

Time-Varying Phase Noise and Channel Estimation in MIMO Systems

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Abstract— Performance of high speed communication systems is negatively affected by oscillator *phase noise* (PN). In this paper joint estimation of channel gains and Wiener PN in *multi-input multi-output* (MIMO) systems is analyzed. The signal model for the estimation problem is outlined in detail. In order to reduce overhead, a low complexity *data-aided least-squares* (LS) estimator for jointly obtaining the channel gains and PN parameters is derived. In order to track PN processes over a frame, a new *decision-directed extended Kalman filter* (EKF) is proposed. Numerical results show that the proposed LS and EKF based PN estimator performances are close to the CRLB and simulation results indicate that by employing the proposed estimators the *bit-error rate* (BER) performance of a MIMO system can be significantly improved in the presence of PN.

I. INTRODUCTION

The throughput demands on point-to-point wireless links are expected to grow in the foreseeable future. For example, microwave backhaul links that interconnect base stations to base station controllers are expected to support data rates of multiple Giga bits per second [1]. However, traditional schemes of increasing bandwidth efficiency, e.g., using higher order modulations, are not capable of meeting this demand. One approach is to use *Multi-input multi-output* (MIMO) systems to significantly enhance the bandwidth efficiency of high speed wireless systems [2]. However, oscillator imperfections that result in *phase noise* (PN) are greatly detrimental to the performance of wireless systems [3]–[6] with a more pronounced impact at higher carrier frequencies, e.g., E-band (60–80 GHz) [7]. Moreover, in the case of MIMO systems, each transmit and receive antenna may be equipped with an independent oscillator. For example, in the case of *line-of-sight* (LoS) MIMO systems, a single oscillator cannot be used for all the transmit or receive antennas since the antennas need to be placed far apart from one another [8]¹. Similarly, in multi-user MIMO or *space division multiple access* (SDMA) systems, multiple

users with independent oscillators transmit their signals to a common receiver [10]. Thus, schemes that can jointly estimate multiple PN processes at the receiver are of particular interest [8], [10].

In single carrier communication systems, PN is multiplicative [6]. *Cramér-Rao lower bounds* (CRLB) and algorithms for estimation of PN in *single-input single-output* (SISO) systems are thoroughly analyzed in [6], [11], [12]. However, these results are not applicable to MIMO systems, where a received signal may be affected by multiple PNs [8]. The effect of PN on the capacity and performance of MIMO systems has been analyzed in [13], where it is demonstrated that PN can greatly limit the performance of multi-antenna systems. In [8], pilot-aided estimation of PN parameters in a MIMO system is investigated. However, the scheme in [8] uses pilot symbols that are transmitted based on *time division multiplexing* (TDM) to track the MIMO PN parameters, which is bandwidth inefficient. In addition, in [8] the MIMO channel is assumed to be known and the estimation performance limits have not been analyzed.

In *orthogonal frequency division multiplexing* (OFDM) systems, PN is convolved with data symbols [14]. Therefore, PN results in *inter-carrier interference* (ICI) and significant performance loss [14]. Considerable research has been carried out to eliminate or reduce the effect of ICI causing PN in MIMO OFDM systems [4], [14]. However, [4], [14] are based on the assumption that the MIMO system is affected by a single PN parameter and are not applicable to LoS and or multi-user MIMO systems.

In this paper, joint estimation of multiple PN and channel gains in a MIMO system is analyzed. A *data-aided least squares* (LS) estimator for jointly obtaining the channel gains and PN is proposed. Next, the pilot and estimated data symbols in combination with a *decision-directed extended Kalman filter* (EKF) are used for tracking the PN processes over a frame. The contributions and organization of this paper can be summarized as follows:

- In Sections II and III, joint estimation of channel gains and PN in a MIMO system is parameterized and new CRLBs for the multi-parameter estimation problem is presented. The CRLBs are then used to benchmark the

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¹For a 4×4 LoS MIMO system operating at 10 GHz and with a transmitter and receiver distance of 2 km, the optimal antenna spacing is 3.8 m [9].

performance of the proposed estimators and to quantify the effect of unknown PN on channel estimation accuracy and vice versa.

