EUROPEAN TRANSACTIONS ON TELECOMMUNICATIONS
Euro. Trans. Telecomms. 2003; **14**:1–7 (DOI: 10.1002/ett.922)

Letter

Telecommunication Transmissions

Multi-user multi-antenna CDMA receivers for asynchronous multipath fading channels

Ruifeng Wang[1] and Steven D. Blostein[2]*

[1] *Netro Corporation, 9461 Willows Road NE, Redmond, WA 98052, USA*
[2] *Department of Electrical and Computer Engineering, Queen's University, Kingston, Ontario, K7L 3N6, Canada*

SUMMARY

In this paper, we investigate joint channel estimation and signal detection approaches for a multi-user CDMA uplink with a base-station antenna array over asynchronous multipath fading channels. Based on the space alternating generalized expectation-maximization (SAGE) algorithm, we develop an iterative multi-user receiver to jointly estimate channel response vectors and bit sequences for all the active users at known fractional time delays, and a BER lower bound is derived. A simplified bit detection method is then proposed for multi-rate CDMA systems. Simulation results show that the new iterative receiver performs equally well for both high- and low-rate users, and the BER performance is close to that of the BER lower bound. Copyright © 2003 AEI.

1. INTRODUCTION

In direct-sequence code division multiple access (DS-CDMA) systems, each user is distinguished by a unique spreading code. When the spreading codes are orthogonal, there will be no multi-access interference (MAI). However, due to asynchronous transmission and multi-path channel propagation, it is impossible to guarantee orthogonality for random relative time delays on uplink channels. Therefore, MAI exists in practical CDMA systems, and significant research efforts have been directed toward its suppression. Two promising approaches are digital beamforming using base-station antenna arrays [1] and multi-user signal detection [2].

Recently, expectation-maximization (EM)-type approaches have been proposed to achieve CDMA signal detection [3], channel estimation [4–6] as well as joint parameter estimation and signal detection for single-antenna [7] and multiple-antenna systems [8].

In this paper, we investigate joint channel estimation and signal detection for asynchronous multi-antenna, multi-path CDMA systems. We apply the space alternating generalized EM (SAGE) algorithm [9] and extend the receiver proposed in Reference [8] to asynchronous multi-path channels, and incorporate multi-path diversity combining. The receiver structure involves multi-user signal decoupling, multi-path signal decoupling and channel response vector estimation. We also derive a BER lower bound.

Our discrete-time problem formulation is presented in Section 2. In Section 3, we apply the SAGE algorithm and obtain an iterative receiver structure for joint channel vector estimation and information symbol detection assuming known time delays. A simplification to wideband

*Correspondence to: Steven D. Blostein, Department of Electrical and Computer Engineering, Queen's University, Kingston, Ontario, K7L 3N6, Canada. E-mail: sdb@ece.queensu.ca

Contract/grant sponsor: Canadian Institute for Telecommunications Research, NCE program of the Government of Canada.

Received 17 August 2001
Revised 25 February 2002
Accepted 25 February 2002

multi-rate CDMA systems is also presented, and simulation results are provided in Section 4.

2. DISCRETE-TIME SYSTEM MODEL

We consider an asynchronous CDMA uplink with an M-element base-station antenna array for a multi-path fading channel. The kth mobile user transmits a block of N-bits with waveform

$$s_k(t) = A_k \sum_{i=1}^{N} b_k(i) c_k(t - iT_b) \tag{1}$$

with amplitude A_k, equiprobable ith bit $b_k(i) \in \{-1, 1\}$, spreading waveform $c_k(t) = \sum_{l=0}^{L-1} c_{kl} p(t - lT_c)$, where $c_{kl} \in \{+1, -1\}$ $(l = 0, \ldots, L-1)$ is the spreading code, $p(t)$ is the chip waveform of duration T_c, T_b is the bit interval and processing gain $L = T_b/T_c$. The spreading waveform has energy normalized to unity, i.e. $\int_0^{T_b} |c_k(t)|^2 = 1$.

