

# Modified Decorrelating Decision-Feedback Detection of BLAST Space-Time System

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*Abstract*-We propose a stable and reduced-complexity detection method for the Bell Labs layered Space-Time (BLAST) coding system. The existing iterative nulling and cancellation algorithm for BLAST has high computational complexity and requires repeated matrix pseudo-inverse calculation which may lead to numerical instability. A square-root algorithm was proposed by other researchers to reduce complexity and improve numerical stability. In this paper, to further reduce complexity, we modify the decorrelating decision-feedback CDMA multiuser detection method and apply it to BLAST. Similar to the square-root algorithm, numerical stable unitary transformations are performed on the Cholesky-decomposed matrices to reorder the detection and cancellation steps. However, our method exploits a symmetry property in the triangularization process, which may further improve the numerical stability and reduce computational complexity over that of the square-root algorithm. In the simulation results, the impact of the reordering on performance is demonstrated.

## I. INTRODUCTION

Bell Labs layered Space-Time (BLAST) is a low-complexity linear space-time coding architecture which is well-suited for high-rate wireless communications [1]. We denote an  $(M, N)$  BLAST system as having  $M$  antennas at the transmitter and  $N$  antennas at the receiver. When  $M \leq N$ , there exists an iterative nulling and interference cancellation algorithm to detect the transmitted information symbols from each transmit antenna [2] [3], instead of the exponentially complex maximum-likelihood search over all possible transmitted symbol combinations.

Since BLAST is a linear space-time coding system, there exist efficient detection algorithms and recently there has been much research activity in extending and improving BLAST [5] [6] [7]. A high-rate linear space-time code of [5] is an extension of BLAST to the case where the number of receive antennas,  $N$ , is less than the number of transmit antennas,  $M$ . The detection algorithms of BLAST are the same as those of its extensions [5] [6] and [7], so it is important to develop an efficient and stable detection algorithm.

The main computation in using the iterative nulling and cancellation algorithm for BLAST symbol detection is the determination of the optimal ordering of the nulling and cancellation steps, and the computation of the corresponding nulling vectors. These steps have computational complexity of order

$O(M^4)$ . When the number of transmit and receive antennas is large, i.e.,  $M \geq 18$ , the repeated use of the pseudo-inverse to calculate the nulling vectors may lead to numerical instability [4].

A square-root algorithm based on QR decomposition of the channel matrix and unitary transformations is used in [4] to avoid the repeated computation of the nulling vectors. Instead, the QR decomposition is computed only once. Not only is computation complexity reduced, but also the numerical robustness is improved by this square-root algorithm.

In this paper, we propose to further reduce the complexity of the algorithm in [4]. Motivated by the decorrelating decision-feedback multiuser detection algorithm originally proposed for code division multiple access (CDMA) systems [9], we interpret an  $(M, N)$  BLAST system as an  $M$ -user CDMA system with spreading factor  $N$ , as first suggested in [3].

## II. SYSTEM MODEL

In the following, we assume that  $N \leq M$  to facilitate simple iterative nulling and cancellation at the receiver.

At the transmitter, the incoming information stream is serial-to-parallel converted to  $M$  sub-streams. Each sub-stream is associated with a transmit antenna. At each time instant, one symbol from each sub-stream is transmitted from its corresponding transmit antenna, resulting in  $M$  symbols transmitted simultaneously.

The wireless channel is assumed to be rich-scattering and flat-fading. The fading between each transmit and receive antenna pair are assumed independent. The channel is also assumed quasi-static, and the channel parameters are assumed to have been estimated at the receiver through a short training sequence before the detection procedure.

The received signal at the  $N$  receive antennas can be organized into a vector after matched filtering and symbol rate sampling:

$$\mathbf{x} = [x_1 \dots x_N]^T \quad (1)$$

The transmitted signal from the  $M$  transmit antennas can also be organized in vector form:

$$\mathbf{s} = [s_1 \dots s_M]^T \quad (2)$$

The received signal  $\mathbf{x}$  can be expressed as a linear combination of the transmitted signal  $\mathbf{s}$ :

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{v} \quad (3)$$

This research has been supported by the Canadian Institute for Telecommunications Research under the NCE program of the Government of Canada. E-mail: {wzha, sdb}@ee.queensu.ca.

where  $\mathbf{H} \in \mathcal{C}^{N \times M}$  is the complex channel matrix, and  $\mathbf{v} \in \mathcal{C}^N$  is the spatially and temporarily white zero-mean Gaussian noise vector collected from the  $N$  receive antennas, with auto-correlation  $\sigma^2 \mathbf{I}$ .