- In Section IV novel algorithms for estimating and tracking the unknown channel gains and time-varying PN, respectively, throughout a frame are proposed. A data-aided LS estimator for jointly obtaining the MIMO channel and PN parameters is proposed. Next, a new EKF based estimator is proposed that is shown to accurately track the PN processes over a frame.
- In Section V simulations are carried out that investigate the performance of a MIMO system using the proposed estimators.

Notation: Superscripts $(\cdot)^*$, $(\cdot)^H$, and $(\cdot)^T$ denote conjugate, conjugate transpose, and transpose operators, respectively. Bold face small letters, e.g., \mathbf{x} , are used for vectors, bold face capital alphabets, e.g., \mathbf{X} , are used for matrices, and $[\mathbf{X}]_{x,y}$ represents the entry in row x and column y of \mathbf{X} . $\mathbf{I}_{X \times X}$, $\mathbf{0}_{X \times X}$, and $\mathbf{1}_{X \times X}$ denote the $X \times X$ identity, all zero, and all 1 matrices, respectively. \odot stands for Schur (element-wise) product, $|\cdot|$ is the absolute value operator, $\angle x$ returns the phase of the complex variable x , $|\mathbf{x}|$ denotes the element-wise absolute value of a vector \mathbf{x} , $\text{diag}(\mathbf{x})$ is used to denote a diagonal matrix, where the diagonal elements are given by vector \mathbf{x} . $\text{diag}(\mathbf{X})$ is used to denote the diagonal elements of matrix \mathbf{X} . $\mathbb{E}[\cdot]$ denotes the expected value of the argument, and $\Re\{\cdot\}$ and $\Im\{\cdot\}$ are the real and imaginary parts of a complex quantity, respectively. Finally, $\mathcal{N}(\mu, \sigma^2)$ and $\mathcal{CN}(\mu, \sigma^2)$ denote real and complex Gaussian distributions with mean μ and variance σ^2 , respectively.

II. SYSTEM MODEL

A MIMO system with N_t and N_r transmit and receive antennas, respectively, is considered. Each frame of length L_f symbols is assumed to consist of a *training sequence (TS)* of length L_t symbols, *data* symbols, and *pilot* symbols that are transmitted every L_p symbols.

In this paper, the following set of assumptions is adopted:

- A1. The TSs broadcast from the N_t transmit antennas are assumed to be mutually orthogonal and of length $L_t = N_t$.
- A2. To ensure generality, each transmit and receiver antenna is assumed to be equipped with an independent oscillator as depicted in Fig. 1. This ensures that the system model is in line with previous work in the literature [8] and is also applicable to various MIMO systems, e.g., LoS MIMO and SDMA MIMO systems [10].
- A3. Quasi-static and frequency flat-fading channels are considered, where the channel gains are assumed to remain constant over the length of one frame.
- A4. Perfect timing synchronization is assumed, which can be achieved by standard synchronization algorithms [11].

Note that Assumption A3 is reasonable for actual wireless links including in LoS microwave backhaul systems, where the antennas are located on stationary towers with almost no obstacles between them, the channels vary much more slowly than the phase noise process. In addition, Assumptions A3 and A4 are in line with previous PN estimation algorithms in SISO and MIMO systems in [6], [8], [11]. More importantly, unlike the results in [6], [8], [11], which assume that the channel gains are estimated and equalized before PN estimation, in this paper we jointly estimate the channel gains and PN in a MIMO system.

The discrete-time baseband received signal at the MIMO receiver, $\mathbf{y}(n) \triangleq [y_1(n), \dots, y_{N_r}(n)]^T$, is given by²

$$\begin{aligned} \mathbf{y}(n) &= \Theta^{[r]}(n) \mathbf{H} \Theta^{[t]}(n) \mathbf{s}(n) + \mathbf{w}(n) \\ &= (\mathbf{A} \odot e^{j\mathbf{B}(n)}) \mathbf{s}(n) + \mathbf{w}(n), \end{aligned} \quad (1)$$

