Antenna Selection For Time-Varying Channels Based on Slepian Subspace Projections

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***Abstract*—Single receive antenna selection (AS) allows single-input single-output (SISO) systems to retain the diversity benefits of multiple antennas with minimum hardware costs. We propose a single receive AS method for time-varying channels, in which practical limitations imposed by next-generation wireless standards such as training, packetization and antenna switching time are taken into account. The proposed method utilizes low-complexity subspace projection techniques spanned by discrete prolate spheroidal (DPS) sequences. It only uses Doppler bandwidth knowledge, and does not need detailed correlation knowledge. Results show that the proposed AS method outperforms ideal conventional SISO systems with perfect CSI but no AS at the receiver and AS using the conventional Fourier estimation/prediction method. A closed-form expression for the symbol error probability (SEP) of phase-shift keying (MPSK) with symbol-by-symbol receive AS is derived.**

I. Introduction

In single receive antenna selection (AS) only one antenna element (AE) at the receiver is selected and connected to the radio-frequency (RF) chain based on the current channel fades [1], [2]. This enables the resulting system to retain most of the diversity benefits of multiple antennas with minimum hardware complexity. We note that AS has been standardized, e.g., in IEEE 802.11n, or is being standardized [3].

Previous studies on AS have focused on designing algorithms and analyzing performance [4]–[7]. To date, only a few studies exist that investigate practical issues such as training and implementation of AS [8]. The impacts of erroneous CSI due to noise on the performance of AS systems are studied in [9]–[13]. However, the mobile wireless channel is time-varying due to user mobility and multipath propagation. This implies that CSI gets rapidly outdated at the receiver. The effects of CSI feedback delay on the performance of AS systems are studied in [14], [15]. In [15], it is shown that CSI feedback delay alters the diversity order. A weighted single-antenna selection rule for time-varying channels which uses the temporal correlation knowledge is proposed in [16]. The general case of selecting a subset of AEs and the problem of training voids have been recently treated in [17]. However, we note that only channel gain estimates obtained from the pilot symbols during the AS training phase are used in the selection and decoding processes in [16] and [17]. This is because channel gain estimates over the data transmission phase are not available, which incurs a loss in signal-to-noise ratio (SNR).

The above observations motivate investigation into practical training-based AS methods for time-varying channels. They use CSI knowledge of the data transmission phase for selection and decoding by employing channel prediction/estimation. In this paper, we propose and analyze the performance of a training-based single receive AS method for time-varying channels that uses the low-training overhead Slepian estimator [18] and predictor [19]. We note that this Slepian estimator/predictor only requires knowledge of the Doppler bandwidth. In contrast, the optimal Wiener predictor requires detailed correlation knowledge, which is difficult to obtain [19]. The paper's contributions are summarized as follows:

- A discrete prolate spheroidal (DPS) based basis expansion model [18], [19] for accurately estimating/predicting time-varying frequency-flat channels is extended to AS.
- A closed-form expression for the SEP of MPSK with receive AS is provided, and verified with simulations.
- Extensive simulation results are presented to compare the performance of the proposed AS method with ideal conventional SISO systems with perfect CSI but no AS at the receiver and AS based on the conventional Fourier basis prediction/estimation method.

II. System Model

Consider a single-antenna transmitter and a K AE receiver equipped with a micro-electromechanical system (MEMS) based antenna switch to connect the selected AE to the RF chain.

As depicted in Fig. 1, in the AS training phase the transmitter transmits $L \geq 2$ training symbols sequentially in time to each AE to improve channel prediction. Consecutive pilots for AE k and AE $k+1$ are also separated in time by a duration of $T_\mathrm{p} \triangleq \alpha T_\mathrm{s}$, where T_s is the symbol duration and $\alpha \geq 2$. Therefore, the duration between two consecutive AS training pilots transmitted for each AE is $T_\mathrm{t} \triangleq K\,T_\mathrm{p} = \alpha\,K\,T_\mathrm{s}$. Time-varying frequency-flat fading and MPSK modulation with average energy $E_\mathrm{s} = 1$ are considered.

The received training signal at AE k, for $1 \leq k \leq K$, is

$$y_k\,[m] = h_k\,[m]\;p_k\,[m] + n_k\,[m]\,, \qquad m \in T_{\mathrm{tr}}^k \tag{1}$$

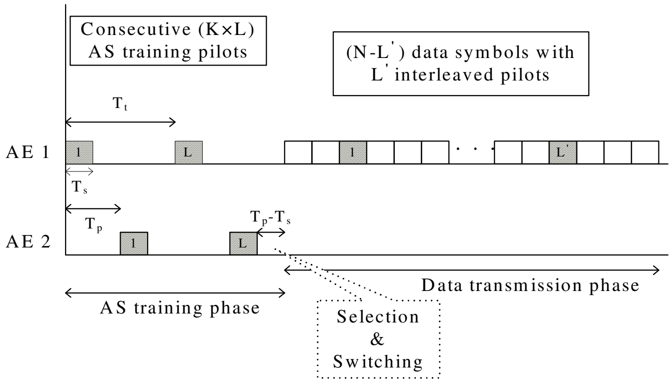

Fig. 1. Antenna selection cycle consists of AS training and data transmission phases. (AE 1 is selected, $K = 2$, $L = 2$, $L' = 2$, and $T_\mathrm{p} = 2\,T_\mathrm{s}$).

