# SNR-INDEPENDENT VELOCITY ESTIMATION FOR MOBILE CELLULAR COMMUNICATIONS SYSTEMS

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## ABSTRACT

Knowledge of the velocity of a mobile terminal is useful for a variety of radio resource management functions being contemplated for future wireless communications systems. Currently, the radial component of velocity of a mobile terminal can be inferred from estimating the maximum Doppler fading bandwidth. Unfortunately, the practicality of current methods are limited since they require an estimate of the signal-to-noise ratio (SNR) of the link. In this paper, we propose two novel autocorrelation function (ACF) based velocity estimators. These estimation methods are then extended to estimate mobile velocity without requiring knowledge of the SNR of the link. Monte-Carlo simulation of the proposed estimators are provided and compared to that of Sampath and Holtzman [1].

## 1. INTRODUCTION

The radial component of a mobile terminal's velocity has a linear relationship with the maximum Doppler frequency $f_m$. Velocity information can be used for radio resource management including improved handoff prediction.

The received signal envelope can be used to measure velocity in a variety of ways, including counting the number of level crossings of a sampled signal [2], recursive maximum likelihood estimation on envelope samples [3], or using an eigen-matrix pencil spectral estimation technique [4]. In [1], Holtzman derives a covariance approximation method.

In addition to envelope information, in-phase or quadrature components of a demodulated signal can also be used to estimate velocity, either directly [2], or using higher order statistics of both quadrature components and envelope [5], or using spectral moments [6]. All these methods [1, 2, 3, 4, 5, 6] assume that the signal-to-noise ratio (SNR) can be estimated from received signals very accurately. However, in practical systems, SNR information is not easily obtainable. In this paper, we propose two ACF estimation algorithms, one based on in-phase or quadrature components and other based on signal envelope information. These methods are then extended to estimate the velocity without SNR information.

This paper is organized as follows: Section 2 describes the short-term isotropic fading model and the application to time-division multiple access (TDMA) systems. Section 3 reviews covariance based estimation of velocity proposed in [1]. In Sections 4 and 5, the proposed SNR-free IQ and envelope-based estimates are derived. Section 6 presents comparisons through Monte-Carlo simulation.

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## 2. SIGNAL MODEL

The general model used for multipath Rayleigh fading is first proposed by Clarke [7]. Let $x_L(k) = x_I(k) + jx_Q(k)$ represent the lowpass equivalent signal at the mobile terminal

$$x_l(k) = \sum_{n=1}^{L} \alpha_n \exp(j(\omega_n k + \theta_n)) \qquad (1)$$

where $\alpha_n$ and $\theta_n$ are the amplitude and phase of the $n^{th}$ multipath component, respectively, $\omega_c$ is the carrier frequency and $\omega_n$ is the Doppler frequency for the $n^{th}$ path and $L$ is the number of paths arriving at the receiver. By the Central Limit Theorem, if $L$ is large, $x_I(k)$ and $x_Q(k)$ can be considered as stationary normal random processes with zero-mean and variance $\sigma^2$. The signal is corrupted by additive white Gaussian noise (AWGN) that is bandlimited at the receiver. Therefore, the received narrowband signal has a complex Gaussian distribution. The envelope is Rayleigh-distributed.

In TDMA systems, for a narrowband transmitted signal, the received low pass signal can be written as

[4]

$$r(t) = \sum_i x_i p(t - iT) \sum_{n=1}^{L} \alpha_n \exp(j\phi_n(t)) + w(t) \quad (2)$$

where $T$ is the symbol duration, $\alpha_n$ and $\phi_n(t)$ are the amplitude and phase for the $n^{th}$ path, $p(t)$ is the pulse shaping filter, $x_i$, $i = 0, 1 \ldots$ are data and $w(t)$ represents narrowband noise.

Assuming that inter-symbol interference can be ignored, the $k^{th}$ received sample is

$$r(k) = x_k \sum_{n=1}^{L} \alpha_n \exp(j\phi_n(kT)) + w(k) \quad (3)$$

If we transmit a training sequence, then $x_k$ is known at the receiver and (3) can be transformed as

$$r(k)/x_k = \sum_{n=1}^{L} \alpha_n \exp(j\phi_n(kT)) + w(k)/x_k \quad (4)$$

where $w(k)$ is a Gaussian process with zero mean, and if symbols $x_k$ have known values 1 or -1, $w(k)/x_k$ have the same statistical properties as $w(k)$.

Therefore, the above equation has the same form as (1), and estimation methods for $f_m$ derived for the general model can be applied to TDMA systems [4].

