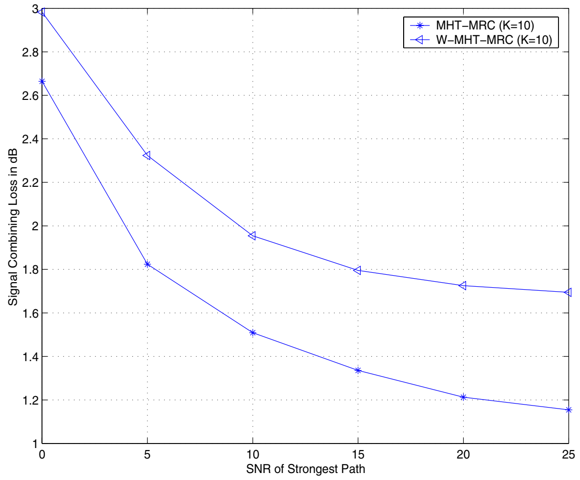

Fig. 2. Signal combining loss as a function of SNR of the strongest path: $K = 10$ users, $L = 4$ paths per user and nominal missed detection probability $P_M = 0.001$

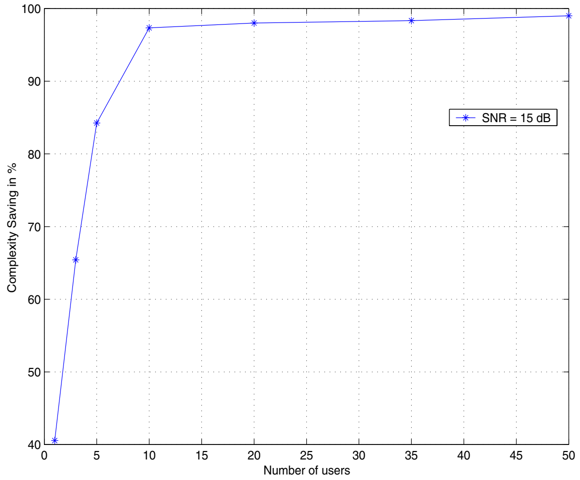

Fig. 3. Complexity savings as a function of number of users: $L = 4$, $P_M = 0.001$ and $SNR = 15dB$

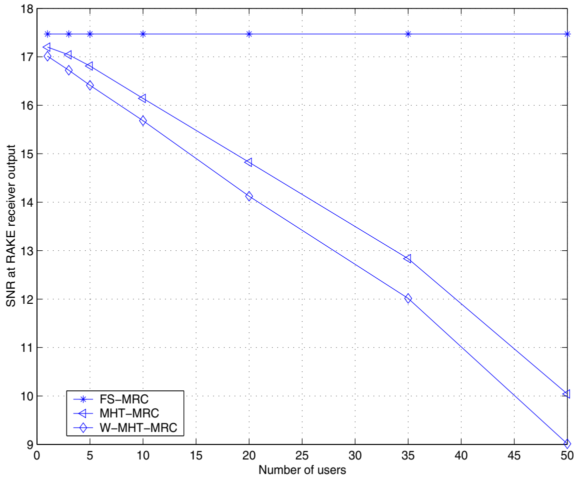

Fig. 4. SNR at the output of RAKE receiver as a function of number of users: $L = 4$, $P_M = 0.001$ and $SNR = 15dB$

the channel estimation error is taken into account, an additional performance loss of less than 1.0 dB is observed. As the number of users increases, the complexity savings increases while the SNR at the RAKE combiner output decreases.

Performance analysis of proposed algorithm in multipath channels was carried out in [12] for case of small number of users and paths using numerical integration. Space limitations prevent the presentation of analysis and it is reported that the signal combining loss is within 1 dB from simulation in [12]. Unfortunately, an exact analysis of signal combining loss is shown to have exponential complexity in the number of signal propagation paths, which motivates future work in obtaining analytical bounds of signal detection performance.

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