where $h_k\,[m]$ is the sampled time-varying channel gain, $p_k\,[m]$ is the pilot, and $n_k\,[m]$ is additive white Gaussian noise (AWGN) with variance N_0 and is independent of $h_k\,[m]$. In (1), the set of time indices when the L AS training pilots are received by AE k

$$T_{\text{tr}}^{k} \triangleq \{\alpha\;[(k-1)+K\;(\ell-1)]\}\,,\qquad 1 \leq \ell \leq L. \tag{2}$$

From (1), channel gain estimates $\left\{\tilde{h}_k\,[m]\;\mid m \in T_{\text{tr}}^{k}\right\}$ can be obtained as

$$\tilde{h}_k\,[m] = y_k\,[m]\;p_k^*\,[m] \triangleq h_k\,[m] + e_k^{\text{n}}\,[m] \tag{3}$$

where $e_k^{\text{n}}\,[m] \triangleq n_k\,[m]\;p_k^*\,[m]$ is the channel estimation error resulting from the AWGN.

Denoting by $\mathcal{I}_{\text{tr}} = \{0, 1, \ldots, M-1\}$ and $\mathcal{I}_{\text{dt}} = \{M, M+1, \ldots, M+N-1\}$ the training and data transmission phases, respectively. Next, the receiver performs Slepian prediction [19] for each AE over $\mathcal{I}_{\text{dt}}$ to obtain $\left\{\hat{h}_k^{\text{SP}}\,[m] \mid m \in \mathcal{I}_{\text{dt}}\right\}$. The receiver then selects and connects the selected AE to the RF chain in a duration of $T_p - T_s$, therefore $M = \alpha\,K\,L$.

We note that solid-state switches can enable switching of antennas between symbols. However, these switches have attenuations in the order of 1 to 3 dB. In contrast, MEMS switches have negligible attenuations, but they enable only per-packet switching. In general as the AS switching times and attenuations decrease, symbol-by-symbol switching may become viable in futuristic systems. Therefore, we consider both symbol-by-symbol and per-packet switching in our analysis since both are relevant.

In each data transmission phase, the transmitter transmits a length-N data packet which consists of $N - L'$ data symbols and L' post-selection pilots, interleaved as [18]

$$\mathcal{P} \triangleq \left\{\left\lfloor(\ell'-1)\,\frac{N}{L'} + \frac{N}{2L'}\right\rfloor \;\middle|\; 1 \leq \ell' \leq L'\right\}. \tag{4}$$

The post-selection pilots are thus received by AE $[\hat{1}]$ at times $m \in T_{\text{dt}}$, where

$$T_{\text{dt}} \triangleq \left\{M - 1 + \left\lfloor(\ell'-1)\;\frac{N}{L'} + \frac{N}{2\,L'}\right\rfloor \;\middle|\; 1 \leq \ell' \leq L'\right\}. \tag{5}$$

Using Slepian estimation [18], refined channel gain estimates $\left\{\hat{h}_{[\hat{1}]}^{\text{SE}}\,[m]\;\mid m \in \mathcal{I}_{\text{dt}}\right\}$ are obtained from the $L_{\text{tot}} \triangleq L + L'$ pilots received by AE $[\hat{1}]$ at times $m \in T_{\text{tot}}^{[\hat{1}]}$, where

$$T_{\text{tot}}^{[\hat{1}]} = T_{\text{tr}}^{[\hat{1}]} \cup T_{\text{dt}} \tag{6}$$

with $T_{\text{tr}}^{[\hat{1}]}$ and T_{dt} given in (2) and (5), respectively. The received signal at AE $[\hat{1}]$ can be expressed as

$$y_{[\hat{1}]}\,[m] = h_{[\hat{1}]}\,[m]\;s\,[m] + n_{[\hat{1}]}\,[m]\,,\qquad m \in \mathcal{I}_{\text{dt}} \tag{7}$$

where

$$s\,[m] = \begin{cases} d\,[m] & m \in \mathcal{I}_{\text{dt}} \backslash T_{\text{dt}} \\ p\,[m] & m \in T_{\text{dt}} \end{cases}. \tag{8}$$

In (8), $d\,[m]$ and $p\,[m]$ denote data and pilot, respectively.

III. Slepian Expansion Model

A. Discrete Prolate Spheroidal (DPS) Sequences

The rate of channel variations is upper bounded by the maximum normalized Doppler bandwidth

$$\nu_{\max} \triangleq \frac{v_{\max}\,f_{\text{c}}}{c}\,T_{\text{s}} \ll \frac{1}{2} \tag{9}$$

where $v_{\max}$ is the user velocity, f_{c} is the carrier frequency, and c is the speed of light. Time-limited snapshots of the band-limited fading process are spanned by the orthogonal DPS sequences $\{u_i\,[m] \mid m \in \mathbb{Z}\}_{i=0}^{M'-1}$, which are defined as [18]

$$\sum_{l=0}^{M'-1} \frac{\sin\left(2\pi\nu_{\max}\,(l-m)\right)}{\pi\,(l-m)}\,u_i\,[l] = \lambda_i\,u_i\,[m]\,,\;\; m \in \mathbb{Z} \tag{10}$$

where $i \in \mathcal{I}_{\text{bl}} = \{0, 1, \ldots, M'-1\}$ and eigenvalues $\{\lambda_i\}_{i=0}^{M'-1}$ decay exponentially for $i \geq \lceil 2\,\nu_{\max}\,M' \rceil + 1$. The DPS sequences are band-limited to the frequency range $\mathcal{W} = (-\nu_{\max}, +\nu_{\max})$ and energy-concentrated in the time interval $\mathcal{I}_{\text{bl}}$.