The elements of  $\mathbf{H}$  are independent of one other due to the rich-scattering environment. The channel matrix  $\mathbf{H}$  can be partitioned into its columns corresponding to the  $M$  transmitted signals, and it is denoted as  $\mathbf{H}^M$ :

$$\mathbf{H}^M = [\mathbf{h}_1 \dots \mathbf{h}_M] \quad (4)$$

#### A. Iterative Nulling and Cancellation Method

We first briefly describe the iterative nulling and cancellation method for symbol detection in BLAST [2]. In this paper, we only consider the zero-forcing (ZF) criterion for nulling to simplify the algorithm descriptions. The formulation using minimum mean-squared error (MMSE) criterion for nulling vector computation is a straightforward extension.

The algorithm consists of the following steps repeated  $M$  times:

For  $K = M$  to 1:

Step 1 Calculate the inverse of the correlation matrix as  $(\mathbf{R}^K)^{-1} = (\mathbf{H}^{K*} \mathbf{H}^K)^{-1}$ .

Step 2 Since the users' signal-to-noise ratios (SNR) are inversely proportional to their respective diagonal entries of  $(\mathbf{R}^K)^{-1}$ , find the smallest diagonal entry. Let  $\alpha$  be the index of the smallest diagonal entry. Reorder  $\mathbf{H}^K$  such that the  $\alpha$ -th column and the last ( $K$ -th) column are interchanged:

$$\mathbf{H}^{K'} = [\mathbf{h}_1 \dots \mathbf{h}_K \dots \mathbf{h}_{K-1} \mathbf{h}_\alpha] \stackrel{\text{def}}{=} [\mathbf{H}^{(K-1)} \mathbf{h}_\alpha] \quad (5)$$

where the deflated channel matrix  $\mathbf{H}^{(K-1)}$  is the same as  $\mathbf{H}^{K'}$  with the last column  $\mathbf{h}_\alpha$  deleted.

Step 3 Calculate the pseudo-inverse matrix  $(\mathbf{H}^{K'})^\dagger$ . Let the nulling vector  $\mathbf{w}$  be the last row of  $(\mathbf{H}^{K'})^\dagger$ . The transmitted signal is detected as the closest point in the signal constellation

$$\hat{s} = \text{dec}(\mathbf{w}\mathbf{x})$$

where  $\text{dec}(\cdot)$  is the slice function, which depends on the modulation used.

Step 4 Perform interference cancellation by subtracting the detected signal from the received signal:

$$\mathbf{x} \leftarrow \mathbf{x} - \hat{s}_M \mathbf{h}_M$$

It has been proved in [3] that choosing the signal with the largest SNR at each step for nulling and cancellation achieves the global optimization that minimizes the probability of symbol errors. So the optimal ordering is the ordering of decreasing SNR.

Since at each step of the above algorithm, a pseudo-inverse of the deflated channel matrix is computed which is of order  $O(M^3)$ , the total complexity of the algorithm is  $\frac{27}{4} M^4$  for the case of  $M = N$ .

#### B. Other Detection Algorithms

A sphere decoding algorithm based on the lattice sphere packing representation of the BLAST system can achieve maximum-likelihood performance [8]. However, its complexity is approximately  $O(2^6 M^6)$ , which limits its application to  $M < 16$ .

An efficient square-root algorithm avoids the repeated computation of nulling vectors by QR decomposition [4]. The optimal ordering and the nulling vectors are all computed by unitary transformations on the QR decomposed matrices. Its computational complexity is  $\frac{29}{3} M^3$  for the case of  $M = N$ .

### III. DECORRELATING DECISION-FEEDBACK METHODS

#### A. Original Decorrelating Decision-Feedback Method

The original decorrelating decision-feedback multiuser detector was used for detecting multiple user signals of a synchronous CDMA system [9]. By making the connection between a BLAST system and a synchronous CDMA system, decorrelating decision-feedback methods can be applied to the BLAST systems as well.

