MINIMUM BER TRANSMIT POWER ALLOCATION AND BEAMFORMING FOR TWO-INPUT MULTIPLE-OUTPUT SPATIAL MULTIPLEXING SYSTEMS

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ABSTRACT

Transmit optimization for two-input multiple-output (TIMO) spatial multiplexing systems is investigated. Minimization of bit error rate (MBER) is employed as the optimization criterion. MBER transmit power allocation for a variety of receiver structures is proposed. An approximate MBER transmit beamforming scheme is also proposed, which eliminates error floors in ill-conditioned TIMO channels. It is shown that the newly proposed transmit optimization schemes together with interference cancellation and detection ordering offer superior performance over existing general precoding methods, with reduced complexity as well as feedback overhead.

1. INTRODUCTION

Wireless communications using multiple transmit and receive antennas, known as multiple-input multiple-output (MIMO) systems, offers a number advantages over single-input single-output (SISO) systems, such as diversity and spatial multiplexing gains [1]. Our goal of this paper is to investigate transmit optimization for MIMO spatial multiplexing systems with two transmit antennas, known also as two-input multiple-output (TIMO). The study of such systems can be motivated in a number of ways: 1) TIMO systems are important in practical scenarios where there are limitations on cost and/or space to install more antennas; 2) a virtual TIMO channel is created when two single-antenna mobiles operate in cooperative communication mode [2]; 3) when transmit antenna selection is employed in MIMO to achieve diversity with reduced cost of transmit radio frequency chains [3], selecting two out of multiple transmit antennas turns MIMO into TIMO; 4) it is easier to analyze TIMO systems than the more general MIMO systems, and these analyses offer insights into MIMO system design and performance.

When channel state information (CSI) is available at the transmitter, system performance can be improved. Transmit optimization is receiver-dependent. Signal reception for spatial multiplexing can employ criteria such as zero-forcing (ZF), minimum mean squared-error (MMSE), successive interference cancellation (SIC), or ordered SIC (OSIC) as, for example, in the case of Vertical Bell Laboratories Layered Space-Time (V-BLAST) [4]. Efforts to optimize MIMO transceiver structures include, e.g., MMSE precoding/decoding [5] and MBER precoding for ZF equalization [6]. These schemes, however, generally require high feedback overhead and/or high complexity processing, e.g., diagonalization of the channel matrix.

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In this paper, we consider complexity and feedback overhead reduction by introducing structural constraints to precoding. For well-conditioned TIMO channels, precoding is constrained to be *transmit power allocation*, i.e., we optimize only the transmit power of signal streams. Minimization of bit error rate (MBER) is employed as the optimization criterion. When the TIMO channel is ill-conditioned, MBER transmit power allocation is shown to experience error floors. To improve performance in ill-conditioned channels, we develop an approximate MBER *transmit beamforming* scheme. It is shown that the proposed transmit optimization schemes offer superior performance over existing precoding methods in general correlated fading channels.

2. TIMO CHANNEL AND SIGNAL RECEPTION

The received signal of a TIMO system with $N_r \geq 2$ receive antennas can be modelled as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \boldsymbol{\eta} = s_1\mathbf{h}_1 + s_2\mathbf{h}_2 + \boldsymbol{\eta}, \tag{1}$$

where $\mathbf{s} = [s_1 \;\; s_2]^T$ denotes the transmitted signal vector; $\mathbf{H} = [\mathbf{h}_1 \vdots \mathbf{h}_2]$ is the $N_r \times 2$ channel matrix, which is assumed to be generally correlated Ricean fading [7]; and $\boldsymbol{\eta}$ is the $N_r \times 1$ additive Gaussian noise vector. For simplicity of analysis purposes, we assume white input and noise, i.e., $\mathbb{E}[\mathbf{s}\mathbf{s}^{\mathcal{H}}] = E_s\mathbf{I}_2$ and $\mathbb{E}[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathcal{H}}] = N_0\mathbf{I}_{N_r}$, input signal-to-noise ratio (SNR) $\gamma_s \overset{\text{def}}{=} E_s/N_0$, and binary phase shift keying (BPSK) modulation.