- $\mathbf{s}(n) \triangleq [s_1(n), \dots, s_{N_t}(n)]^T$, $s_\ell(n)$ is the n th transmitted symbol that corresponds to the ℓ th transmit antenna and consists of both pilots and data symbols,
- \mathbf{H} is the $N_r \times N_t$ channel matrix with $h_{k,\ell}$ denoting the quasi-static *unknown* channel gain from the ℓ th transmit to the k th receive antenna, which is assumed to be distributed as $h_{k,\ell} \sim \mathcal{CN}(\mu_{h_{k,\ell}}, \sigma_{h_{k,\ell}}^2)$ from frame to frame,
- $\Theta^{[r]}(n) \triangleq \text{diag}(e^{j\theta_1^{[r]}(n)}, \dots, e^{j\theta_{N_r}^{[r]}(n)})$ and $\Theta^{[t]}(n) \triangleq \text{diag}(e^{j\theta_1^{[t]}(n)}, \dots, e^{j\theta_{N_t}^{[t]}(n)})$ are $N_r \times N_r$ and $N_t \times N_t$ matrices, respectively, $\theta_k^{[t]}(n)$ and $\theta_k^{[r]}(n)$ correspond to the n th sample of the PN at the ℓ th transmit and k th receive antenna, respectively,
- $\mathbf{A} \triangleq [\alpha_1, \dots, \alpha_{N_r}]^T$ is the $N_r \times N_t$ channel magnitude matrix with $\alpha_k \triangleq [\alpha_{k,1}, \dots, \alpha_{k,N_t}]^T$, $\alpha_{k,\ell} \triangleq |h_{k,\ell}|$,
- $\mathbf{B}(n) \triangleq [\beta_1(n), \dots, \beta_{N_r}(n)]^T$ is an $N_r \times N_t$ matrix with $\beta_k(n) \triangleq [\beta_{k,1}(n), \dots, \beta_{k,N_t}(n)]^T$, $\beta_{k,\ell}(n) \triangleq \theta_\ell^{[t]}(n) + \theta_k^{[r]}(n) + \angle h_{k,\ell}$ denotes the overall phase contributions from the oscillators and channel corresponding to the ℓ th transmit and k th receive antenna, and
- $\mathbf{w}(n) \triangleq [w_1(n), \dots, w_{N_r}(n)]^T$ is the vector of *additive white Gaussian noise (AWGN)*, where $w_k(n) \sim \mathcal{CN}(0, \sigma_w^2)$.

For free-running oscillators, it is found that the PN process can be modeled as a Wiener process [3], [4]. Therefore, $\theta_\ell^{[t]}$ is modeled as [3], [4]

$$\theta_\ell^{[t]}(n) = \theta_\ell^{[t]}(n-1) + \Delta_\ell^{[t]}(n), \quad (2)$$

where a similar model is used for $\theta_k^{[r]}$. In (2), the phase innovations for the ℓ th transmit antenna, $\Delta_\ell^{[t]}(n)$ (k th receive antenna, $\Delta_k^{[r]}(n)$) is assumed to be $\Delta_\ell^{[t]}(n) \sim \mathcal{N}(0, \sigma_{\Delta_\ell^{[t]}}^2)$

²Throughout this paper indices $\ell = 1, \dots, N_t$, $k = 1, \dots, N_r$, and $n = 1, \dots, L_f$ are used to denote transmit antennas, receive antennas, and symbols, respectively.

$(\Delta_k^{[r]}(n) \sim \mathcal{N}(0, \sigma_{\Delta_k^{[r]}}^2))$ and also mutually independent, $\forall \ell$ ($\forall k$). As shown in [5], for most practical oscillators the PN innovation variance is small, e.g., $\sigma_{\Delta}^2 = (10^{-3}, 10^{-5}) \text{ rad}^2$.

III. CRAMÉR-RAO LOWER BOUNDS

In this section new expressions for the *Fisher's information matrices (FIMs)* for *data-aided estimation (DAE)* of PN and channel gains in an $N_r \times N_t$ MIMO system is derived.

Corresponding to (1), the vector of parameters of interest, $\boldsymbol{\lambda}$, is given by $\boldsymbol{\lambda} \triangleq [\boldsymbol{\lambda}_1^T, \dots, \boldsymbol{\lambda}_{N_r}^T]^T$, where $\boldsymbol{\lambda}_k \triangleq [\boldsymbol{\alpha}_k^T, \boldsymbol{\beta}_k^T(n)]^T$ is the parameter vector for the k th receive antenna.