The signal received at the base-station antenna array from K users, each with P propagation paths and transmitting N bits is given by vector function

$$\mathbf{x}(t) = \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{p=1}^{P} A_k \alpha_{k,p} \mathbf{a}(\theta_{k,p}) \times c_k(t - iT_b - \tau_{k,p}) b_k(i) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{a}(\theta_{k,p})$ is the M-dimensional channel vector. For performance evaluation purposes, we parameterize this vector by direction-of-arrival (DOA) $\theta_{k,p}$ for the pth path of the kth user, $\alpha_{k,p}$ and $\tau_{k,p}$ represent the channel attenuation and relative time delay for the kth user through the pth propagation path respectively. We assume $\tau_{k,p} \in [0, T_b)$ for $k \in \{1, \ldots, K\}$ and $p \in \{1, \ldots, P\}$. The channel fading attenuations $\alpha_{k,p}(t)$ for $k \in \{1, \ldots, K\}$ and $p \in \{1, \ldots, P\}$ are assumed to be mutually independent. The vector function $\mathbf{n}(t)$ models additive white Gaussian noise (AWGN) with zero mean and covariance matrix $\sigma^2 \mathbf{I}_M$, where $\mathbf{I}_M$ is an $M \times M$ identity matrix. We assume that DOAs and channel attenuations for all users remain unchanged over the N-bit duration. We denote the channel vector for the pth path of the kth user as

$$\mathbf{f}_{k,p} = A_k \alpha_{k,p} \mathbf{a}(\theta_{k,p}) \tag{3}$$

In this asynchronous system, we choose $2T_b$ to be the observation interval to collect samples for each bit. Using the notation in Reference [10], we obtain the discrete-time system model for $2ML$ received samples in the interval $2T_b$ as

$$\mathbf{x}(i) = \sum_{k=1}^{K} H_k \mathbf{b}_k^w(i) + \mathbf{n}(i) \tag{4}$$

where $\mathbf{b}_k^w(i) = [b_k(i-1) \;\; b_k(i) \;\; b_k(i+1)]^T$, $H_k = \sum_{p=1}^{P} H_{k,p}$, and $\mathbf{n}(i)$ is an AWGN vector with zero-mean and covariance matrix $\frac{\sigma^2}{L} \mathbf{I}_{2ML}$, where $\mathbf{I}_{2ML}$ is a $2ML \times 2ML$ identity matrix. In Equation (4), matrix H_k incorporates both spatial and temporal characteristics of our channel. For $k \in \{1, \ldots, K\}$ and $p \in \{1, \ldots, P\}$,

$$H_{k,p} = \left[C_{k,p}^{-1} \;\vdots\; C_{k,p}^{0} \;\vdots\; C_{k,p}^{1} \right] \begin{bmatrix} \mathbf{f}_{k,p} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{f}_{k,p} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{f}_{k,p} \end{bmatrix} \equiv C_{k,p} F_{k,p} \tag{5}$$

where the $2ML \times M$ matrix of time-shifted codes $C_{k,p}^n$, for $n = -1, 0, 1$, is given by

$$C_{k,p}^n = \begin{bmatrix} \mathbf{c}_{k,p}^n & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{c}_{k,p}^n \end{bmatrix}$$

where $\mathbf{c}_{k,p}^n$, for $n = -1, 0, 1$, is the kth user's spreading code vector with time delay corresponding to the pth path.

Equation (4) can be expressed compactly as

$$\mathbf{x}(i) = \mathcal{H} \mathbf{b}^w(i) + \mathbf{n}(i) \tag{6}$$

where $\mathcal{H} = [H_1 \vdots \cdots \vdots H_K]$ and $\mathbf{b}^w(i) = [\mathbf{b}_1^w(i)^T \cdots \mathbf{b}_K^w(i)^T]^T$. The desired bit sequence can be obtained by maximizing the likelihood function, for $i \in \{1, \ldots, N\}$, as

$$\begin{aligned} \hat{\mathbf{b}}^w(i) &= \text{sign}\{\mathbf{b}^w(i) + \mathcal{R}^{-1} \mathcal{H}^H \mathbf{n}(i)\} \\ &= \text{sign}\{\mathbf{b}^w(i) + \mathbf{w}(i)\} \end{aligned} \tag{7}$$

where $\text{sign}\{a\} = 1$ if $a \geqslant 0$ or -1 if $a < 0$, $\mathcal{R} = \mathcal{H}^H \mathcal{H}$, and where the (k,j)th 3×3 sub-matrix in $\mathcal{R}$, $\mathcal{R}_{kj}$, is given by

$$\mathcal{R}_{kj} = H_k^H H_j = \sum_{p=1}^{P} F_{k,p}^H C_{k,p}^H \sum_{q=1}^{P} C_{j,q} F_{j,q}$$

In Equation (7), $\mathbf{w}(i)$ is an output AWGN vector with zero mean and covariance matrix $\frac{\sigma^2}{L} \mathcal{R}^{-1}$. The detector structure based on Equation (7) is illustrated in Figure 1.