2.1. TIMO Signal Reception

2.1.1. ZF Receiver

With ZF equalization, the transmitted signal is estimated as

$$\hat{\mathbf{s}} = \mathbf{H}^{\dagger}\mathbf{r} = \mathbf{s} + \mathbf{H}^{\dagger}\boldsymbol{\eta}. \tag{2}$$

The decision-point SNR of the k-th signal stream is obtained as

$$\gamma_{Z,k} = \gamma_s \left[\left(\mathbf{H}^{\mathcal{H}}\mathbf{H} \right)^{-1} \right]^{-1}_{k,k} \overset{\text{def}}{=} \gamma_s g^2_{Z,k}, \qquad k = 1, 2, \tag{3}$$

where $g^2_{Z,k} \overset{\text{def}}{=} [(\mathbf{H}^{\mathcal{H}}\mathbf{H})^{-1}]^{-1}_{k,k}$ denotes the power gain of k-th stream. The power gains can be calculated as

$$g^2_{Z,1} = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_2\|^2}, \quad g^2_{Z,2} = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_1\|^2}, \tag{4}$$

where $\Delta_{\mathbf{H}} \overset{\text{def}}{=} \|\mathbf{h}_1\|^2\|\mathbf{h}_2\|^2 - |\mathbf{h}_2^{\mathcal{H}}\mathbf{h}_1|^2$.

2.1.2. *SIC Receiver*

Without loss of generality (w.l.o.g.), we assume that stream $k = 1$ is detected first. Assuming ZF equalization is employed, the power gain of detecting s_1 is the same as that of the ZF receiver, i.e.,

$$g_{S,1}^2 = g_{Z,1}^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_2\|^2}. \quad (5)$$

Assuming that $\hat{s}_1 = s_1$, the interference due to the first stream is then regenerated and subtracted, i.e.,

$$\mathbf{r}' = \mathbf{r} - \hat{s}_1 \mathbf{h}_1 = s_2 \mathbf{h}_2 + \boldsymbol{\eta}. \quad (6)$$

The detection of s_2 in (6) with ZF equalization is given by $\hat{s}_2 = \mathbf{h}_2^\dagger \mathbf{r}' = s_2 + \frac{\mathbf{h}_2^{\mathcal{H}} \boldsymbol{\eta}}{\|\mathbf{h}_2\|^2}$, which is equivalent to maximal ratio combining (MRC), with power gain

$$g_{S,2}^2 = \|\mathbf{h}_2\|^2. \quad (7)$$

2.1.3. *OSIC Receiver*

To improve SIC performance, the streams can be reordered based on SNR at each stage. The SNR-based ordering scheme [4] detects the stream with largest decision-point SNR first, or, equivalently, detects the stream with largest power gain first. From (4), the stream to be detected first is

$$k_1 = \arg\max_l \gamma_{Z,l} = \arg\max_l g_{Z,l}^2 = \arg\max_l \|\mathbf{h}_l\|^2.$$

i.e., SNR-based ordering is equivalent to norm-based ordering in TIMO systems[1]. Therefore, we obtain the power gains as

$$g_{O,1}^2 = \frac{\Delta_{\mathbf{H}}}{\min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\}}, \quad g_{O,2}^2 = \min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\}. \quad (8)$$

The average BER of the above receivers can be calculated as [8][2]

$$\bar{P}(\gamma_s; g_1^2, g_2^2) = \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_1^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_2^2}\right), \quad (9)$$

where $\mathcal{Q}(x) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy$. The gain g_k^2 depends on the receiver structure and is given in (4), (5), (7) and (8).