Proposition 1: The FIM for joint DAE of channel gains and PN parameters over the observation, $\mathbf{y} \triangleq [\mathbf{y}_1^T, \dots, \mathbf{y}_{N_r}^T]^T$ with $\mathbf{y}_k \triangleq [y_k(n-L_t+1), \dots, y_k(n)]^T$, is a $2N_t N_r \times 2N_t N_r$ matrix, where its $2N_t \times 2N_t$ submatrices, $\mathbf{FIM}_{k,\bar{k}}$, for $k, \bar{k} = 1, \dots, N_r$, are given by (3) at the bottom of this page. In (3),

- $\mathbf{U}_k \triangleq \left[\mathbf{0}_{(k-1)L_t \times N_t}^T, (\mathbf{E}_k \mathbf{S})^T, \mathbf{0}_{(N_r-k)L_t \times N_t}^T \right]^T$ is an $N_r L_t \times N_t$ matrix,
- $\mathbf{E}_k \triangleq \text{diag}(e^{j\beta_{k,1}(n)}, \dots, e^{j\beta_{k,N_t}(n)})$ is an $N_t \times N_t$ matrix,
- $\mathbf{S} \triangleq [s_1, \dots, s_{N_t}]^T$ is an $N_t \times L_t$ matrix with $s_\ell \triangleq [s_\ell(n-L_t+1), \dots, s_\ell(n)]^T$
- $\boldsymbol{\Upsilon}_k \triangleq \text{diag}(\alpha_{k,1}, \dots, \alpha_{k,N_t})$ is an $N_t \times N_t$ matrix, and
- the ℓ th row, $\bar{\ell}$ th column, for $\ell, \bar{\ell} = 1, \dots, N_t$, elements of the $N_t \times N_t$ matrices $\mathbf{\Pi}_{11}$, $\mathbf{\Pi}_{12}$, $\mathbf{\Pi}_{21}$, $\mathbf{\Pi}_{22}$ are given by $[\mathbf{\Pi}_{11}]_{\ell,\bar{\ell}}(k, \bar{k}) \triangleq \text{Tr} \left\{ \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \alpha_{k,\ell}} \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \alpha_{\bar{k},\bar{\ell}}} \right\}$,
 $[\mathbf{\Pi}_{12}]_{\ell,\bar{\ell}}(k, \bar{k}) \triangleq \text{Tr} \left\{ \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \alpha_{k,\ell}} \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \beta_{\bar{k},\bar{\ell}}(n)} \right\}$,
 $[\mathbf{\Pi}_{21}]_{\ell,\bar{\ell}}(k, \bar{k}) \triangleq \text{Tr} \left\{ \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \beta_{k,\ell}(n)} \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \alpha_{\bar{k},\bar{\ell}}} \right\}$,
 $[\mathbf{\Pi}_{22}]_{\ell,\bar{\ell}}(k, \bar{k}) \triangleq \text{Tr} \left\{ \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \beta_{k,\ell}(n)} \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \beta_{\bar{k},\bar{\ell}}(n)} \right\}$,
 respectively, and
- the $L_t \times L_t$ submatrices, $\boldsymbol{\Sigma}_{\mathbf{y}_{k,\bar{k}}}$, of the $N_r L_t \times N_r L_t$ covariance matrix, $\boldsymbol{\Sigma}_{\mathbf{y}}$, are given by (4) at the bottom of this page. Note that $\frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \alpha_{k,\ell}}$, and $\frac{\partial \boldsymbol{\Sigma}_{\mathbf{y}}}{\partial \beta_{k,\ell}(n)}$ can be easily determined and are given in [15].

Proof: See [15].

Remark 1: The CRLB for estimation of channel gains and PN parameters is given by the diagonal elements of the inverse of the FIM matrix. Given the structure of the FIM, it is difficult to find a closed-form expression for the CRLB for an $N_r \times N_t$ MIMO system.

Remark 2: The FIM is not block diagonal. Therefore, the estimation of channel magnitudes and PN parameters in a MIMO system are coupled with one another, i.e., channel estimation accuracy is affected by the presence of PN and vice versa. This result indicates that channel and PN estimation need to be carried out jointly in a MIMO system.

IV. CHANNEL AND PHASE NOISE ESTIMATION

In this section, the data-aided LS and decision-directed EKF channel and PN estimators are derived in detail.