B. Slepian Estimator

The M'-length Slepian sequences $\left\{u_i\,[m] \mid m \in \mathcal{I}_{\text{bl}}\right\}_{i=0}^{M'-1}$ are the restrictions of the DPS sequences on $\mathcal{I}_{\text{bl}}$. The Slepian estimator approximates the $M' \times 1$ true channel vector $\boldsymbol{h} \triangleq \left[h\,[0]\,, h\,[1]\,, \ldots, h\,[M'-1]\right]^T$ by projecting it onto the subspace spanned by the D length-M' Slepian basis vectors $\{\boldsymbol{u}_i\}_{i=0}^{D-1}$ as [19]

$$\boldsymbol{h} \approx \hat{\boldsymbol{h}}^{\text{SE}} = \boldsymbol{U}\,\hat{\boldsymbol{\gamma}} = \sum_{i=0}^{D-1} \hat{\gamma}_i\,\boldsymbol{u}_i \tag{11}$$

where $\boldsymbol{U} \triangleq \left[\boldsymbol{u}_0, \ldots, \boldsymbol{u}_{D-1}\right]$ is an $M' \times D$ matrix, $\boldsymbol{u}_i \triangleq \left[u_i\,[0]\,, u_i\,[1]\,, \ldots, u_i\,[M'-1]\right]^T$, and D is given by [19]

$$D = \underset{d\,\in\,\{1,\ldots,J\}}{\operatorname{argmin}} \left(\frac{1}{2\,\nu_{\max}\,J}\sum_{i=d}^{J-1}\lambda_i + \frac{d}{J}\,N_0\right). \tag{12}$$

In (12), J is the number of interleaved pilots in the length-M' block. The coefficient vector $\hat{\boldsymbol{\gamma}} \triangleq \left[\hat{\gamma}_0, \hat{\gamma}_1, \ldots, \hat{\gamma}_{D-1}\right]^T$ is estimated using the J pilots $\{p[l] \mid l \in \mathcal{J}\}$, received at times $l \in \mathcal{J}$, via [18]

$$\hat{\gamma} = \boldsymbol{G}^{-1} \sum_{l \in \mathcal{J}} y[l]\; p^*[l]\; \boldsymbol{f}^*[l] \tag{13}$$

where $y[l]$ is the received signal and

$$\boldsymbol{G} = \sum_{l \in \mathcal{J}} \boldsymbol{f}\;[l]\; \boldsymbol{f}^\dagger[l]\,. \tag{14}$$

In (13) and (14) $\boldsymbol{f}[l] \triangleq \left[u_0[l], \ldots, u_{D-1}[l]\right]^T$.

C. Slepian Predictor

The Slepian predictor approximates the true channel gain $h[m]$ by projecting it onto the subspace spanned by the D minimum-energy (ME) extension of the Slepian sequences $\{u_i[m] \mid m \in \mathbb{Z} \setminus \mathcal{I}_{\text{bl}}\}_{i=0}^{M'-1}$ as [19]

$$\hat{h}^{\text{SP}}[m] \;= \boldsymbol{f}^T[m]\; \hat{\gamma} = \sum_{i=0}^{D-1} \hat{\gamma}_i\, u_i[m], \quad m \in \mathbb{Z} \setminus \mathcal{I}_{\text{bl}} \tag{15}$$

where $\{u_i[m] \mid m \in \mathbb{Z} \setminus \mathcal{I}_{\text{bl}}\}_{i=0}^{M'-1}$ can be calculated from (10).

IV. Receive Antenna Selection Algorithm

We propose the following training-based "one out of K" per-packet receive AS algorithm for time-varying channels:

1) Each AE is trained using $L \geq 2$ pilot symbols such that the duration between two consecutive pilots is $T_{\text{t}} = \alpha\, K\, T_{\text{s}}$.
2) The receiver then:
   a) Performs channel prediction over the data time interval $\mathcal{I}_{\text{dt}}$, based on the sent pilots, via (15)

   $$\hat{h}_k^{\text{SP}}[m] \;= \boldsymbol{f}^T[m]\; \hat{\boldsymbol{\gamma}}_k = \sum_{i=0}^{D-1} \hat{\gamma}_{k,i}\, u_i[m] \tag{16}$$

   where $\hat{\boldsymbol{\gamma}}_k \triangleq \left[\hat{\gamma}_{k,0}, \hat{\gamma}_{k,1}, \ldots, \hat{\gamma}_{k,D-1}\right]^T$ is obtained via (13) $\left(\text{with } T_{\text{tr}}^k \text{ replacing } \mathcal{J}\right)$.