2.2. Ill-Conditioned TIMO Channels

Since the $\mathcal{Q}(\cdot)$ function decreases rapidly in its argument, the average BER in (9) is dominated by the term with smaller power gain. In the extreme case with vanishing power gain, the system experiences an error floor. We refer to this as an *ill-conditioned* TIMO channel. From the power gains given in (4), (5), (7) and (8), we know that the channel is ill-conditioned when either $\Delta_{\mathbf{H}} \approx 0$ or $\min\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\} \approx 0$. It can be shown that, w.l.o.g., we can assume $\mathbf{h}_2 \approx a \cdot \mathbf{h}_1$ with $a \in \mathbb{C}$ [9]. The ill-conditioned TIMO channel can be modelled as

$$\mathbf{H} \approx \mathbf{h}_1 [1 \ \ a], \quad (10)$$

which is also an example of a "pinhole" channel [10]. The least-squares (LS) estimate of a can be found to be

$$\hat{a}_{LS} = \mathbf{h}_1^\dagger \mathbf{h}_2 = \frac{\mathbf{h}_1^{\mathcal{H}} \mathbf{h}_2}{\|\mathbf{h}_1\|^2}. \quad (11)$$

[1]We note that this does not apply to general MIMO with $N_t \geq 3$.

[2]This is a lower bound for SIC and OSIC due to the neglecting of error propagation, which is also an accurate approximate at moderate-to-high SNR's.

3. MBER TRANSMIT POWER ALLOCATION FOR TIMO

Denote the power allocated to the k-th stream as p_k^2 ($k = 1, 2$). The received signal can be written as

$$\mathbf{r} = p_1 s_1 \mathbf{h}_1 + p_2 s_2 \mathbf{h}_2 + \boldsymbol{\eta}. \quad (12)$$

We assume that the total transmit power is constrained via $p_1^2 + p_2^2 = 2$.

The average BER (9) can be straightforwardly generalized to

$$\bar{P}(\gamma_s; \{g_k^2\}; \{p_k^2\}) = \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_1^2 p_1^2}\right) + \frac{1}{2}\mathcal{Q}\left(\sqrt{2\gamma_s g_2^2 p_2^2}\right). \quad (13)$$

To minimize (13) under transmit power constraint, no closed-form solution exists. However, taking the approach in [11], we approximate the objective function to obtain a closed-form solution with performance very close to the MBER solution. Using the approximate BER formula given in [12][3], an approximate MBER solution is obtained as [11]

$$p_k^2 = \gamma_s^{-1} g_k^{-2} \left(\ln g_k^2 + \nu\right)_+, \quad (k = 1, 2), \quad (14)$$

where $(x)_+ \stackrel{\text{def}}{=} \max\{0, x\}$, and ν is chosen to satisfy power constraint. Note the fact that the total transmit power $p_1^2 + p_2^2$ is a piecewise-linear function in ν, with breakpoints at $-\ln g_1^2$ and $-\ln g_2^2$. W.l.o.g., assuming $g_1^2 \geq g_2^2$, we can simplify (14) as

$$\begin{cases} p_1^2 = 2, \ p_2^2 = 0 & \text{if } \ln\left(\frac{g_1^2}{g_2^2}\right) \geq 2\gamma_s g_1^2 \\ p_1^2 = \frac{\ln g_1^2 + \nu_a}{\gamma_s g_1^2}, \ p_2^2 = \frac{\ln g_2^2 + \nu_a}{\gamma_s g_2^2} & \text{otherwise} \end{cases}, \quad (15)$$

where $\nu_a = \frac{2\gamma_s g_1^2 g_2^2 - g_1^2 \ln g_2^2 - g_2^2 \ln g_1^2}{g_1^2 + g_2^2}$. We note that in the first case of (15), the solution $p_1^2 = 2$ and $p_2^2 = 0$ implies that the stream with weaker power gain is dropped, and all available power is allocated to the stronger stream. This occurs when either $g_1^2 \gg g_2^2$ or γ_s is small.

For OSIC, we need to examine the effect of MBER power allocation on ordering. W.l.o.g., we assume $\|\mathbf{h}_1\| \geq \|\mathbf{h}_2\|$. In norm-based ordering, s_1 is detected first. Denote the corresponding power gains as $\alpha_1^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_2\|^2}$, and $\alpha_2^2 = \|\mathbf{h}_2\|^2$. Consider the opposite detection ordering. Denote the resulting power gains as $\beta_1^2 = \frac{\Delta_{\mathbf{H}}}{\|\mathbf{h}_1\|^2}$ and $\beta_2^2 = \|\mathbf{h}_1\|^2$. Note that $\alpha_1^2 \alpha_2^2 = \beta_1^2 \beta_2^2$. By assumption ($\|\mathbf{h}_1\| \geq \|\mathbf{h}_2\|$), we have $\beta_1^2 \leq \alpha_1^2 \leq \beta_2^2$, and $\beta_1^2 \leq \alpha_2^2 \leq \beta_2^2$. At moderate-to-high SNR ($\gamma_s \gg 1$), it can be shown that [9]

$$\bar{P}(\gamma_s; \alpha_1^2, \alpha_2^2) \leq \bar{P}(\gamma_s; \beta_1^2, \beta_2^2).$$