## 3. COVARIANCE METHOD

The following velocity estimation method is proposed in [1], where $V$ is defined as a measure of the squared deviations

$$V = \frac{1}{N} \sum_{i=0}^{N-1} (z_{i+1}^2 - z_i^2)^2 \quad (5)$$

$z_i$'s are samples of the envelope and N is the number of samples. If N is large and ergodicity applies, then V can be replaced by the ensemble average

$$\begin{aligned} E[V] &= 2[var(z_i^2) - cov(z_{i+1}^2, z_i^2)] \\ &= 8\sigma^4(1 - J_0^2(2\pi f_m)) + 8\sigma_n^4(1 - sinc^2(B_n T_s)) \\ &\quad +8\sigma_n^2\sigma^2(1 - J_0(2\pi f_m) sinc(B_n T_s)) \quad (6) \end{aligned}$$

where $\sigma_n^2 = 0.5 N_0 B_n$ is the averaged power of the in-band noise, and $\sigma^2$ is the averaged signal power. The power spectral density of the AWGN is $N_0/2$ and normalized Doppler frequency $f_m = F_m/F_s$. $B_n$,$F_s$,$T_s$ and $F_m$ are bandwidth of receiver filter, sampling rate, sampling interval and continuous Doppler frequency shift, respectively.

Assuming that $\sigma^2$ and $\sigma_n^2$ are known and $V$ is estimated from (5), we can estimate $f_m$ by solving (6).

## 4. IQ BASED ACF ESTIMATION

For Rayleigh fading channels, the in-phase component's ACF of the received signal is given by [7]

$$\begin{aligned} \phi_k &= E[x_I(i+k) x_I(i)] \\ &= \sigma^2 J_0(2\pi f_m k) + \sigma_n^2 sinc(B_n T_s k) \quad (7) \end{aligned}$$

where $J_0$ is the zero-order Bessel function of the first kind and $x_I(i)$ is the $i^{th}$ sample of the in-phase component. We define the ratios

$$\begin{aligned} c_1 &= \phi_1/\phi_0 \\ c_2 &= \phi_2/\phi_0 \quad (8) \end{aligned}$$

Inserting (7) into (8), we obtain

$$\begin{aligned} c_1 &= \frac{snr J_0(2\pi f_m) + sinc(B_n T_s)}{1 + snr} \\ c_2 &= \frac{snr J_0(4\pi f_m) + sinc(2 B_n T_s)}{1 + snr} \quad (9) \end{aligned}$$

where $snr = \frac{\sigma^2}{\sigma_n^2}$. If the SNR is known, $f_m$ can be estimated from $c_1$ over an invertible region of $J_0(.)$.

$$f_m = \frac{1}{2\pi} J_0^{-1} \left[ \frac{c_1(1 + snr) - sinc(B_n T_s)}{snr} \right] \quad (10)$$

However, if the SNR is not known, both equations in (9) can be combined to determine $f_m$ via

$$c_3 J_0(4\pi f_m) - c_4 J_0(2\pi f_m) = c_5 \quad (11)$$

where

$$\begin{aligned} c_3 &= sinc(B_n T_s) - c_1 \quad (12) \\ c_4 &= sinc(2 B_n T_s) - c_2 \quad (13) \\ c_5 &= c_2 sinc(B_n T_s) - c_1 sinc(2 B_n T_s) \quad (14) \end{aligned}$$

If $\hat{c_1} = \frac{\hat{\phi_1}}{\hat{\phi_0}}$ and $\hat{c_2} = \frac{\hat{\phi_2}}{\hat{\phi_0}}$ are estimates of $c_1$ and $c_2$ from $N$ samples, $\hat{f_m}$ can be obtained by solving (11) numerically, where $\phi_0$, $\phi_1$, and $\phi_2$ are estimated via

$$\begin{aligned} \hat{\phi_0} &= \frac{1}{N} \sum_{i=1}^{N} E[x_i x_i] \\ \hat{\phi_1} &= \frac{1}{N-1} \sum_{i=1}^{N-1} E[x_i x_{i+1}] \\ \hat{\phi_2} &= \frac{1}{N-2} \sum_{i=1}^{N-2} E[x_i x_{i+2}] \quad (15) \end{aligned}$$

The accuracy of the above $f_m$ estimate depends on the accuracy of estimators $\hat{c_1}$ and $\hat{c_2}$ . Since $x_I(k)$ is a stationary ergodic normal process, it can be shown that $\hat{\phi_0}$, $\hat{\phi_1}$ and $\hat{\phi_2}$ are maximum likelihood estimators (MLE). According to the invariance property of MLE, it can be shown that $f_m$ estimated from (11) is also an MLE.