   b) Selects AE $[\hat{1}]$ which maximizes the post-processing SNR over $\mathcal{I}_{\text{dt}}$ as

   $$[\hat{1}] = \underset{1 \leq k \leq K}{\operatorname{argmax}} \sum_{m=M}^{M+N-1} \left|\hat{h}_k^{\text{SP}}[m]\right|^2. \tag{17}$$

3) The transmitter then sends out a length-N data packet consisting of $N - L'$ data symbols plus L' post-selection pilot symbols interleaved according to (4).
4) To decode data the receiver obtains refined channel gain estimates $\left\{\hat{h}_{[\hat{1}]}^{\text{SE}}[m] \mid m \in \mathcal{I}_{\text{dt}}\right\}$, based on the L_{tot} pilots, via

   $$\hat{\boldsymbol{h}}_{[\hat{1}]}^{\text{SE}} = \boldsymbol{U}'\, \hat{\boldsymbol{\gamma}}_{[\hat{1}]} = \sum_{i=0}^{D-1} \hat{\gamma}_{[\hat{1}],i}\, \boldsymbol{u}'_i \tag{18}$$

   where $\hat{\boldsymbol{h}}_{[\hat{1}]}^{\text{SE}} \triangleq \left[\hat{h}_{[\hat{1}]}^{\text{SE}}[M], \ldots, \hat{h}_{[\hat{1}]}^{\text{SE}}[M+N-1]\right]^T$ is of size $N \times 1$, $\boldsymbol{U}' \triangleq \left[\boldsymbol{u}'_0, \ldots, \boldsymbol{u}'_{D-1}\right]$ is the $N \times D$ submatrix of the complete $(M+N) \times D$ Slepian sequences matrix $\boldsymbol{U}$, and $\boldsymbol{u}'_i \triangleq \left[u_i[M], \ldots, u_i[M+N-1]\right]^T$.

V. Symbol Error Probability (SEP) Analysis

In this section, we analyze the proposed receive AS algorithm from Section IV as well as the symbol-by-symbol receive AS.

A. Prediction and Estimation CSI Models

We first define the CSI uncertainty model for Slepian estimation as

$$\hat{h}_k^{\text{SE}}[m] = h_k[m] + e_k^{\text{SE}}[m], \qquad 1 \leq k \leq K,\; m \in \mathcal{I}_{\text{dt}} \tag{19}$$

where the true channel gain $h_k[m]$ is correlated over time and modeled as a zero-mean circularly symmetric complex Gaussian random variable (RV) with unit-variance. The estimation error $e_k^{\text{SE}}[m]$ and $h_k[m]$ are assumed to be mutually uncorrelated.

From (19), the variance of the channel gain estimate $\hat{h}_k^{\text{SE}}[m]$ can be expressed as

$$\sigma^2_{\hat{h}_k^{\text{SE}}}[m] = \sigma^2_{h_k}[m] + \sigma^2_{e_k^{\text{SE}}}[m] = 1 + \text{MSE}_k^{\text{SE}}[m] \tag{20}$$

where $\text{MSE}_k^{\text{SE}}[m]$ is the mean-square-error (MSE) per sample for the Slepian estimator of AE k. Its expression is not included here due to space constraints. We refer to [18]. We also note that the CSI model for the Slepian predictor can be obtained from (19) and (20) by replacing superscript $(\cdot)^{\text{SE}}$ by $(\cdot)^{\text{SP}}$.

B. SEP Analysis

1) Per-Packet Basis Selection SEP: The SEP of an MPSK symbol received at time m of a system employing the per-packet receive AS algorithm in Sec. IV is given by

$$\begin{aligned} \text{SEP}_m(\eta) \;\; = \;\; & \frac{1}{\pi} \sum_{k=1}^{K} \int_0^{\frac{\text{M}-1}{\text{M}}\pi} \int_0^{\infty} \int_0^{\infty} \exp\left(\frac{-x'\, b_k^{\text{SE}}[m]}{\sin^2(\theta)}\right) \\ & \times f_{X'_k, Y'_k}(x', y') \prod_{\substack{l=1 \\ l \neq k}}^{K} F_{Y'_l}(y')\, \mathrm{d}x' \mathrm{d}y' \mathrm{d}\theta \end{aligned} \tag{21}$$

where $\eta \triangleq \frac{E_s}{N_0}$ is the average SNR per branch, $b_k^{\text{SE}}[m] \triangleq \frac{\left(\zeta_k^{\text{SE}}[m]\right)^2 \sin^2\left(\frac{\pi}{\text{M}}\right)}{\left(1 - \zeta_k^{\text{SE}}[m]\right) + \frac{1}{\eta}}$, $\zeta_k^{\text{SE}}[m] \triangleq \frac{1}{1 + \sigma^2_{e_k^{\text{SE}}}[m]} = \frac{1}{1 + \text{MSE}_k^{\text{SE}}[m]}$, and $f_{X'_k, Y'_k}(x', y')$ is the joint probability distribution of the exponentially distributed RV $X'_k \triangleq \left|\hat{h}_k^{\text{SE}}[m]\right|^2$ and RV $Y'_k \triangleq \sum_{m=M}^{M+N-1} \left|\hat{h}_k^{\text{SP}}[m]\right|^2$ with CDF $F_{Y'_k}(y')$. Deriving a closed-form expression for $\text{SEP}_m(\eta)$ in (21) is analytically intractable since closed-form expressions for $f_{X'_k, Y'_k}(x', y')$ and $F_{Y'_k}(y')$ do not exist. Therefore, Monte Carlo averaging techniques [20] are used to evaluate SEP. The detailed analysis is omitted here for brevity.