The received signal vector  $\mathbf{x}$  is correlated with the conjugate transpose of the channel matrix. This correlation is analogous to the matched filter bank front-end of a CDMA multiuser receiver. The correlator output  $\mathbf{y} \in \mathcal{C}^M$  is:

$$\mathbf{y} = \mathbf{H}^* \mathbf{x} = \mathbf{R} \mathbf{s} + \mathbf{z} \quad (6)$$

where  $\mathbf{R} = \mathbf{H}^* \mathbf{H}$  is a  $M \times M$  cross-correlation matrix, and  $\mathbf{z}$  is a zero-mean Gaussian noise vector with auto-correlation  $\sigma^2 \mathbf{R}$ .

The cross-correlation matrix can be Cholesky decomposed as  $\mathbf{R} = \mathbf{L} \mathbf{L}^*$ , where  $\mathbf{L}$  is a lower triangular matrix and  $\mathbf{L}^*$  is its conjugate transpose. A filter with impulse response  $\mathbf{L}^{-1}$  is applied to the correlator outputs  $\mathbf{y}$  of (6) to whiten the noise:

$$\check{\mathbf{y}} = \mathbf{L}^{-1} \mathbf{y} = \mathbf{L}^* \mathbf{s} + \mathbf{n} \quad (7)$$

Since  $\mathbf{L}^*$  is upper triangular, the  $k$ -th component of  $\check{\mathbf{y}}$  can be expressed as:

$$\check{y}_k = L_{k,k}^* s_k + \sum_{i=k+1}^M L_{k,i}^* s_i + n_k \quad (8)$$

which contains only interference from  $(M - k)$  signals.

The last component  $\check{y}_M$  contains no interference, so a decision for this transmitted signal can be made first:  $\hat{s}_M = \text{dec}(\check{y}_M)$ . The next signal can be detected by subtracting the interference contribution from the  $M$ -th signal using the previous decision, i.e.,  $\hat{s}_{M-1} = \text{dec}(\check{y}_{M-1} - L_{M-1,M}^* \hat{s}_M)$ . This procedure is repeated until all signals are detected.

The above decorrelating decision-feedback method first cancels the interference using the feedback of previous decisions, and then makes a decision on the current signal. The detection and decision-feedback are performed in decreasing

order of received signal energies in the original decorrelating decision-feedback CDMA multiuser detector in [9]. The Cholesky decomposition is calculated only once, so repeated calculation of the pseudo-inverse is avoided. Is the decreasing energy ordering for decorrelating decision-feedback the same as that of BLAST, i.e., the decreasing SNR ordering?

In the BLAST system, the received energies correspond to the column norms of the channel matrix  $\mathbf{H}$ . In a CDMA system, the cross-correlation between the different user codes can be designed to be equal, so decreasing energy ordering is the same as decreasing SNR ordering. However, in the BLAST system, the ‘‘spreading codes’’ values are actually channel gains, which are random and generally do not have equal cross-correlations. So we propose a modification to the original decorrelating decision-feedback method to obtain the optimal ordering.

### B. Modified Decorrelating Decision-Feedback Method

The original cross-correlation matrix  $\mathbf{R}$ , or its corresponding Cholesky decomposition matrices  $\mathbf{L}$  and  $\mathbf{L}^*$ , have to be reordered for optimal detection ordering. In this subsection, we propose a modified decorrelating decision-feedback detector where the detected signal has the largest SNR at every step.

The inverse of the cross-correlation matrix is  $\mathbf{R}^{-1} = \mathbf{L}^{-*}\mathbf{L}^{-1}$ , where  $\mathbf{L}^{-1}$  can be easily calculated from the lower triangular matrix  $\mathbf{L}$  by back-substitution, and  $\mathbf{L}^{-*}$  is the conjugate transpose of  $\mathbf{L}^{-1}$ . The signal to be detected with the largest SNR corresponds to the signal with the smallest diagonal entry of  $\mathbf{R}^{-1}$ . Note that we do not need to calculate  $\mathbf{R}^{-1}$  to find the smallest diagonal entry, since the diagonal entries of  $\mathbf{R}^{-1}$  are equal to the column norms of  $\mathbf{L}^{-1}$ .