3. ITERATIVE MULTI-USER RECEIVER

In this section, we investigate the problem of joint signal detection and channel array response vector estimation

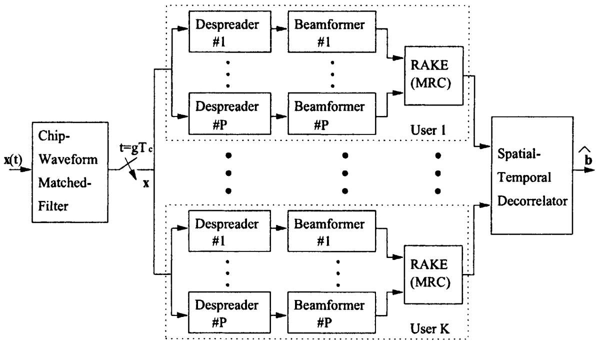

Figure 1. Spatial-temporal decorrelating detector when channel parameters are known.

assuming that we know the time delays at the receiver. In the case where time delay information cannot be accurately estimated, the following methods can be generalized in a manner as found in Reference [11] with a corresponding reduction in the maximum number of supported users. A receiver structure was previously developed from the SAGE algorithm for synchronous single fading channels [8]. We extend this derivation to the case of asynchronous multi-path fading channels. The resulting receiver structure, as will be derived, is fundamentally different from that of Reference [8].

3.1. Iterative multi-user receiver

The observed incomplete data is the set of vectors $\{\mathbf{x}(i);\, i = 1, \ldots, N\}$. In applying SAGE, we choose user index k as the index set to detect the information bit sequences. Thus, the admissible hidden-data space, or complete data, for index k and $i = 1, \ldots, N$ is given by

$$\mathbf{x}_k^S(i) = H_k \mathbf{b}_k^w(i) + \mathbf{n}(i) = \sum_{p=1}^{P} H_{k,p} \mathbf{b}_k^w(i) + \mathbf{n}(i) \tag{8}$$

Given the spatial channel estimation results at the jth iteration, $\hat{F}_{k,p}^j$, for $k = 1, \ldots, K$ and $p = 1, \ldots, P$, and detected symbols, $\hat{\mathbf{b}}_k^{w^j}(i)$, at the jth iteration, the conditional expectation of $\mathbf{x}_k^S(i)$, for $k = 1, \ldots, K$ and $i = 1, \ldots, N$, is estimated as

$$\hat{\mathbf{x}}_k^{Sj}(i) = \mathbf{x}(i) - \sum_{k_1=1, k_1 \neq k}^{K} \sum_{p=1}^{P} \hat{H}_{k_1,p}^j \hat{\mathbf{b}}_{k_1}^{w^j}(i) \tag{9}$$

where we have denoted $\hat{H}_{k,p}^j = C_{k,p} \hat{F}_{k,p}^j$ for $k = 1, \ldots, K$ and $p = 1, \ldots, P$.

The maximization results at the next iteration are given by

$$\left[\hat{F}_k^{j+1}, \hat{\mathbf{b}}_k^{w^{j+1}}\right] = \arg \max_{H_k, \mathbf{b}_k^w(i)} \Omega\left(\hat{\mathbf{x}}_k^{Sj}(1), \ldots, \hat{\mathbf{x}}_k^{Sj}(N)\right)$$

where $\Omega(\hat{\mathbf{x}}_k^{Sj}(1), \ldots, \hat{\mathbf{x}}_k^{Sj}(N))$ is the log-likelihood function of $\hat{\mathbf{x}}_k^S(i)$ at the jth iteration.

To estimate $F_{k,p}$, for $k = 1, \ldots, K$ and $p = 1, \ldots, P$, we further decouple the complete data $\mathbf{x}_k^S(i)$ by choosing path index p as the index subset. The corresponding admissible hidden-dataset for user index k, path index p, and bit i, $1 \leqslant i \leqslant N$, is given by

$$\mathbf{x}_{k,p}^S(i) = H_{k,p} \mathbf{b}_k^w(i) + \mathbf{n}(i) \tag{10}$$

Using Equation (9), the conditional expectation of $\mathbf{x}_{k,p}^S(i)$, for $i = 1, \ldots, N$, is obtained as

$$\begin{aligned} \hat{\mathbf{x}}_{k,p}^{Sj}(i) &= \hat{\mathbf{x}}_k^{Sj}(i) - \sum_{p_1=1, p_1 \neq p}^{P} \hat{H}_{k,p_1}^j \hat{\mathbf{b}}_k^{w^j}(i) \\ &= \mathbf{x}(i) - \sum_{p_1=1, p_1 \neq p}^{P} \hat{H}_{k,p_1}^j \hat{\mathbf{b}}_k^{w^j}(i) \\ &\quad - \sum_{k_1=1, k_1 \neq k}^{K} \sum_{p=1}^{P} \hat{H}_{k_1,p}^j \hat{\mathbf{b}}_{k_1}^{w^j}(i) \end{aligned} \tag{11}$$