2) Symbol-By-Symbol AS SEP For $\mathtt{M}$*PSK:* Receive AS is on an instantaneous symbol-by-symbol basis according to

$$[\hat{1}]_m = \underset{1\leq k\leq K}{\operatorname{argmax}} \left|\hat{h}_k^{\text{SP}}[m]\right|^2, \tag{22}$$

with corresponding channel gain estimate $\hat{h}^{\text{SE}}_{[\hat{1}]_m}[m]$ used to decode the $\mathtt{M}$PSK symbol received at time m.

Theorem 1 The SEP of an $\mathtt{M}$PSK symbol received at time m in a time-varying channel for a system with one transmit and K receive antennas employing selection criterion (22) with channel gain estimate $\hat{h}^{\text{SE}}_{[\hat{1}]_m}[m]$ to decode an $\mathtt{M}$PSK symbol received at time m is given by

$$\begin{aligned}
\mathit{SEP}'_m(\eta) &= \frac{1}{\pi}\sum_{k=1}^{K}\sum_{r=0}^{K-1}\sum_{\substack{l_0,\ldots,l_r=1\\ l_0=1,\, l_1\neq\ldots\neq l_r\neq k}}^{K}\frac{(-1)^r}{r!\left(4\sigma^2_{k,c_1}[m]\right)}\\
&\quad\times\frac{1}{\sigma^2_{k,c_2}[m]\left(1-\left[\rho^2_{k,c_1c_2}[m]+\rho^2_{k,c_1s_2}[m]\right]\right)}\\
&\quad\int_0^{\frac{\mathtt{M}-1}{\mathtt{M}}\pi}\int_0^\infty\int_0^\infty \exp\left(\frac{-x\, b_k^{SE}[m]}{\sin^2(\theta)}\right.\\
&\quad -y\sum_{j=1}^{r}\zeta^{SP}_{l_j}[m]-\left[\frac{x}{\sigma^2_{k,c_1}[m]}+\frac{y}{\sigma^2_{k,c_2}[m]}\right]\\
&\quad\left.\times\frac{1}{2\left(1-\left[\rho^2_{k,c_1c_2}[m]+\rho^2_{k,c_1s_2}[m]\right]\right)}\right)\\
&\quad\times I_0\left(\frac{\sqrt{\rho^2_{k,c_1c_2}[m]+\rho^2_{k,c_1s_2}[m]}}{\left(1-\left[\rho^2_{k,c_1c_2}[m]+\rho^2_{k,c_1s_2}[m]\right]\right)}\right.\\
&\quad\left.\times\frac{\sqrt{xy}}{\sigma_{k,c_1}[m]\,\sigma_{k,c_2}[m]}\right)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\theta
\end{aligned}\tag{23}$$

where the notation $\sum\limits_{\substack{l_0,\ldots,l_r=1\\ l_0=1,\, l_1\neq\ldots\neq l_r\neq k}}^{K}$ compactly denotes $\sum\limits_{l_0=1}^{1}\sum\limits_{\substack{l_1=1\\(l_1\neq k)}}^{K}\sum\limits_{\substack{l_2=1\\(l_2\neq k, l_2\neq l_1)}}^{K}\cdots\sum\limits_{\substack{l_r=1\\(l_r\neq k,\, l_r\neq l_1,\ldots,\, l_r\neq l_{r-1})}}^{K}$, $\zeta^{\text{SP}}_{l_j}[m] \triangleq \frac{1}{1+\sigma^2_{e^{\text{SP}}_{l_j}}[m]} = \frac{1}{1+\text{MSE}^{\text{SP}}_{l_j}}$, $b_k^{\text{SE}}[m] \triangleq \frac{\left(\zeta_k^{\text{SE}}[m]\right)^2\sin^2\left(\frac{\pi}{\mathtt{M}}\right)}{\left(1-\zeta_k^{\text{SE}}[m]\right)+\frac{1}{\eta}}$ with $\eta = \frac{E_s}{N_0}$ denoting the average SNR per branch, and $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. In (23), $\rho_{k,c_1c_2}[m]$ and $\rho_{k,c_1s_2}[m]$ denote the correlation coefficients of $(X_{k,c_1}[m], X_{k,c_2}[m])$ and $(X_{k,c_1}[m], X_{k,s_2}[m])$, respectively, where $X_k \triangleq \left|\hat{h}_k^{\text{SE}}[m]\right|^2 = X_{k,c_1}[m]+jX_{k,s_1}[m]$ and $Y_k \triangleq \left|\hat{h}_k^{\text{SP}}[m]\right|^2 = X_{k,c_2}[m]+jX_{k,s_2}[m]$, and $(X_{k,c_1}[m], X_{k,s_1}[m])$ and $(X_{k,c_2}[m], X_{k,s_2}[m])$ are i.i.d. zero-mean Gaussian RVs with variances $\sigma^2_{k,c_1}[m] = \sigma^2_{k,s_1}[m]$ and $\sigma^2_{k,c_2}[m] = \sigma^2_{k,s_2}[m]$, respectively.