In other words, for TIMO at moderate-to-high SNR, norm-based ordering, which is optimal for TIMO without power allocation, is also optimal for the MBER power allocation scheme, in the sense of minimizing average BER.

Remark 3.1 *Feedback Overhead and Complexity Issues:* For a TIMO system using a general precoding method, either the channel or precoding matrix is required at the transmitter. The proposed power allocation scheme requires only transmit power information. Precoding schemes require diagonalization of a channel matrix [5, 6]. Using power allocation, operations performed at the transmitter are trivial.

[3]The BER can approximated as $P_b(\gamma) \approx \frac{1}{5}\exp\{-c\gamma\}$, where c is a constellation-specific constant. Therefore, extension of the results in what follows is straightforward.

Remark 3.2 *Application Scenarios:* A TIMO configuration is appropriate for the uplink of a wireless system, where each mobile terminal is equipped with two transmit antennas while the basestation may have more antennas. The downlink, on the other hand, has a multiple-input two-output (MITO) structure. To exploit the inherent transmit diversity in such a MITO system, transmit processing is necessary [5]. Alternatively, transmit antenna selection [3] may be employed to reduce costly transmit radio frequency chains. Selecting two out of multiple transmit antennas results in a two-input two-output system, which belongs to the general TIMO family, and our proposed power allocation applies.

Remark 3.3 *Performance in Ill-Conditioned Channels:* W.l.o.g., we assume $|a| \leq 1$ in (10). The power gains of OSIC can be obtained as $g_{O,1}^2 \approx 0$ and $g_{O,2}^2 = |a|^2 \|\mathbf{h}_1\|^2$. Applying power allocation (14), we obtain $p_1^2 = 0$, and $p_2^2 = 2$. Average BER can be approximated as

$$\bar{P}(\gamma_s; \mathbf{h}_1, a) \approx \frac{1}{10} + \frac{1}{10} \exp\left\{-2\gamma_s |a|^2 \|\mathbf{h}_1\|^2\right\},$$

which experiences error floors. This motivates our study of general precoding for ill-conditioned TIMO channels.

4. PRECODING FOR ILL-CONDITIONED TIMO

W.l.o.g., assume $\|\mathbf{h}_1\| \geq \|\mathbf{h}_2\|$, or, equivalently, $|a| \leq 1$. Denote the precoding matrix

$$\mathbf{P}_b = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}.$$

The transmit power constraint is given by

$$\operatorname{tr}\left(\mathbf{P}_b \mathbf{P}_b^{\mathcal{H}}\right) = |p_{11}|^2 + |p_{12}|^2 + |p_{21}|^2 + |p_{22}|^2 = 2.$$

The received signal is $\mathbf{r} = \mathbf{H}\mathbf{P}_b\mathbf{s} + \boldsymbol{\eta}$. With ZF equalization, the estimate of the transmitted signal

$$\begin{aligned} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} &= \mathbf{H}^\dagger \mathbf{r} = \mathbf{H}^\dagger \mathbf{H} \mathbf{P}_b \mathbf{s} + \mathbf{H}^\dagger \boldsymbol{\eta} \\ &= \frac{1}{1+|a|^2} \left(\begin{bmatrix} 1 & a \\ a^* & |a|^2 \end{bmatrix} \begin{bmatrix} p_{11}s_1 + p_{12}s_2 \\ p_{21}s_1 + p_{22}s_2 \end{bmatrix} \right. \\ &\quad \left. + \frac{\mathbf{h}_1^{\mathcal{H}} \boldsymbol{\eta}}{\|\mathbf{h}_1\|^2} \begin{bmatrix} 1 \\ a^* \end{bmatrix} \right). \end{aligned} \tag{16}$$