In the case $x \ll 1$, Bessel function $J_0(x) \approx 1 - \frac{x^2}{4}$ and an approximated closed-form expression can be obtained from (11):

$$\hat{f_m} \approx \sqrt{\frac{c_3 - c_4 - c_5}{c_3\pi^2 + 4c_4\pi^2}} \quad (16)$$

An SNR estimate can also be obtained from (9) as

$$snr = \frac{sinc(2B_nT_s) - c_2}{c_2 - J_0(4\pi f_m)} \quad (17)$$

## 5. ENVELOPE-BASED ACF

The IQ-based estimates in the previous section are based on the assumption that we can coherently demodulate the received signal. Alternatively, envelope-based ACF estimation can be used. From [7],

$$\begin{aligned} \phi_{zz}(k) &= E[z(i+k)z(i)] \\ &= \frac{\pi}{4}\Omega_p F[-0.5, -0.5; 1, \rho^2(k)] \end{aligned} \quad (18)$$

where $\rho^2(k) = \frac{4}{\Omega_p^2}E^2(x_I(i+k)x_I(i))$, $x_I$ is the in-phase component, $z(k)$ is the envelope at the $k^{th}$ sampling instant, $\Omega_p$ is the averaged received power and $F(.)$ is the hypergeometric function.

By considering AWGN and approximating $F(.)$ by the first two terms, we can generalize the above expression to obtain

$$\phi_{zz}(k) \approx \frac{\pi}{4}\Omega_p(1 + \frac{1}{4}[\frac{\sigma^2 J_0(2\pi f_m k) + \sigma_n^2 sinc(B_nT_sk)}{\sigma^2 + \sigma_n^2}]^2)$$

Defining similar ratios as (8),

$$\begin{aligned} e_1 &= (\phi_{zz}(1))(\phi_{zz}(0)) \\ &= \frac{4}{5}[1 + \frac{1}{4}\frac{(snrJ_0(2\pi f_m) + sinc(B_nT_s))^2}{(1+snr)^2}] \\ e_2 &= (\phi_{zz}(2))(\phi_{zz}(0)) \\ &= \frac{4}{5}[1 + \frac{1}{4}\frac{(snrJ_0(4\pi f_m) + sinc(2B_nT_s))^2}{(1+snr)^2}] \end{aligned} \quad (19)$$

If SNR is known, $f_m$ can be estimated from $e_1$ directly

$$f_m = \frac{1}{2\pi}J_0^{-1}[\frac{\sqrt{5e_1 - 4}(1+snr) - sinc(B_nT_s)}{snr}] \quad (20)$$

Next we consider $f_m$ estimation without SNR, in which both equations in (19) are combined to obtain the identical form as (11) with $c_3$,$c_4$ and $c_5$ replaced by

$$\begin{aligned} e_3 &= sinc(B_nT_s) - \sqrt{53_1 - 4} \\ e_4 &= sinc(2B_nT_s) - \sqrt{53_2 - 4} \\ e_5 &= \sqrt{5e_2 - 4}sinc(B_nT_s) - \sqrt{5e_1 - 4}sinc(2B_nT_s) \end{aligned} \quad (21)$$

In (21), $e_1$ and $e_2$ can be estimated by $\hat{e_1} = \frac{\hat{\phi_{zz}(1)}}{\hat{\phi_{zz}(0)}}$ and $\hat{e_2} = \frac{\hat{\phi_{zz}(2)}}{\hat{\phi_{zz}(0)}}$, where $\hat{\phi_{zz}(0)}$ ,$\hat{\phi_{zz}(1)}$ and $\hat{\phi_{zz}(2)}$ can be estimated by (15), with envelope samples replacing the in-phase components. Doppler frequency $\hat{f_m}$ is obtained by solving

$$e_3J_0(4\pi f_m) - e_4J_0(2\pi f_m) = e_5 \quad (22)$$

For small $f_m$, a closed-form approximation analogous to (16) can be obtained with $e_3$, $e_4$ and $e_5$ replacing $c_3$, $c_4$ and $c_5$, respectively.

We can also estimate the snr from (19) via

$$snr = \frac{sinc(B_nT_s) - \sqrt{5e_1 - 4}}{\sqrt{5e_1 - 4} - J_0(2\pi f_m)} \quad (23)$$

We remark that we have also derived an alternative estimator based on Lee's expression for the envelope ACF [8], which turns out to have very similar performance and computation complexity.