At the jth iteration,

$$\begin{aligned} &\Omega\left(\hat{\mathbf{x}}_{k,p}^{Sj}(1), \ldots, \hat{\mathbf{x}}_{k,p}^{Sj}(N)\right) \\ &= -\frac{1}{\sigma^2/L} \sum_{i=1}^{N} \left(\hat{\mathbf{x}}_{k,p}^{Sj}(i) - H_{k,p} \mathbf{b}_k^{w^j}(i)\right)^H \\ &\quad \times \left(\hat{\mathbf{x}}_{k,p}^{Sj}(i) - H_{k,p} \mathbf{b}_k^{w^j}(i)\right) \end{aligned} \tag{12}$$

By maximizing the likelihood, we estimate the spatial-temporal channel matrix at the $(j+1)$th stage, $\hat{H}_{k,p}^{j+1}$, for $k = 1, \ldots, K$ and $p = 1, \ldots, P$ (see Appendix A for details),

$$\hat{H}_{k,p}^{j+1} = \left[\sum_{i=1}^{N} \hat{\mathbf{x}}_{k,p}^{Sj}(i) \hat{\mathbf{b}}_k^{wjH}(i)\right] \left[\sum_{i=1}^{N} \hat{\mathbf{b}}_k^{wj}(i) \hat{\mathbf{b}}_k^{wjH}(i)\right]^{-1} \tag{13}$$

Since $\hat{H}_{k,p} = C_{k,p} F_{k,p}$ with known $C_{k,p}$, we may employ a maximum-likelihood criterion to obtain unknown matrix $F_{k,p}$, which under a Gaussian-error model is equivalent to a least-squares criterion, i.e. minimizing $J(\theta) = (\hat{H}_{k,p} - C_{k,p} F_{k,p})^H (\hat{H}_{k,p} - C_{k,p} F_{k,p})$. From Equation (5), if the time delays are non-zero, the columns of $C_{k,p}$ are full-rank and

$$\hat{F}_{k,p}^{j+1} = \left(C_{k,p}^H C_{k,p}\right)^{-1} C_{k,p}^H \hat{H}_{k,p}^{j+1} \tag{14}$$

The above channel array response vector estimates are then used to detect each user's information sequence at the $(j+1)$ th stage via

$$\hat{\mathbf{b}}_k^{w^{j+1}}(i) = \mathrm{sign}\left[\left(\hat{\mathcal{R}}_{kk}^{j+1}\right)^{-1} \hat{H}_k^{j+1^H} \hat{\mathbf{x}}_k^{Sj}(i)\right]$$

where $\hat{\mathcal{R}}_{kk}^{j+1} = \hat{H}_k^{j+1^H} \hat{H}_k^{j+1}$
and

$$\hat{H}_k^{j+1} = \sum_{p=1}^{P} C_{k,p} \hat{F}_{k,p}^{j+1}$$

After initial channel estimation using at least one training bit per user, the iterations for joint channel estimation and symbol detection are now summarized:
For $j = 1, 2, \ldots;\ k = (j \text{ modulo } K)$

E-step:
For $i = 1, \ldots, N$, apply Equation (9):

$$\hat{\mathbf{x}}_k^{Sj}(i) = \mathbf{x}(i) - \sum_{k_1=1, k_1 \neq k}^{K} \sum_{p=1}^{P} C_{k_1,p} \hat{F}_{k_1,p}^{j} \hat{\mathbf{b}}_{k_1}^{wj}(i) \qquad (15)$$

For $p = 1, \ldots, P$, apply Equation (11):

$$\hat{\mathbf{x}}_{k,p}^{Sj}(i) = \hat{\mathbf{x}}_k^{Sj}(i) - \sum_{p_1=1, p_1 \neq p}^{P} C_{k,p_1} \hat{F}_{k,p_1}^{j} \hat{\mathbf{b}}_k^{wj}(i) \qquad (16)$$