Proof: The proof is given in the appendix. ■

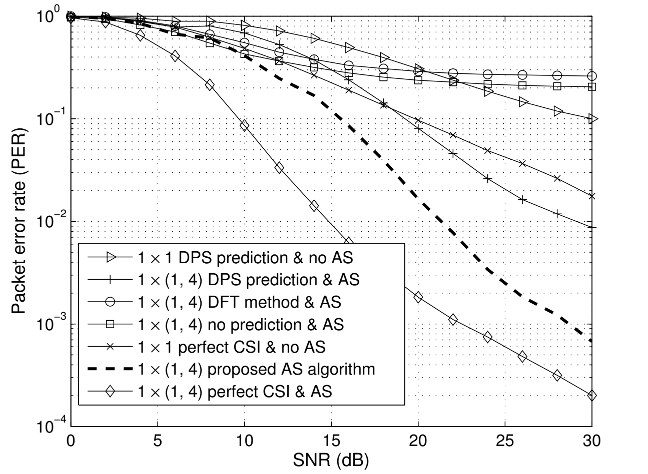

Fig. 2. PER performance of the proposed AS algorithm for a $1\times(1,4)$ system. (4PSK, data packet length $N = 40$, training pilots $L = 2$, post-selection pilots $L' = 2$, and $T_p = 3\,T_s$).

VI. Simulations

We now present numerical results to gain further insight into the previous analysis and study performance over time-varying channels. A system with one transmit and K receive antennas out of which only one is selected, denoted by $1\times(1,K)$, is simulated. The system operates at carrier frequency $f_{\text{c}} = 2\,\text{GHz}$ and the user moves with velocity $v_{\max} = 100\,\text{km/h}$. The packet consists of $N = 40$ $\mathtt{M}$PSK symbols each of duration $T_{\text{s}} = 20.57\ \mu\text{s}$ [18]. These parameters give a Doppler bandwidth $\nu_{\max} = 3.8\times10^{-3}$. The realizations of the time-varying channel are generated using plane-wave propagation principles [21], i.e.,

$$h[m] = \sum_{p=0}^{P-1}\frac{1}{\sqrt{P}}\exp\left(j\left(2\pi\nu_{\max}\cos\alpha_p m+\psi_p\right)\right) \tag{24}$$

where the number of propagation paths is set to $P = 30$, and the path angles α_p and ψ_p are independent and uniformly distributed over $[-\pi\ \pi)$. They are also assumed constant over an AS cycle but change independently from cycle to cycle.

The packet error rate (PER) of the proposed receive AS algorithm as a function of average SNR for a $1\times(1,4)$ system is shown in Fig. 2. For comparison, we also show the PER performance of (i) a 1×1 system with perfect CSI and no AS, (ii) a 1×1 system employing Slepian basis expansion channel prediction and no AS, (iii) a $1\times(1,4)$ system employing AS without channel prediction. We note that the antenna with the highest channel gain estimate $\tilde{h}_k[m]$ in (3) is selected since no channel prediction is used, (iv) a $1\times(1,4)$ system employing discrete Fourier transform (DFT) channel prediction [18] with AS according to the maximum total post-processing SNR selection criterion, as in (17), and DFT channel estimation [18] for data decoding, and (v) a $1\times(1,4)$ system employing Slepian channel prediction and AS according to (17), with the predicted channel gains $\left\{\hat{h}_k^{\text{SP}}[m]\ \middle|\ m\in\mathcal{I}_{\text{dt}}\right\}$ used not

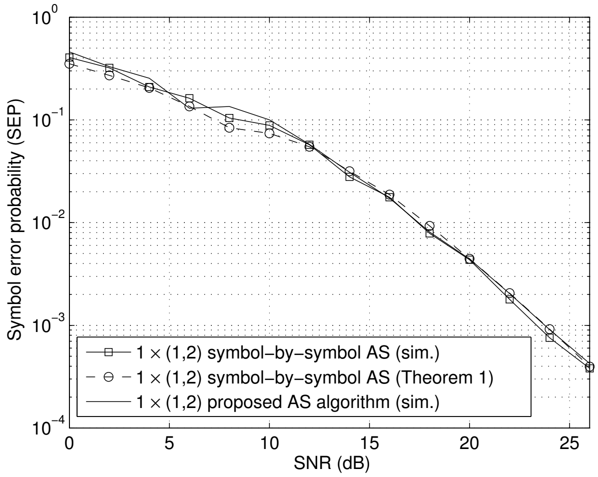

Fig. 3. SEP for the 20-*th* 4PSK data symbol as a function of the average SNR for a $1 \times (1, 2)$ system. (Data packet length $N = 40$, training pilots $L = 2$, post-selection pilots $L' = 2$, and $T_p = 5T_s$).