## 6. SIMULATION RESULTS

Our simulations are based on the isotropic scattering environment outlined in the model in Section 2. In the following, the carrier frequency $f_c$ is 2GHz and 50 Monte-Carlo trials are used.

For both IQ and envelope-based ACF estimation, we generate noiseless fading signals using [9]. A front-end bandpass filter with bandwidth $B_n$ is used at the receiver. To solve (11) we need to invert the Bessel function, which requires that $F_s$ and $B_n$ be chosen carefully to ensure that there exists a unique root of the nonlinear equation: we set $F_s = 1600Hz$, $B_n = 800Hz$, and limit the velocity to within 185 km/h. There is therefore some dependence of velocity on estimator performance. At low velocity, say less than $100km/h$, estimation is quite accurate. With increasing velocity, estimation error increases, but only moderately.

Fig. 1 shows bias encoutered in SNR estimation using (17) and (23). Low bias is obtained for SNRs less than about 20 dB. Also, the envelope-based method is inferior to the IQ-based method. Figures 2 and 3 depict

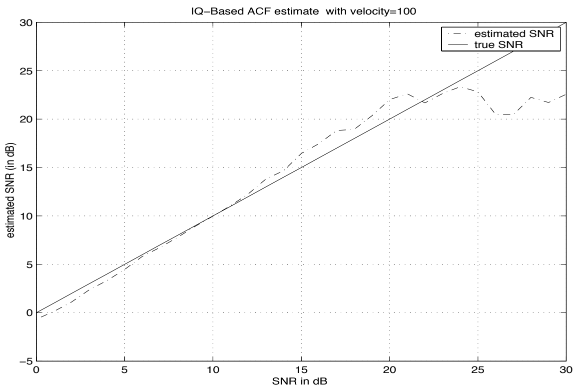


Fig. 1. Mean SNR estimate versus true SNR. IQ-Based ACF Estimate. True mobile velocity is 100km/h.

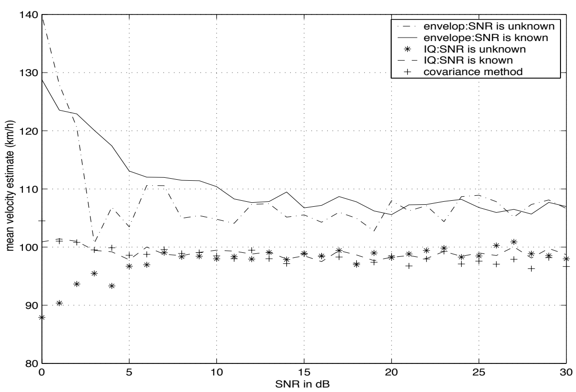


Fig. 2. Mean velocity estimate versus SNR. True mobile velocity is 100km/h.

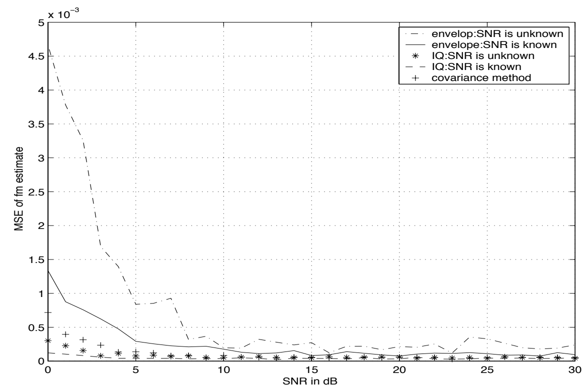


Fig. 3. Mean-square error of $f_m$ estimate versus SNR. True mobile velocity is 100km/h.

bias and mean-squared-error of the following estimators as a function of SNR: IQ-based estimation with (i) SNR known, (ii) SNR unknown, (iii) envelope-based estimation with SNR known, (iv) SNR unknown and (v) the covariance estimator [1] which requires known SNR. For known SNR, we see that IQ-based estimation has the best performance, followed by the convariance estimator. Also, the estimators that assume known SNR outperform the proposed SNR-free estimators, which is expected since additional ACF estimates provide the missing information. For 100 km/h as well as at other velocities tested, we also observe that IQ-based estimators generally outperform the envelope-based estimators.

In conclusion, while the proposed ACF-based estimator of mobile velocity using signal envelope, while not as accurate as that of [1], the practical advantage of not requiring knowledge of link SNR.

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