M-step:
For $p = 1, \ldots, P$, apply Equations (14) and (13):

$$\hat{F}_{k,p}^{j+1} = \left(C_{k,p}^H C_{k,p}\right)^{-1} C_{k,p}^H \left[\sum_{i=1}^{N} \hat{\mathbf{x}}_{k,p}^{Sj}(i) \hat{\mathbf{b}}_k^{wjH}(i)\right] \times \left[\sum_{i=1}^{N} \hat{\mathbf{b}}_k^{wj}(i) \hat{\mathbf{b}}_k^{wjH}(i)\right]^{-1} \qquad (17)$$

$$\hat{H}_k^{j+1} = \sum_{p=1}^{P} C_{k,p} \hat{F}_{k,p}^{j+1}$$

$$\hat{\mathcal{R}}_{kk}^{j+1} = \hat{H}_k^{j+1^H} \hat{H}_k^{j+1}$$

For $i = 1, \ldots, N$

$$\hat{\mathbf{b}}_k^{w^{j+1}}(i) = \mathrm{sign}\left[\left(\hat{\mathcal{R}}_{kk}^{j+1}\right)^{-1} \hat{H}_k^{j+1^H} \hat{\mathbf{x}}_k^{Sj}(i)\right] \qquad (18)$$

$$\begin{aligned} \hat{F}_{k',p}^{j+1} &= \hat{F}_{k',p}^{j}, \quad k' \neq k \\ \hat{\mathbf{b}}_{k'}^{w(j+1)}(i) &= \hat{\mathbf{b}}_{k'}^{wj}(i), \quad k' \neq k \end{aligned}$$

Since the SAGE algorithm guarantees that the likelihood function increases monotonically with each iteration, the proposed iterative spatial-temporal receiver converges to a fixed stationary point or local/global maxima depending on the initial guess of the unknown parameters. In Reference [8], we have observed that using an array of highly correlated antenna signals accelerates the receiver's convergence to its local BER minimum. A BER lower bound is derived in Appendix B. From Equations (15) and (16), we observe that the computational complexity increases linearly with the number of the users in the system.

Remark: Compared to the synchronous single path case [8], the key issue for an asynchronous multi-path receiver algorithm is multi-path identification and estimation. Equations (15) and (16) propose a two-stage decoupling of the received signal. The first step, Equation (15), decouples the signals according to user codes. The multi-path signals for each user are then resolved in the second step, Equation (16), and are used to form a maximum-likelihood estimates of the vector channel impulse response used for optimum RAKE combining.

3.2. Simplified bit detection algorithm for wideband systems

The above receiver structure obtained can be applied to third-generation (3G) multi-rate CDMA such as WCDMA and cdma2000. To do this, we define rate-ratio $r = T_l/T_h$, where T_h and T_l are the bit durations for high-rate and low-rate users respectively. That is, when a low-rate user transmits one bit, a high-rate user transmits r bits.

Assuming that the multi-path delays lie within T_h, the observation interval is, $T_h + T_l$. Thus, $\hat{\mathcal{R}}_{kk}^{-1}$ in Equation (18) is a $(r+2) \times (r+2)$ matrix for high-rate users. Since in 3G systems r may be quite large, i.e., $r \geqslant 32$, the complexity for the matrix inversion in Equation (18) becomes prohibitive. To simplify the receiver, we propose to omit the decorrelation in Equation (18), i.e. we replace Equation (18) by

$$\hat{\mathbf{b}}_k^{w^{j+1}} = \mathrm{sign}\left[\hat{H}_k^{j+1^H} \hat{\mathbf{x}}_k^{Sj}(i)\right] \qquad (19)$$

This can be justified in wideband CDMA systems since the differences between the relative time delays for different paths are larger than the chip interval and inter-path correlation is very low. In other words, the despreading process alone is enough to resolve the received signals from different paths for each user. Therefore, it is expected that simplified bit detection, Equation (19), without a multi-path decorrelator would have only slight performance degradation. Using the simplified bit detector, the detection algorithm at each iteration for each user is equivalent to conventional single-user detection, i.e. the decision variable at each iteration for each user is obtained by despreading, beamforming and then RAKE combining.

4. SIMULATIONS

We perform simulations where we insert one training bit at the start of each sequence to obtain initial channel estimates. A uniform linear array with half-wavelength spacing is used at the base-station and DOAs are uniformly distributed in $[-60°, 60°]$. The time delay of the first path for each user is uniformly distributed in $(0, T_b/2)$ and the time delay of the second path for each user is uniformly distributed in $(T_b/2, T_b)$. Multi-path channels are assumed to be Rayleigh-distributed with same covariance for all paths.