We find the smallest column norm of  $\mathbf{L}^{-1}$ , and then reorder the columns of  $\mathbf{L}^{-1}$  by interchanging the smallest column-norm column with the last ( $M$ -th) column. The rows of  $\mathbf{L}$ , corresponding to columns of  $\mathbf{L}^{-1}$ , as well as both the corresponding rows and columns of  $\mathbf{R}$  are interchanged in the same way. Interchanging two columns of a matrix can be performed by post-multiplication by a unitary permutation matrix  $\mathbf{P}$ , and interchanging two rows of a matrix can be performed by pre-multiplication by a unitary permutation matrix  $\mathbf{P}^*$ , so the matrices after reordering are:

$$\mathbf{P}^*\mathbf{R}\mathbf{P} = (\mathbf{P}^*\mathbf{L})(\mathbf{L}^*\mathbf{P}) \quad (9)$$

In the following, we exploit the fact that there exists a unitary matrix  $\mathbf{\Sigma}$  that transforms  $\mathbf{L}^*\mathbf{P}$  into upper triangular form [10]. Similarly, its conjugate transpose  $\mathbf{\Sigma}^*$  transforms  $\mathbf{P}^*\mathbf{L}$  into lower triangular form:

$$\mathbf{P}^*\mathbf{R}\mathbf{P} = (\mathbf{P}^*\mathbf{L}\mathbf{\Sigma}^*)(\mathbf{\Sigma}\mathbf{L}^*\mathbf{P}) \quad (10)$$

The following symmetry property (Claim 1) is very useful to lower triangularize the reordered inverse matrix  $\mathbf{L}^{-1}\mathbf{P}$ .

**Claim 1:** *Let  $\mathbf{\Sigma}$  be the unitary matrix that transforms  $\mathbf{L}^*\mathbf{P}$  to upper triangular form, then the reordered inverse*

*matrix  $\mathbf{L}^{-1}\mathbf{P}$  is transformed to lower triangular form by the same  $\mathbf{\Sigma}$ .*

**Proof:**

$$\mathbf{I} = \mathbf{P}^*\mathbf{P} = (\mathbf{P}^*\mathbf{L})(\mathbf{L}^{-1}\mathbf{P}) = (\mathbf{P}^*\mathbf{L}\mathbf{\Sigma}^*)(\mathbf{\Sigma}\mathbf{L}^{-1}\mathbf{P}) \quad (11)$$

Since  $\mathbf{\Sigma}\mathbf{L}^*\mathbf{P}$  is upper triangular, its conjugate transpose  $\mathbf{P}^*\mathbf{L}\mathbf{\Sigma}^*$  is in lower triangular form. Since  $\mathbf{\Sigma}\mathbf{L}^{-1}\mathbf{P}$  is the inverse of  $\mathbf{P}^*\mathbf{L}\mathbf{\Sigma}^*$ , it has to also be lower triangular.  $\square$

Instead of finding the unitary transformation based on the  $\mathbf{L}^{-1}\mathbf{P}$  directly, we may use  $\mathbf{L}^*\mathbf{P}$  to find  $\mathbf{\Sigma}$  to increase numerical stability.

In addition to finding the smallest norm, reordering and triangularization using (9), (10) and (11) for the first step, the following Claims 2 and 3 ensure that it is sufficient to use deflated Cholesky factors  $\mathbf{L}^{(M-1)}$  and  $(\mathbf{L}^{(M-1)})^{-1}$  for the next step.

**Claim 2:** *We can reduce  $\mathbf{P}^*\mathbf{L}\mathbf{\Sigma}^*$  to matrix  $\mathbf{L}^{(M-1)}$  as*

$$\begin{bmatrix} \mathbf{L}^{(M-1)} & \mathbf{0} \\ \times & \times \end{bmatrix} = \mathbf{P}^*\mathbf{L}\mathbf{\Sigma}^* \quad (12)$$

where  $\times$  represents entries that are irrelevant,  $\mathbf{L}^{(M-1)}\mathbf{L}^{(M-1)*} = \mathbf{R}^{(M-1)}$ , and  $\mathbf{R}^{(M-1)} = \mathbf{H}^{(M-1)*}\mathbf{H}^{(M-1)}$  is the cross-correlation matrix for the reordered and deflated Channel matrix  $\mathbf{H}^{(M-1)}$  using the notation in (5).