A. Data-Aided Estimation (DAE)

Since the maximum-likelihood estimator for joint estimation of channel and phase noise has a very high computational complexity, in this subsection an LS estimator for joint estimation of \mathbf{A} and $\mathbf{B}(n)$, $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}^{\text{[DAE]}}$, respectively, is determined. Using the received signal model in (1), the LS estimator is given by

$$\hat{\mathbf{P}}(n) = \left(\hat{\mathbf{A}} \odot e^{j\hat{\mathbf{B}}(n)} \right) = \mathbf{Y} \mathbf{S}^H (\mathbf{S} \mathbf{S}^H)^{-1} = \frac{\mathbf{Y} \mathbf{S}^H}{L_t}, \quad (5)$$

where $\mathbf{Y} \triangleq [\mathbf{y}_1, \dots, \mathbf{y}_{N_r}]^T$, $\mathbf{P}(n) \triangleq \mathbf{A} \odot e^{j\mathbf{B}(n)}$, and \mathbf{S} is defined in (3). The third equality in (5) follows from Assumption A1, i.e., $\mathbf{S} \mathbf{S}^H = L_t \mathbf{I}_{N_t \times N_t}$. Using (5), estimates of the channel and PN matrices are determined by

$$\hat{\mathbf{A}} = |\mathbf{Y} \mathbf{S}^H| / L_t, \quad \hat{\mathbf{B}}^{\text{[DAE]}}(n) = \angle \{ \mathbf{Y} \mathbf{S}^H / L_t \}, \quad (6)$$

respectively. Simulation results in Section V show that the performance of the proposed LS estimator in (6) is close to the CRLB over a wide range of SNR values.

B. Decision-Directed Estimation

This section presents an EKF to track $N_r N_t$ PN parameters, $\beta_{k,\ell}(n)$, in decision directed mode. Using (2), the

$$\mathbf{FIM}_{k,\bar{k}} = \begin{bmatrix} 2\Re \{ \mathbf{U}_k^H \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \mathbf{U}_{\bar{k}} \} + \mathbf{\Pi}_{11}(k, \bar{k}) & -2\Im \{ \mathbf{U}_k^H \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \mathbf{U}_{\bar{k}} \boldsymbol{\Upsilon}_{\bar{k}} \} + \mathbf{\Pi}_{12}(k, \bar{k}) \\ 2\Im \{ \boldsymbol{\Upsilon}_k \mathbf{U}_k^H \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \mathbf{U}_{\bar{k}} \} + \mathbf{\Pi}_{21}(k, \bar{k}) & 2\Re \{ \boldsymbol{\Upsilon}_k \mathbf{U}_k^H \boldsymbol{\Sigma}_{\mathbf{y}}^{-1} \mathbf{U}_{\bar{k}} \boldsymbol{\Upsilon}_{\bar{k}} \} + \mathbf{\Pi}_{22}(k, \bar{k}) \end{bmatrix}, \quad (3)$$

$$\boldsymbol{\Sigma}_{\mathbf{y}_{k,\bar{k}}} = \begin{cases} \sum_{l=1}^{N_t} \alpha_{k,l}^2 \mathbf{s}_l \mathbf{s}_l^H \odot (\sigma_{\Delta_k^{[r]}}^2 \boldsymbol{\Psi} + \sigma_{\Delta_k^{[t]}}^2 \boldsymbol{\Psi}) + \sum_{l=1}^{N_t} \sum_{\substack{\bar{l}=1 \\ \bar{l} \neq l}}^{N_t} |\alpha_{k,l}| |\alpha_{\bar{k},\bar{l}}| e^{j\beta_{k,l}(n)} e^{-j\beta_{\bar{k},\bar{l}}(n)} \mathbf{s}_l \mathbf{s}_{\bar{l}}^H \\ \quad \odot (\sigma_{\Delta_k^{[r]}}^2 \boldsymbol{\Psi}) + \sigma_w^2 \mathbf{I}_{L_t \times L_t}, & k = \bar{k} \\ \sum_{l=1}^{N_t} |\alpha_{k,l}| |\alpha_{\bar{k},l}| e^{j\beta_{k,l}(n)} e^{-j\beta_{\bar{k},l}(n)} \mathbf{s}_l \mathbf{s}_l^H \odot (\sigma_{\Delta_k^{[t]}}^2 \boldsymbol{\Psi}), & k \neq \bar{k} \end{cases} \quad (4)$$