From (16), we observe that s_1 and s_2 are coupled in z_1 and z_2, and $z_2 = a^* z_1$. Therefore, it suffices to consider

$$z_1' \stackrel{\text{def}}{=} (1+|a|^2) z_1 = (p_{11} + a p_{21}) s_1 + (p_{12} + a p_{22}) s_2 + \frac{\mathbf{h}_1^{\mathcal{H}} \boldsymbol{\eta}}{\|\mathbf{h}_1\|^2}.$$

W.l.o.g., we assume $\Re(p_{11} + ap_{21}) \geq 0$, $\Re(p_{12} + ap_{22}) \geq 0$, and $\Re(p_{11} + ap_{21}) \geq \Re(p_{12} + ap_{22})$. As a result, s_1 is detected first, and s_2 is then detected after performing interference cancellation. An approximate MBER precoding solution is found to be [9]

$$\mathbf{P}_b = \sqrt{\frac{2}{5}} \left(1 + |a|^2\right)^{-1/2} \begin{bmatrix} 1 \\ a^* \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix}, \tag{17}$$

which has rank one, and can be viewed as power allocation (vector $[2, 1]$) followed by transmit beamforming (vector $[1, a^*]^T$) pointing to the approximate MBER direction. We refer to this scheme as *OSIC with MBER transmit beamforming*. Average BER can be approximated by [9]

$$\bar{P}_{IC}^{BF}(\gamma_s; \mathbf{h}_1, a) \approx \frac{3}{20} \exp\left\{-\frac{4}{5}\gamma_s \|\mathbf{h}_1\|^2 (1 + |a|^2)\right\}, \tag{18}$$

which does not experience error floor effects.

Remark 4.1 It is of interest to compare power allocation with the proposed transmit beamforming in ill-conditioned TIMO channels. Using the same notation as in Section 2 and similar argument as presented in this section, a power allocation solution that approximately minimizes error rate can be found to be $p_1 = 2\,|\Re(a)|\, p_2,\ p_2 = \sqrt{2}\left(1 + 4[\Re(a)]^2\right)^{-1/2}$ [9]. The average BER for power allocation in ill-conditioned channels can be approximated as [9]

$$\bar{P}_{IC}^{PA}(\gamma_s; \mathbf{h}_1, a) \approx \frac{3}{20} \exp\left\{-\frac{2\gamma_s \|\mathbf{h}_1\|^2 [\Re(a)]^2}{1 + 4[\Re(a)]^2}\right\}. \tag{19}$$

It is obvious that (19) experiences error floors when $\Re(a) \approx 0$. Therefore, power allocation alone cannot eliminate error floor effects in ill-conditioned TIMO channels, whereas power allocation together with beamforming can.

Remark 4.2 *4-PAM- versus QPSK-mixing:* We note that the power allocation vector in (17) pre-mixes two BPSK streams into a 4-PAM stream. It is also possible to pre-mix two BPSK streams into a two-dimensional constellation, e.g., QPSK as in [13]. Performance and complexity issues of these two different pre-mixing schemes will be considered in future work.

Remark 4.3 *Feedback Overhead and Complexity Issues:* From (17), only an estimate of a is required at the transmitter, which can be obtained using (11). Operations performed at the transmitter are also trivial.

5. NUMERICAL RESULTS AND DISCUSSIONS

Figs. 1 and 2 compare BER performance of the proposed MBER power allocation method for ZF, SIC and OSIC receivers with two existing precoding methods in well-conditioned and ill-conditioned channels, respectively. In addition, comparison of OSIC with MBER transmit beamforming as proposed in Section 4 is also made. In our simulations, we adopt the spatial fading correlation model for general non-isotropic scattering given in [7]. The following parameters are chosen: $N_r = 4$ receive antennas; transmit and receive antenna spacings expressed in wavelengths are 0.5 and 10, respectively; angles of arrival/departure of the deterministic component are $\pi/6$ and 0, respectively; angle spread $10°$; and BPSK modulation is used for the purposes of comparison with [6].