We consider a dual-rate system with rate-ratio $r = 4$, two high-rate and two low-rate users. Gold sequences of length 31 are assigned to the high-rate users. For low-rate users, we obtain length-124 spreading sequences by repeating each length-31 Gold sequence four times. We transmit a 100-bit information sequence from each high-rate user and a 25-bit information sequence for each low-rate user simultaneously. We apply four iterations of the receiver bit detection algorithm. Figures 2 and 3 show the BER performance of the proposed iterative receivers for high-rate and low-rate users respectively. As expected for wideband CDMA, the simplified receiver of Section 3.2 achieves similar performance to that of a detector having a multi-path decorrelator. Note that simplified suboptimum bit sequence detection may outperform the multi-path decorrelation detector in certain SNR regions since (a) the correlation matrix in Equation (18) is a function of imperfect channel estimates, and (b) a decorrelation detector has a well-known noise-enhancement property [2]. Note also from Figures 2 and 3 that the BER performance for high-rate and low-rate users is close to that of their lower bounds and that the two lower bounds are equivalent.

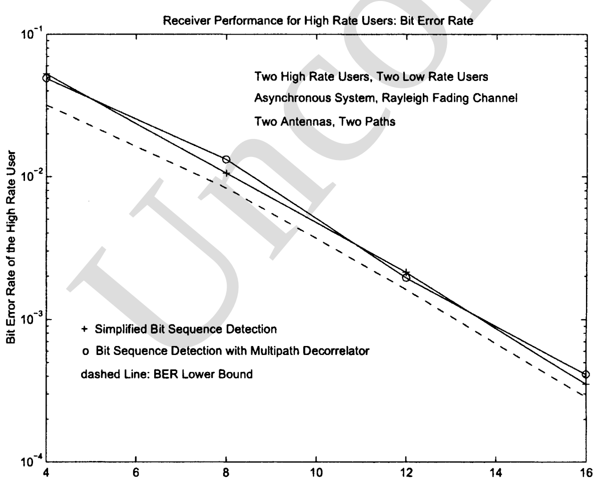

Figure 2. Bit Error Rate (BER) of the desired high-rate user.

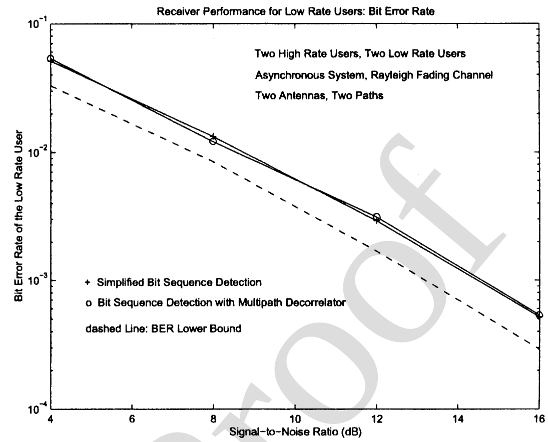

Figure 3. Bit Error Rate (BER) of the desired low-rate user.

5. CONCLUSIONS

In this paper, we have developed a multi-user receiver structure for joint channel vector estimation and bit sequence detection in asynchronous multi-path fading channels and derived a BER lower bound. The detection and estimation is based on the SAGE algorithm, and typically requires about four iterations to converge to a locally optimum point. The results are then fed to a RAKE multi-path combiner. We also proposed a method to lower signal detection complexity for multi-rate wideband CDMA systems, while maintaining comparable BER performance. Simulation results confirm that the BER performance for high-rate and low-rate users is similar and close to their respective BER lower bounds.

APPENDIX A: DERIVATION OF $\hat{H}_{k,p}$ IN EQUATION (13)

Discarding terms independent of $H_{k,p}$ in Equation (12), the likelihood function is rewritten (j superscripts omitted) as

$$\Omega = \frac{1}{\sigma^2/L} \sum_{i=1}^{N} \Big[\mathbf{b}_k^{wT}(i) H_{k,p}^H H_{k,p} \mathbf{b}_k^w(i) - 2\hat{\mathbf{x}}_{k,p}^{SH}(i) H_{k,p} \mathbf{b}_k^w(i) \Big] \quad (20)$$

We denote $H_{k,p}(m,n)$, for $m = 1, \ldots, 2ML$ and $n = 1, 2, 3$, as the (m,n)th component of the matrix $H_{k,p}$ and $\hat{\mathbf{x}}^S_{k,p}(i) = (\hat{x}^S_{k,p}(i,1) \cdots \hat{x}^S_{k,p}(i, 2ML))^T$. For notational consistency, we also denote

$$\left(b^w_k(i,1) b^w_k(i,2) b^w_k(i,3)\right) = \mathbf{b}^w_k(i)$$

The derivative of Equation (20) with respect to $H_{k,p}$, in matrix form, is

$$\begin{bmatrix} \frac{\partial \Omega}{\partial H_{k,p}(1,1)} & \frac{\partial \Omega}{\partial H_{k,p}(1,2)} & \frac{\partial \Omega}{\partial H_{k,p}(1,3)} \\ \vdots & \vdots & \vdots \\ \frac{\partial \Omega}{\partial H_{k,p}(2ML,1)} & \frac{\partial \Omega}{\partial H_{k,p}(2ML,2)} & \frac{\partial \Omega}{\partial H_{k,p}(2ML,3)} \end{bmatrix} \quad (21)$$