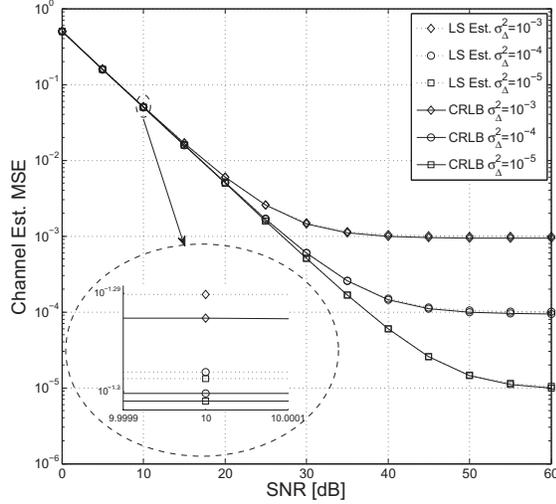


Fig. 1. MSE for DAE of channel, $\mathbf{P}(n)$ in (5), for a 2×2 MIMO system for different PN variances.

unknown *state vector*, $\phi(n) \triangleq [\beta_1^T(n), \dots, \beta_{N_r}^T(n)]^T$, is given by

$$\phi(n) = \phi(n-1) + \delta(n). \quad (7)$$

The state noise vector, $\delta(n) \triangleq [\Delta_{1,1}(n), \dots, \Delta_{1,N_t}(n), \dots, \Delta_{N_r,1}(n), \dots, \Delta_{N_r,N_t}(n)]^T$, is distributed as $\delta(n) \sim \mathcal{N}(\mathbf{0}_{N_t N_r \times 1}, \mathbf{Q})$. The state noise covariance matrix, $\mathbf{Q} = \mathbb{E}\{\delta(n)\delta(n)^T\}$, is an $N_t N_r \times N_t N_r$ matrix where its submatrices $\mathbf{Q}_{k,\bar{k}}$, for $k, \bar{k} = 1 \dots, N_r$ can be determined as

$$\mathbf{Q}_{k,\bar{k}} = \begin{cases} \sigma_{\Delta_k^{[r]}}^2 \mathbf{1}_{N_t \times N_t} + \text{diag}(\sigma_{\Delta_1^{[t]}}^2, \dots, \sigma_{\Delta_{N_t}^{[t]}}^2), & k = \bar{k} \\ \text{diag}(\sigma_{\Delta_1^{[t]}}^2, \dots, \sigma_{\Delta_{N_t}^{[t]}}^2), & k \neq \bar{k}. \end{cases} \quad (8)$$

The *observation* equation for the Kalman filter is given by (1). Let us define $\mathbf{z}(n) \triangleq [z_1(n), \dots, z_{N_r}(n)]^T = (\mathbf{A} \odot e^{j\mathbf{B}(n)}) \mathbf{s}(n)$. The $N_r \times N_r N_t$ Jacobian matrix for the EKF is evaluated by computing the first order partial derivative of $\mathbf{z}(n)$ with respect to the state vector $\phi(n)$, i.e., $\mathbf{D}(\phi(n)) = \partial \mathbf{z}(n) / \partial \phi(n)$.

Using (7) and (8), the EKF equations can be written as

$$\hat{\phi}_{(n|n-1)} = \hat{\phi}_{(n-1|n-1)}, \quad (9a)$$

$$\mathbf{M}_{(n|n-1)} = \mathbf{M}_{(n-1|n-1)} + \mathbf{Q}, \quad (9b)$$

$$\mathbf{D}(\hat{\phi}_{(n|n-1)}) = \mathbf{D}(\phi(n))|_{\phi(n)=\hat{\phi}_{(n|n-1)}}, \quad (9c)$$

$$\mathbf{K}(n) = \mathbf{M}_{(n|n-1)} \mathbf{D}^H(\hat{\phi}_{(n|n-1)}) \quad (9d)$$

$$\times \left(\mathbf{D}(\hat{\phi}_{(n|n-1)}) \mathbf{M}_{(n|n-1)} \mathbf{D}^H(\hat{\phi}_{(n|n-1)}) + \mathbf{W} \right)^{-1},$$