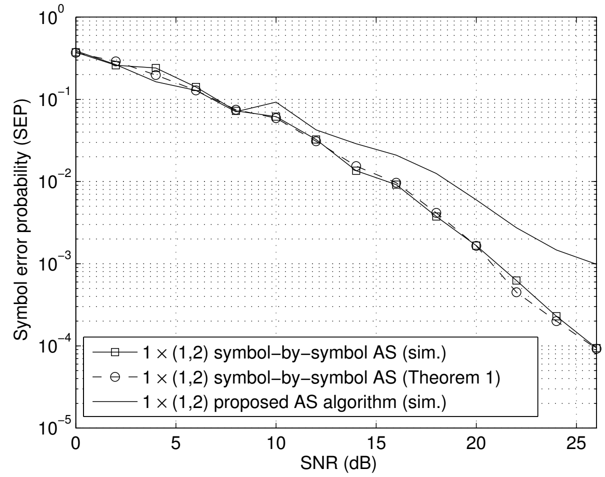

Fig. 4. SEP for the first 4PSK data symbol as a function of the average SNR for a $1 \times (1, 2)$ system. (Data packet length $N = 40$, training pilots $L = 2$, post-selection pilots $L' = 2$, and $T_p = 5T_s$).

only for selection but also data decoding. Inspection of Fig. 2 reveals that the $1 \times (1, 4)$ system employing the proposed AS algorithm achieves an SNR performance gain in excess of 9 dB over the 1×1 system with perfect CSI and no AS at a PER $= 10^{-2}$. To highlight the importance of channel estimation, the performance of the same proposed $1 \times (1, 4)$ system is about 6 dB worse than $1 \times (1, 4)$ system employing AS with perfect CSI at the same PER of 10^{-2}. Also, error-floors exist for the systems employing AS either with DFT basis expansion model (BEM) or without channel prediction. In contrast, no error-floors arise with Slepian BEM.

The SEP of the 20-*th* and first 4PSK symbols as a function of average SNR for $1 \times (1, 2)$ systems employing the proposed receive AS algorithm and the symbol-by-symbol receive AS scheme analyzed in *Theorem 1* are depicted in Figs. 3 and 4, respectively. It can be observed that the curves in Fig. 3 are close to each other. A gap can be observed in Fig. 4 between the curves at moderate to high SNRs. This is because the channel prediction for the first symbol is much better than channel prediction for the 20-*th* symbol, which clearly affects the SEP. Similarly, there is a slight upward shift of the proposed AS method's SEP curve in Fig. 4, due to the fact that the first symbol is located far from the post-selection pilots $\mathcal{P} = \{11, 31\}$. From Figs. 3 and 4 and from other simulations (not included), we also observe that the SEP of the first few symbols in a packet for a system which uses symbol-by-symbol AS (*Theorem 1*) is lower than that of the AS algorithm proposed in Sec. IV, while the SEPs of remaining symbols are close to one another.

VII. Conclusions

Receive antenna selection (AS) for time-varying fading for a system consisting of a single-antenna transmitter and a K-antenna receiver is considered. A receive AS method which uses the low-complexity reduced-rank Slepian basis expansion channel predictor and estimator is proposed. It takes into account practical constraints imposed by next-generation wireless standards such as training and packet reception for AS. Further, it only uses Doppler bandwidth knowledge, and does not need detailed correlation knowledge. A closed-form expression for the SEP of $\mathtt{M}$PSK with receive AS is derived. It is shown that, in spite of the aforementioned realistic limitations, the proposed AS scheme outperforms ideal conventional SISO systems with perfect channel knowledge but no AS at the receiver and conventional complex basis based estimation. Although the focus was on single carrier communication over time-varying frequency-flat channels, the proposed AS scheme may be extendible to OFDM systems. The extension to the case where subsets of more than one receive antenna are selected in time-varying frequency-selective channels remains as an important topic for future research.

Appendix

A. *Proof of Theorem* 1

The maximum-likelihood (ML) soft estimate for the symbol received by AE $[\hat{1}]_m$ at time m is

$$\begin{aligned} r_{[\hat{1}]_m}[m] &= \left(\hat{h}^{\mathrm{SE}}_{[\hat{1}]_m}[m]\right)^* y_{[\hat{1}]_m}[m] \\ &= \left|\hat{h}^{\mathrm{SE}}_{[\hat{1}]_m}[m]\right|^2 d[m] - \left(\hat{h}^{\mathrm{SE}}_{[\hat{1}]_m}[m]\right)^* d[m] \\ &\quad \times e^{\mathrm{SE}}_{[\hat{1}]_m}[m] + \left(\hat{h}^{\mathrm{SE}}_{[\hat{1}]_m}[m]\right)^* n_{[\hat{1}]_m}[m]. \quad (25) \end{aligned}$$

Conditioned on $\hat{h}^{\mathrm{SE}}_{[\hat{1}]_m}[m]$ and $d[m]$, $r_{[\hat{1}]_m}[m]$ in (25) is a complex Gaussian RV whose conditional mean $\mu_{r_{[\hat{1}]_m}}[m]$ and variance $\sigma^2_{r_{[\hat{1}]_m}}[m]$ are given by

$$\mu_{r_{[\hat{1}]_m}}[m] = \left|\hat{h}^{\text{SE}}_{[\hat{1}]_m}[m]\right|^2 d[m]\,\zeta^{\text{SE}}_{[\hat{1}]_m}[m] \tag{26}$$

$$\begin{aligned}\sigma^2_{r_{[\hat{1}]_m}}[m] &= \left|\hat{h}^{\text{SE}}_{[\hat{1}]_m}[m]\right|^2 |d[m]|^2 \left(1-\zeta^{\text{SE}}_{[\hat{1}]_m}[m]\right)\\ &\quad +N_0\left|\hat{h}^{\text{SE}}_{[\hat{1}]_m}[m]\right|^2 \end{aligned}\tag{27}$$

where $\zeta^{\text{SE}}_{[\hat{1}]_m}[m] \triangleq \frac{1}{1+\sigma^2_{e^{\text{SE}}_{[\hat{1}]_m}}[m]} = \frac{1}{1+\text{MSE}^{\text{SE}}_{[\hat{1}]_m}}$.