**Proof:** This proof is similar to that of [4]. By substituting (12) into (10), we obtain:

$$\begin{aligned} \mathbf{P}^*\mathbf{R}\mathbf{P} &= \begin{bmatrix} \mathbf{L}^{(M-1)} & \mathbf{0} \\ \times & \times \end{bmatrix} \begin{bmatrix} \mathbf{L}^{(M-1)*} & \times \\ \mathbf{0} & \times \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{L}^{(M-1)}\mathbf{L}^{(M-1)*} & \times \\ \times & \times \end{bmatrix} \end{aligned} \quad (13)$$

By (5), it is also true that:

$$\begin{aligned} \mathbf{P}^*\mathbf{R}\mathbf{P} &= \begin{bmatrix} \mathbf{H}^{(M-1)*} \\ \mathbf{h}_\alpha^* \end{bmatrix} \begin{bmatrix} \mathbf{H}^{(M-1)} & \mathbf{h}_\alpha \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{H}^{(M-1)*}\mathbf{H}^{(M-1)} & \times \\ \times & \times \end{bmatrix} \end{aligned} \quad (14)$$

So the upper triangular matrix  $\mathbf{L}^{(M-1)*}$  retains all the information contained in the deflated channel matrix.  $\square$

**Claim 3:** *The inverse lower triangular matrix  $\mathbf{\Sigma}\mathbf{L}^{-1}\mathbf{P}$  can be expressed in reduced form as*

$$\mathbf{\Sigma}\mathbf{L}^{-1}\mathbf{P} = \begin{bmatrix} (\mathbf{L}^{(M-1)})^{-1} & \mathbf{0} \\ \times & \times \end{bmatrix} \quad (15)$$

where  $(\mathbf{L}^{(M-1)})^{-*}(\mathbf{L}^{(M-1)})^{-1} = (\mathbf{R}^{(M-1)})^{-1}$ .

**Proof:** Let

$$\begin{bmatrix} \mathbf{B} & \mathbf{0} \\ \times & \times \end{bmatrix} = \mathbf{\Sigma}\mathbf{L}^{-1}\mathbf{P}$$

Then by (11):

$$\begin{aligned}
\mathbf{I} &= (\mathbf{P}^* \mathbf{L} \Sigma^*) (\Sigma \mathbf{L}^{-1} \mathbf{P}) \\
&= \begin{bmatrix} \mathbf{L}^{(M-1)} & \mathbf{0} \\ \times & \times \end{bmatrix} \begin{bmatrix} \mathbf{B} & \times \\ \mathbf{0} & \times \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{L}^{(M-1)} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \quad (16)
\end{aligned}$$

From the above equation

$$\mathbf{L}^{(M-1)} \mathbf{B} = \mathbf{I}^{(M-1)} \quad (17)$$

So  $\mathbf{B} = (\mathbf{L}^{(M-1)})^{-1}$ . Since  $(\mathbf{R}^{(M-1)})^{-1} = (\mathbf{L}^{(M-1)})^{-*} (\mathbf{L}^{(M-1)})^{-1}$ ,  $\mathbf{B}$  retains all the information to calculate  $(\mathbf{R}^{(M-1)})^{-1}$  and to find the largest SNR for the next step.  $\square$

The complete algorithm of the modified decorrelating decision-feedback detection for BLAST systems can be described by the following steps:

Initialization:

Cholesky decompose  $\mathbf{R} = \mathbf{L}\mathbf{L}^*$  and invert matrix  $\mathbf{L}$  by back-substitution. Let  $\mathbf{L}^M = \mathbf{L}$ .

Iterations:

For  $K = M$  to 1

1. Find the column of  $(\mathbf{L}^K)^{-1}$  with the smallest column norm, and reorder it to the last column via the transformation  $(\mathbf{L}^K)^{-1} \mathbf{P}$ . Similarly reorder the columns of  $\mathbf{L}^{K*}$  by  $\mathbf{L}^{K*} \mathbf{P}$ .
2. Find a unitary matrix  $\Sigma$  that transforms  $\mathbf{L}^{K*} \mathbf{P}$  to upper triangular form  $\Sigma \mathbf{L}^{K*} \mathbf{P}$ . Similarly compute lower triangular matrix  $\Sigma (\mathbf{L}^K)^{-1} \mathbf{P}$ .  $\Sigma$  can be realized by a series of Givens rotations [10].