For $m = 1, \ldots, 2ML$ and $n = 1, 2, 3$, we obtain

$$\frac{\partial \Omega}{\partial H_{k,p}(m,n)} = \frac{2}{\sigma^2/L} \sum_{i=1}^{N} H^*_{k,p}(m) \mathbf{b}^w_k(i) b^w_k(i,n) - \left(\hat{x}^S_{k,p}(i,m)\right)^* b^w_k(i,n) \quad (22)$$

where $H^*_{k,p}(m) = (H^*_{k,p}(m,1)\ H^*_{k,p}(m,2)\ H^*_{k,p}(m,3))$ is a 3-dimensional row vector for $m = 1, \ldots, 2ML$. After some straightforward operations, Equation (21) can be expressed in the compact form

$$\frac{\partial \Omega}{\partial H_{k,p}} = \frac{2}{\sigma^2/L} \sum_{i=1}^{N} H^*_{k,p} \mathbf{b}^w_k(i) \mathbf{b}^{wT}_k(i) - \left(\hat{\mathbf{x}}^S_{k,p}(i)\right)^* \mathbf{b}^{wT}_k(i) \quad (23)$$

Equating Equation (23) to zero, and noting that $\mathbf{b}^w_k(i)$ is a real vector, we obtain Equation (13).

APPENDIX B: DERIVATION OF BER LOWER BOUND

From Equation (15) and (18), we obtain the decision variable for the kth user

$$\hat{\mathbf{y}}^{j+1}_k = \left(\hat{\mathcal{R}}^{j+1}_{kk}\right)^{-1} \hat{H}^{j+1^H}_k \times \left[\mathbf{x}(i) - \sum_{k_1=1, k_1 \neq k}^{K} \sum_{p=1}^{P} \hat{H}^j_{k_1} \hat{\mathbf{b}}^{wj}_{k_1,p}(i)\right] \quad (24)$$

From Equation (24), it is observed that the MAI is subtracted at each iteration. To obtain a BER lower bound, we assume that all MAI has been eliminated at the final iteration in the receiver and the channel array response vectors of user k are known perfectly. Thus, the decision variable of the kth user is given by $\mathbf{y}_k = \mathbf{b}^w_k + \mathbf{w}_k(i)$ where $\mathbf{w}_\mathbf{k}(i)$ is a zero mean AWGN vector with covariance matrix $\frac{\sigma^2}{L}\mathcal{R}^{-1}_{kk}$. To compute the BER of the desired bit $b_k(i)$ which is the second component in vector $\mathbf{b}^w_k(i)$, we first express the corresponding average SNR as $\bar{\gamma}_k = \frac{1}{\frac{\sigma^2}{L}E[(\mathcal{R}^{-1}_{kk})_2]}$, where $(\mathcal{R}^{-1}_{kk})_2$ is the second diagonal component of matrix $\mathcal{R}^{-1}_{kk}$ and $E[\cdot]$ represents the expectation over the Rayleigh-distributed channel attenuation. Since the self-correlation of the spreading code between the current bit interval and the previous bit interval due to multi-path delays is very small compared to that between current bit intervals due to different path delays, we obtain‘tpb 3pt

$$(\mathcal{R}^{-1}_{kk})_2 = \frac{1}{\sum_{p=1}^{P} \sum_{q=1}^{P} \alpha^*_{k,p} \alpha_{k,q} \mathbf{a}(\theta_{k,p}) \mathbf{a}(\theta_{k,q}) \rho^{0,0}_{kk,pq}}$$

where $\rho^{0,0}_{kk,pq}$ is the self-correlation of the spreading code between path p and path q for the kth user at the current bit interval. Since we have assumed that the channel attenuations are mutually independent, we have

$$E[(\mathcal{R}^{-1}_{kk})_2] = \frac{1}{MA^2_k \sum_{p=1}^{P} E[\alpha^2_{k,p}] \rho^{0,0}_{kk,pp}}$$

where $\alpha^2_{k,p} = \alpha^*_{k,p}\alpha_{k,p}$. Because $\rho^{0,0}_{kk,pp}$ is identically distributed for $p = 1, \ldots, P$, the average SNR for user k is obtained as

$$\bar{\gamma}_k = \frac{MLA^2_k \rho^{0,0}_{kk,11}}{\sigma^2} \sum_{p=1}^{P} E\left[\alpha^2_{k,p}\right] \quad (25)$$