$$\hat{\phi}_{(n|n)} = \hat{\phi}_{(n|n-1)} \quad (9e)$$

$$+ \Re \left\{ \mathbf{K}(n) \left[\mathbf{y}(n) - \left(\hat{\mathbf{A}} \odot e^{j\hat{\mathbf{B}}(n)} \right) \mathbf{s}(n) \right] \right\},$$

$$\mathbf{M}_{(n|n)} = \Re \left\{ \mathbf{M}_{(n|n-1)} - \mathbf{K}(n) \mathbf{D}(\hat{\phi}_{(n|n-1)}) \mathbf{M}_{(n|n-1)} \right\}, \quad (9f)$$

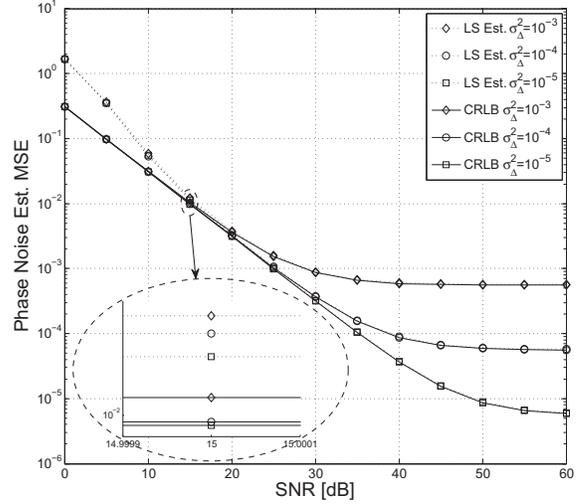


Fig. 2. MSE for DAE of PN for a 2×2 MIMO system for different PN variances.

where $\hat{\phi}_{(n|n-1)} = [\hat{\beta}_1^T(n|n-1), \dots, \hat{\beta}_{N_r}^T(n|n-1)]^T$ is the predicted state vector at the n th symbol, $\mathbf{W} = \sigma_w^2 \mathbf{I}_{N_r \times N_r}$, $\hat{\mathbf{B}}(n) = \mathbf{B}(n)|_{\phi(n)=\hat{\phi}_{(n|n-1)}}$, $\mathbf{K}(n)$ is the $N_t N_r \times N_r$ Kalman gain matrix, $\hat{\mathbf{A}}$ is given in (6), and $\mathbf{M}_{(n|n)}$ is the $N_t N_r \times N_t N_r$ filtering error covariance matrix. In order to ensure convergence of the EKF, the state vector is initialized with the LS estimates and $\mathbf{M}(L_t|L_t) = 0.1 \mathbf{I}_{N_r N_t \times N_r N_t}$.

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section $L_t = N_t$, the observation length in the decision-directed mode, $L_o = N_t$, and $\sigma_{\Delta_\ell^{[t]}}^2 = \sigma_{\Delta_k^{[r]}}^2 = \sigma_{\Delta}^2$, $\forall k, \ell$, and $\sigma_w^2 = 1/\text{SNR}$. The MIMO channel matrix is generated as a sum of LoS and *non-LoS* (*NLoS*) components using the model in [9]. K denotes the Rician factor. The antenna spacing at the transmitters and receiver are set to their optimum values [9], the carrier frequency is set to 70 GHz, and $K = 2$ dB. Walsh-Hadamard codes with *binary phase-shift keying* (*BPSK*) are used for the TSSs. The phase unwrapping algorithm in [6] is applied here. Without loss of generality, only the *mean-square errors* (*MSEs*) for estimation of channel gain and phase noise for the first antenna element are presented³.

A. Estimation Performance

Figs. 1 and 2 show the CRLB and MSE for DAE of MIMO channels, $\mathbf{P}(n)$, in (5) and time-varying PN, respectively, versus SNR. The CRLB in Section III is numerically evaluated for PN variances $\sigma_{\Delta}^2 = [10^{-3}, 10^{-4}, 10^{-5}] \text{ rad}^2$. Note that the PN variance, $\sigma_{\Delta}^2 = 10^{-3} \text{ rad}^2$, corresponds to very strong PN [5]. The CRLB results in Fig. 1 show that in the presence of PN, estimation of the MIMO channel

³Note that similar results are obtained for the estimation of the parameters for the remaining antennas and are not presented here to avoid repetition.

