A Hybrid Random-Greedy Approach to User Selection for MU-MIMO Based on Pairwise Metrics

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***Abstract*—This paper proposes a hybrid user group selection method in multi-user multi-input multi-output (MU-MIMO) systems for downlink transmission. The proposed scheme provides additional trade offs between performance and complexity. In this method, initial groups of users are first randomly selected from a pool until a performance threshold of a simplified metric is satisfied. The proposed metric computes the minimum geometric angle among users' subspaces. Then, the user group is enlarged by maximizing sum rate among the users in greedy fashion based on ta subspace angle proposed performance metric. Monte Carlo simulation results compare the performance of the proposed method with existing techniques as a function of the initially chosen group size and selection pool size and reveals potential for lowered complexity and/or improved performance.**

***Index Terms*—Multi-user MIMO, user selection, epsilon-greedy, subspace angle**

I. Introduction

To improve the sum-rate performance of multi-user multi-input multi-output (MU-MIMO) systems, numerous precoding algorithms have been employed including optimal strategies such as dirty paper coding (DPC) [1]. However, since DPC requires significant encoding and decoding complexity, especially when the number of users is large, implementation of DPC is challenging. Instead, low-complexity precoding methods have been studied. To address complexity and maintain high performance in MU-MIMO systems, block digitalization (BD) precoding is a popular alternative due to its simpler implementation [2]. The BD precoder is a multi-antenna generalization of the zero-forcing strategy that cancels mutual inter-user interference transmitted to users in the cell [2].

When the number of users in a cell is large, user selection is essential for a base station (BS) to serve larger numbers of users in downlink (DL) transmission. In principle, the best selection strategy would be to search over all possible subsets of users resulting in complexity that grows exponentially with user group size. In view of this, greedy algorithms have been proposed to solve this problem. In [3], users are iteratively selected to achieve highest total throughput including the previously selected users. Although this method may perform well, implementation complexity is high due to frequent use of the singular value decomposition (SVD). In [4], a greedy low-complexity scheduling algorithm is proposed where the product of squared row norms of effective channels is employed as the selection metric. In [5], a low-complexity greedy user selection algorithm is proposed based on the angle between subspaces of users. This method can reduce complexity in user selection in which the product of eigenvalues can be obtained iteratively and recursively using the relationship between principal angles and eigenvalues. In [6], a user scheduling scheme is proposed to maximize the sum rate by investigating user channel characteristics in a subspace approach and designing user-scheduling metrics from a geometric viewpoint. However, its use of BD precoding also requires frequent use of SVD to precode the data stream.

The methods presented above are based on greedy algorithms that significantly reduce complexity over exhaustive search. On the other hand, in reinforcement learning problems such as the multi-arm bandit, random selection may yield better results [7]. In reinforcement learning, if a greedy action is selected, current knowledge of the values of the actions is said to be *exploited*. Instead, if a non-greedy action is selected, then *exploration* occurs, enabling improved reward in an enlarged search space. In other words, exploitation is the action that maximizes expected reward in a given step, but exploration may enlarge the search space to produce a greater total reward in the long run [7]. Inspired by both exploitation and exploration processes, an epsilon-greedy method is proposed in order to randomly select users based on a reinforcement learning formulation that is shown to improve performance for cognitive radio applications [8].

The above discussion motivates the hybrid user selection method proposed here: a random user selection process that incorporates a priori problem knowledge is combined with greedy user-incremental maximization of the sum rate in downlink transmission of a MU-MIMO system. In general, analytical determination of the CDF of the minimum geometric angle among user subspaces by their multi-antenna receivers is not tractable. A threshold of the empirical CDF obtained by Monte Carlo simulation is therefore used instead. We select a group of initial users among randomly chosen groups in the user selection pool. This subspace angle criterion ensures that users with relatively low mutual interference are chosen to initialize greedy selection. Then, the group is enlarged from users selected from the pool by a greedy algorithm based on the minimum *geometric angle* among subspaces of user channels. The process is summarized as follows:

- To initially select users, L_r users are randomly from the pool, where L_r is less than the target group size L, and represents a design parameter. If the pair-wise minimum geometric angle among user subspace pairs is below a threshold, the process terminates. Otherwise,

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another group of L_r users is selected until the threshold requirement is met, typically stopping when the group performance minimum angle pairwise metric is within the top 10% of all groups of size L_r. Details are described by Algorithm 1.
- Then, the rest of the users, $L - L_r$, are incrementally chosen from the pool, based on greedily maximizing sum rate in which the minimum geometric angle among the subspace of user effective channels is used as a metric. Details are described by Algorithm 2.
- The choice of initial group size, L_r, random selection threshold as well as greedy selection, the combination of Algorithm 1 and Algorithm 2, provide trade-offs between performance and complexity.