Assuming that $E[\alpha^2_{k,p}]$ are identical for all P paths of user k, the BER lower bound for the kth user can be obtained [12] (pp. 781)

$$P_k = [(1-\mu)/2]^P \sum_{p=0}^{P-1} \binom{P-1+p}{p} [(1+\mu)/2]^p \quad (26)$$

where $\mu = \sqrt{\frac{\bar{\gamma}_k}{(1+\bar{\gamma}_k)}}$

REFERENCES

1. Q2 Paulraj AJ, Papadias CB. Space-time processing for wireless communications. *IEEE Signal Processing Magazine* 1997; 49–83. Q1
2. Moshavi S. Multi-user detection for DS-CDMA communications. *IEEE Communications Magazine* 1996; **34**(10):124–136.
3. Nelson LB, Poor HV. Iterative multiuser receivers for CDMA channels: an EM-Based approach. *IEEE Transactions on Communications* 1996; **44**(12):1700–1710.

4. Fawer U, Aazhang B. A multiuser receiver for code division multiple access communications over multipath channels. *IEEE Transactions on Communications* 1995; **43**(2/3/4):1556–1565.
5. Ertin E, Mitra U, Siwamogsatham S. Iterative techniques for DS/CDMA multipath channel estimation. In 1998 Allerton Conference, Monticello, IL., Sept. 1998.
6. Fleury BH, Tschudin M, Heddergott R, Dahlhaus D, Pedersen KI. Channel parameter estimation in mobile radio environments using the SAGE algorithm. *IEEE Journal on Selected Areas in Communications* 1999; **17**(3):434–450.
7. Sharfer I, Hero AO. Optimum multiuser CDMA detector using grouped coordinate ascent and the DWT. Proceedings IEEE 1997 Workshop in Signal Processing Advances in Wireless Communications, Paris, April 1997.
8. Wang R, Blostein SD. A spatial-temporal decorrelating receiver for CDMA systems with base-station antenna arrays. *IEEE Transactions on Communications* 2001; **49**(2):329–340.
9. Fessler JA, Hero AO. Space-Alternating generalized EM algorithm. *IEEE Transactions Signal Processing* 1994; **42**(10):2664–2677.
10. Madhow U. Blind adaptive interference suppression for the near-far resistant acquisition and demodulation of direct-sequence CDMA signals. *IEEE Transactions Signal Processing* 1997; **45**(1):124–136.
11. Zha W, Blostein SD. Improved CDMA Multiuser Receivers Robust to Timing Errors. Proceedings IEEE International Conference on Acoustic, Speech and Signal Proceedings 2001.
12. Proakis JG. *Digital Communications*, 3rd edn. McGraw-Hill, Inc., 1995.

AUTHORS' BIOGRAPHIES

Ruifeng Wang (S'96-M'00) received the B.S. and M.S. degrees from Northern Jiaotong University, Beijing, China, in 1986 and 1989 respectively, and the Ph.D. from Queen's University, Kingston, ON, Canada, in 1999, all in Electrical Engineering.

He is currently with Netro Corporation, Redmond, W.A. Before joining Netro, he was a wireless systems engineer with R&D department, COM DEV, and with Wireless Local Technologies Group, AT&T Wireless, sebsequently. His research interests lie in space-time signal processing and radio resource management for wireless communications.

Steven D. Blostein received his B.S. degree in Electrical Engineering from Cornell University in 1983, and the M.S. degree and Ph.D. in Electrical and Computer Engineering from the University of Illinois at Urbana-Champaign, in 1985 and 1988 respectively. He has been on the Faculty at Queen's University since 1988 and currently holds the position of professor in the Department of Electrical and Computer Engineering. His current interests lie in statistical signal processing, wireless communications and video image communications. He is a senior member of IEEE and served as associate editor for *IEEE Transactions on Image Processing* from 1996 to 2000. He currently leads the Multirate Wireless Data Access Major Project sponsored by the Canadian Institute for Telecommunications Research and five industrial affiliates.

Author Query Form (ETT/922)

Special Instructions: Author please write responses to queries directly on Galley proofs and then fax back. Alternatively please list responses in an e-mail.

Q1: Please check the short title.

Q2: Author: Please provide volume no.