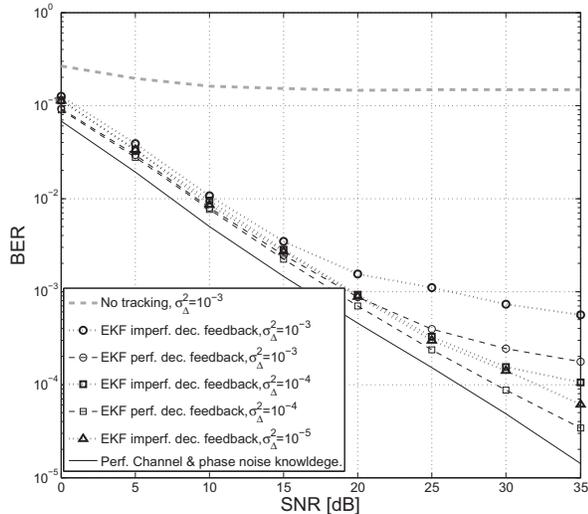


Fig. 3. Perfect and Imperfect decision feedback BER of a 2×2 MIMO system for PN variances, $\sigma_{\Delta}^2 = [10^{-3}, 10^{-4}, 10^{-5}] \text{ rad}^2$, with BPSK and $L_p = 10$

suffers from an error floor, which is directly related to the variance of PN innovations. This result can be anticipated since even at very high SNR, where the effect of the AWGN noise, \mathbf{w}_k , is negligible, the noise corresponding to the PN limits estimation accuracy. The same noise also limits the estimation accuracy of the n th symbol's PN parameters in the data-aided case and results in an error floor in Fig. 2.

B. System Performance

The proposed LS estimator and training symbols at the start of each frame are used to estimate the MIMO channel gains and phases. Next, the proposed EKF estimator is applied in both perfect and imperfect decision feedback modes to track the phase parameters over a frame. The frame length is set to $L_f = 1000$ BPSK symbols and new channels are generated for each frame. Pilot spacing is set to $L_p = 10$, which corresponds to a synchronization overhead of 10%. An MMSE linear receiver given by

$$\hat{\mathbf{s}}(n) = \left(\hat{\mathbf{P}}^H(n) \hat{\mathbf{P}}(n) + \sigma_w^2 \mathbf{I}_{N_t \times N_t} \right)^{-1} \hat{\mathbf{P}}^H(n) \mathbf{y}(n), \quad (10)$$

is used to equalize the effect of phase noise and channel gains.

Fig. 3 depicts the *bit error rate (BER)* of a 2×2 MIMO system using the proposed LS estimator *without* phase tracking and with the proposed EKF phase tracking in the presence of various PN variances. The scenario with perfect channel knowledge and synchronization is also plotted as a benchmark. The results in Fig. 3 demonstrate that without phase tracking, the MIMO system performance deteriorates significantly. However, by combining the proposed LS and EKF estimators, the BER performance of a MIMO system is shown to improve immensely even in the presence of very strong PN, e.g., $\sigma_{\Delta}^2 = 10^{-3} \text{ rad}^2$. More importantly, it is demonstrated that the BER of a MIMO system using the

combination of the proposed channel and PN estimators is close to the ideal case of perfect channel and PN knowledge (a performance gap of 3dB with SNR=20dB). Finally, Fig. 3 shows that the overall MIMO system's BER suffers from an error floor at high SNRs. This result is anticipated, since at high SNR the performance of the MIMO system is dominated by PN instead of AWGN.

VI. CONCLUSION

In this paper, estimation of channel and PN in MIMO systems is analyzed. The MSE of the LS estimator is shown to be close to the derived CRLB over a wide range of SNR values. In order to track the time-varying PN over a frame using the pilot and estimated data symbols, a new decision-directed EKF based estimator is proposed. By employing both the known pilot symbols and the received data symbols, the resulting decision-directed EKF does not result in error propagation. Next, simulations show that the performance of a MIMO system using the proposed LS and EKF estimator is close to the idealistic setting of perfect channel and PN estimation, e.g., at an SNR of 20 dB for a 2×2 MIMO system, there is a performance gap of 3dB between the two systems.

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