The SEP of an $\mathtt{M}$PSK symbol received at time m conditioned on $\left\{\hat{h}^{\text{SP}}_k[m]\right\}_{k=1}^K$, $[\hat{1}]_m$, and $\hat{h}^{\text{SE}}_{[\hat{1}]_m}[m]$ $\text{SEP}'_m\left(\left\{\hat{h}^{\text{SP}}_k[m]\right\}_{k=1}^K, [\hat{1}]_m, \hat{h}^{\text{SE}}_{[\hat{1}]_m}[m]\right)$, denoted for brevity by $\text{SEP}'_m(\Omega)$, is

$$\begin{aligned}\text{SEP}'_m(\Omega) &= \frac{1}{\pi}\int_0^{\frac{\mathtt{M}-1}{\mathtt{M}}\pi} \exp\left(\frac{-\left|\mu_{r_{[\hat{1}]_m}}[m]\right|^2 \sin^2\left(\frac{\pi}{\mathtt{M}}\right)}{\sigma^2_{r_{[\hat{1}]_m}}[m]\sin^2(\theta)}\right)\mathrm{d}\theta\\ &= \frac{1}{\pi}\int_0^{\frac{\mathtt{M}-1}{\mathtt{M}}\pi}\exp\left(\frac{-\left|\hat{h}^{\text{SE}}_{[\hat{1}]_m}[m]\right|^2 b^{\text{SE}}_{[\hat{1}]_m}[m]}{\sin^2(\theta)}\right)\mathrm{d}\theta\end{aligned}\tag{28}$$

where $b^{\text{SE}}_k[m] \triangleq \frac{\left(\zeta^{\text{SE}}_k[m]\right)^2 \sin^2\left(\frac{\pi}{\mathtt{M}}\right)}{\left(1-\zeta^{\text{SE}}_k[m]\right)+\frac{1}{\eta}}$, and the last equality follows from substituting (26) and (27).

Now averaging over the index $[\hat{1}]_m$ to get $\text{SEP}'_m\left(\left\{\hat{h}^{\text{SP}}_k[m]\right\}_{k=1}^K, \left\{\hat{h}^{\text{SE}}_k[m]\right\}_{k=1}^K\right)$, denoted for brevity by $\text{SEP}'_m(\Xi)$, yields

$$\begin{aligned}\text{SEP}'_m(\Xi) &= \sum_{k=1}^K \Pr\left([\hat{1}]_m = k \,\middle|\, \left\{\hat{h}^{\text{SP}}_k[m]\right\}_{k=1}^K\right)\\ &\quad\times \text{SEP}'_m\left([\hat{1}]_m = k, \hat{h}^{\text{SE}}_{[\hat{1}]_m}[m]\right)\\ &= \frac{1}{\pi}\sum_{k=1}^K\left(\prod_{\substack{l=1\\l\neq k}}^K \Pr\left(\left|\hat{h}^{\text{SP}}_l[m]\right|^2 < \left|\hat{h}^{\text{SP}}_k[m]\right|^2 \,\middle|\right.\right.\\ &\quad\left.\left.\left\{\hat{h}^{\text{SP}}_k[m]\right\}_{k=1}^K\right)\right)\int_0^{\frac{\mathtt{M}-1}{\mathtt{M}}\pi}\\ &\quad\exp\left(\frac{-\left|\hat{h}^{\text{SE}}_k[m]\right|^2 b^{\text{SE}}_k[m]}{\sin^2(\theta)}\right)\mathrm{d}\theta.\end{aligned}\tag{29}$$

The expression for SEP, when averaging over fading (i.e., Ξ), becomes

$$\begin{aligned}\text{SEP}'_m(\eta) &= \frac{1}{\pi}\sum_{k=1}^K\int_0^{\frac{\mathtt{M}-1}{\mathtt{M}}\pi}\int_0^\infty\int_0^\infty \exp\left(\frac{-x\,b^{\text{SE}}_k[m]}{\sin^2(\theta)}\right)\\ &\quad\times f_{X_k,Y_k}(x,y)\prod_{\substack{l=1\\l\neq k}}^K F_{Y_l}(y)\,\mathrm{d}x\mathrm{d}y\mathrm{d}\theta\end{aligned}\tag{30}$$

where $f_{X_k,Y_k}(x,y)$ is the joint PDF of the two correlated exponentially distributed RVs $X_k \triangleq \left|\hat{h}^{\text{SE}}_k[m]\right|^2 = X_{k,c_1}[m] + jX_{k,s_1}[m]$ and $Y_k \triangleq \left|\hat{h}^{\text{SP}}_k[m]\right|^2 = X_{k,c_2}[m] + jX_{k,s_2}[m]$ given in [22], and $F_{Y_l}(y)$ is the CDF of the exponentially distributed RV $Y_l \triangleq \left|\hat{h}^{\text{SP}}_l[m]\right|^2$.

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