Detection:

Perform decorrelating decision-feedback detection in Eq. (7) with the reordered matrices.

### C. Implementation Issues

In a practical implementation, we do not need to calculate the cross-correlation matrix  $\mathbf{R}$  first to get its Cholesky decomposition matrix  $\mathbf{L}$ . Rather we can perform a QR decomposition on the channel matrix  $\mathbf{H}$  directly to get  $\mathbf{L}$ . Let the QR decomposition be  $\mathbf{H} = \mathbf{Q}\mathbf{F}$ , where  $\mathbf{Q}$  is orthonormal and  $\mathbf{F}$  is upper triangular. Since  $\mathbf{Q}^* \mathbf{Q} = \mathbf{I}$ , we have  $\mathbf{H}^* \mathbf{H} = (\mathbf{F}^* \mathbf{Q}^*) (\mathbf{Q} \mathbf{F}) = \mathbf{F}^* \mathbf{F}$ . Then we can conclude that  $\mathbf{L} = \mathbf{F}^*$ .

Although both algorithms utilize a QR decomposition, and our research was partially inspired by [4], our modified decorrelating decision-feedback method is different from the square-root algorithm [4]. The square-root algorithm uses both the matrix  $\mathbf{Q}$  for nulling vector calculation and the inverse matrix  $\mathbf{F}^{-1}$  for optimal ordering. The square-root algorithm performs repeated triangularization on the inverse matrix  $\mathbf{F}^{-1}$ . Even with back-substitution, this may lead to instability, so [4] adopted a computationally complex series of transformations to avoid computing  $\mathbf{F}^{-1}$  directly by inverting.

Our method utilizes symmetry properties, so we can perform repeated triangularization on the conjugate transpose matrix  $\mathbf{L}^*$ , while the inverse matrix  $\mathbf{L}^{-1}$  is triangularized by symmetry by the same transformation. As the accuracy requirement on  $\mathbf{L}^{-1}$  is relaxed, it can be computed by simple back-substitution. Another possible advantage of the proposed method is that there should be less rounding error effects in a fixed-point implementation, since normally  $\mathbf{L}$  has larger entry values than  $\mathbf{L}^{-1}$ .

The dominant computation of the modified decorrelating decision-feedback receiver is in the QR decomposition, the matrix inversion and the reordering and triangularization of matrices  $\mathbf{L}^*$  and  $\mathbf{L}^{-1}$ .

The computation complexity for QR decomposition is  $2M^2(N - M/3)$  [10]. The computational complexity to calculate  $\mathbf{L}^{-1}$  by back-substitution is  $M^3/3$ . At the  $i$ -th step, finding the smallest column norm takes  $i^2/2$  operations, and triangularization of the two matrices takes  $2Mi$ . So the total complexity for the  $M$  steps is

$$\sum_{i=1}^M (\frac{1}{2}i^2 + 2Ni) = \frac{1}{6}M^3 + M^3 = \frac{7}{6}M^3 \quad (18)$$

Thus, the total complexity for the algorithm is:

$$\frac{1}{2}M^3 + 2M^2N \quad (19)$$

When  $M = N$ , it is  $\frac{5}{2}M^3$ , which is less than  $\frac{29}{3}M^3$  of the square-root algorithm [4] and  $\frac{27}{4}M^4$  of the iterative nulling and cancellation algorithm [2].

If instead of zero-forcing nulling, MMSE-nulling is required, then the decorrelating decision-feedback receiver can be modified to a MMSE decision-feedback receiver either by Cholesky decomposition on matrix  $(\mathbf{R} + \alpha \mathbf{I})$ , where  $\alpha > 0$ , or by QR decomposition on the augmented channel matrix directly as in [4].