Fig. 1 shows average BER's in an uncorrelated Rayleigh fading channel. To clarify the plot, performances of ZF with power allocation and SIC without power allocation are not shown since they are nearly identical to that of MMSE precoding/decoding; OSIC without power allocation (also not shown) has performance close to that of ZF with MBER precoding. We observe that at a BER of 10^{-3}, the proposed power allocation scheme offers 0.6, 1.4 and 0.8 dB gains over ZF, SIC and OSIC receivers, respectively. SIC and OSIC with power power allocation outperform precoding schemes. We can also see that OSIC with MBER beamforming, though designed for ill-conditioned channels, outperforms OSIC without power allocation at SNR's larger than 5 dB.

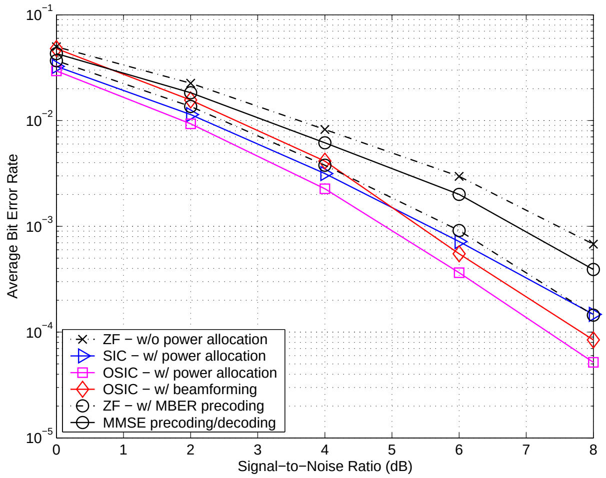

Fig. 1. Average BER performance in uncorrelated Rayleigh fading TIMO channel ($N_t = 2$, $N_r = 4$).

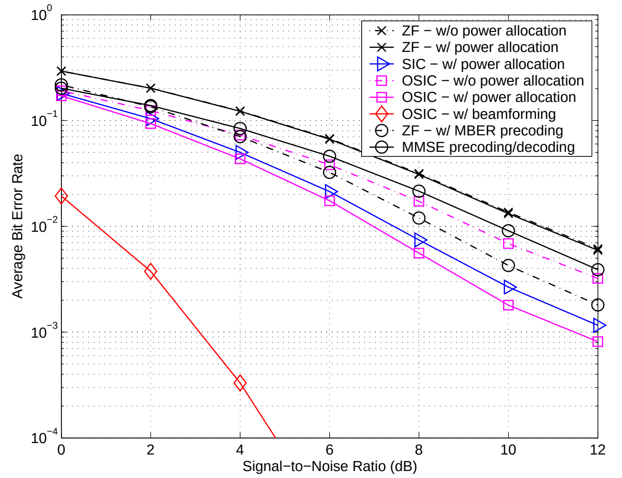

Fig. 2. Average BER performance in correlated Ricean fading TIMO channel ($N_t = 2$, $N_r = 4$, $K = 8$ dB).

Fig. 2 illustrates average BER's in a correlated Ricean fading channel. Performance of SIC without power allocation (not shown) is nearly identical to that of MMSE precoding/decoding. Again, SIC and OSIC with power allocation outperform precoding schemes. We also observe that the proposed OSIC with MBER beamforming offers significant gain over power allocation and precoding schemes shown: at a BER of 10^{-3}, 8.5 dB SNR gain over OSIC with power allocation. This is as excepted since in Ricean fading, due to the existence of a line-of-sight (LOS) component, the channel matrix is likely to be ill-conditioned.

6. CONCLUSIONS

MBER transmit power allocation and beamforming for TIMO spatial multiplexing are proposed in this paper. It is shown that SIC and OSIC with MBER power allocation outperform existing precoding schemes in Rayleigh fading channels, and the proposed OSIC with MBER transmit beamforming eliminates error floors and offers superior performance over power allocation as well as general precoding schemes in correlated Ricean fading channels. Compared with more general precoding methods, the proposed schemes reduce both complexity and feedback overhead, and improve error rate performance.

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