## IV. SIMULATION RESULTS

Throughout the simulations,  $q$ -QAM constellations are used. The average energy per bit is fixed to 1, so the average energy per symbol is  $\bar{E}_s = 2(q - 1)/3$  as in [8]. The channel matrix is simulated as zero-mean complex Gaussian with variance 0.5 per dimension. The additive zero-mean white Gaussian noise (AWGN) is complex, with variance  $\sigma^2$  per dimension, where  $\sigma^2$  is subject to the following equation [8]:

$$\sigma^2 = \frac{M \bar{E}_s}{2 \log_2 q} 10^{-\frac{SNR}{10}} \quad (20)$$

Although the performance of the iterative nulling and cancellation detector, the square-root algorithm and the proposed modified decorrelating decision-feedback detector with optimal ordering are the same, the impact of the reordering on the performance was not quantified in [2] and [4]. In Figs. 1 and 2,

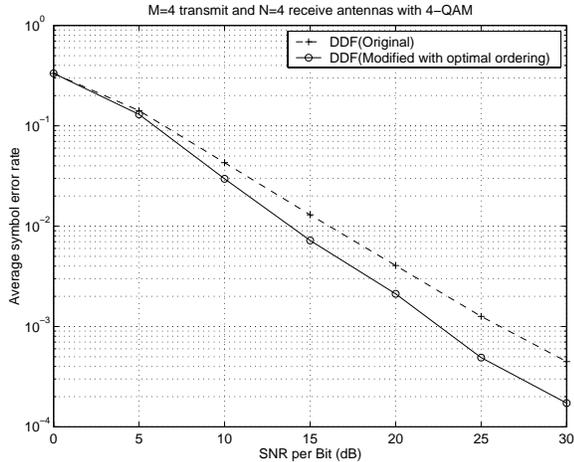


Fig. 1. Average symbol error rate of the original and modified decorrelating decision-feedback (DDF) detectors for BLAST system with  $M = 4$  transmit antennas,  $N = 4$  receive antennas and 4-QAM modulation.

we compare the average symbol error rates (SER) of the original decorrelating decision-feedback (DDF) detector and the modified decorrelating decision-feedback detector with optimal ordering.

In Fig. 1, the number of transmit and receiver antennas are  $M = N = 4$ , and 4-QAM modulation is used. The SER of the optimal ordered decorrelating decision-feedback detector is lower than the original one. In the simulations, the frequency of occurrence that the optimal order is the same as the original order is less than 30% of all the channel realizations.

In Fig. 2, simulation results for  $M = N = 8$  antennas, and 16-QAM modulation are shown. As the number of transmit and receive antennas increases, the frequency of occurrence that the optimal order is the same as the original order is decreased to less than 1% of all the channel realizations. This explains the larger performance improvement of the modified decorrelating decision-feedback detector over the original decorrelating decision-feedback detector in Fig. 2 versus that in Fig. 1.

## V. CONCLUSION

A modified decorrelating decision-feedback detection method is proposed and applied to the BLAST space-time system. The repeated computation of pseudo-inverse is avoided by decorrelating decision-feedback detection. By exploiting the symmetry in triangularizing the conjugate transpose and the inverse matrix, increased numerical stability and decreased computational complexity are achieved. Although the proposed algorithm, the square-root algorithm and the iterative nulling and cancellation algorithm have the same performance when the numerical precision is infinite, future research should be conducted to compare their fixed-point DSP implementations.

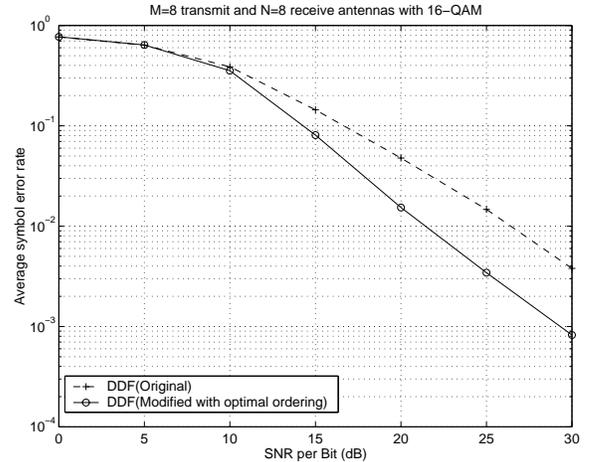


Fig. 2. Average symbol error rate of the original and modified decorrelating decision-feedback (DDF) detectors for BLAST system with  $M = 8$  transmit antennas,  $N = 8$  receive antennas and 16-QAM modulation.

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