In the sequel, we use the following notational conventions: boldface upper case letters $\mathbf{X}$ denote matrices and lowercase underlined letters $\underline{x}$ denote vectors. Capital letters in serif font, e.g., A, denote a set of users. Superscripts $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote conjugate, transpose, and conjugate-transpose, respectively. $\mathbb{E}\{\cdot\}$ denotes expectation, Tr$\{\cdot\}$ denotes matrix trace, $(.)^\perp$ denotes orthogonal complement and $\|(.)\|_2$ denotes the Euclidean norm of $(.)$.

II. System Model

A MU-MIMO system is considered where a single BS with N antennas transmits data signals to L selected users from the pool of K users in downlink (DL) transmission and each user has M antennas. It is assumed that $M < N$ and $\frac{N}{M}$ is an integer. Considering a subset of users, Γ, that has been scheduled for DL transmission, we define $\mathbf{H}_k \in \mathbb{C}^{M\times N}$, $\underline{x}_k \in \mathbb{C}^{M\times 1}$ and $\underline{y}_k \in \mathbb{C}^{M\times 1}$ as the narrowband, flat-fading channel to the kth user, the vector of transmitted symbols and the received signal at the kth user, respectively. Without loss of generality, we assume that the elements of $\mathbf{H}_k$ are independent and identically distributed unit variance complex Gaussian random variables and that each user obtains perfect knowledge of the channel matrix. The channel matrix is quantized and fed back by each user to the base station over error free, a zero delay and limited feedback channel [9]. The received signal at kth user can be therefore expressed as [10]

$$\underline{y}_k = \underbrace{\mathbf{H}_k\ \mathbf{W}_k\ \underline{x}_k}_{\text{Desired signal}} + \underbrace{\sum_{l\in\Gamma, l\neq k} \mathbf{H}_k\ \mathbf{W}_l\ \underline{x}_l}_{\text{Inter-user interference}} + \underbrace{\underline{n}_k}_{\text{Noise}}, \quad (1)$$

where $\underline{n}_k \in \mathbb{C}^{M\times 1}$ denotes the circularly-symmetric complex Gaussian noise vector with zero mean and covariance matrix $\mathbf{I}_M$ and $\mathbf{W}_k \in \mathbb{C}^{N\times M}$ denotes the linear precoder matrix at the kth user, respectively; and $rank(H_k) = \min(N, M) = M$. It is worth mentioning that on the right hand side of Equation (1), the first and second terms are the desired signal and inter-user interference for kth user, respectively. We assume that $\mathbb{E}\{\underline{x}_k \underline{x}_k^H\} = \mathbf{I}_M$ and user power allocation is given by

$$\sum_{k=1}^{|\Gamma|} \text{Tr}\ (\Omega_k) \leq P, \quad (2)$$

where $\Omega_k = \left(\mathbf{W}_k\mathbf{W}_k^H\right)$ and P denotes the average power (SNR) constraint in DL transmission. We denote the average power allocated to user k by p_k, i.e., $p_k = \text{Tr}\big(\Omega_k\big)$.

The quantization codebook is fixed beforehand and employed by each user and known to the BS. This codebook $\mathcal{C}$ consists of 2^B matrices in $\mathbb{C}^{M\times N}$, i.e., $(\mathbf{V}_1, \mathbf{V}_2, ..., \mathbf{V}_{2^B})$, where B is the number of feedback bits allocated per user [9]. Let $\hat{\mathbf{H}}_k$ be the quantization of the channel matrix $\mathbf{H}_k$ selected from the codebook $\mathcal{C}$ by the following metric:

$$\hat{\mathbf{H}}_k = \underset{\mathbf{V}\ \in\ \mathcal{C}}{\arg\min}\ d^2(\mathbf{H}_k, \mathbf{V}), \quad (3)$$

where, $d(\mathbf{H}_k, \mathbf{V})$ is chordal distance given by [11]

$$d(\mathbf{H}_k, \mathbf{V}) = \sqrt{\sum_{i=1}^{M} \sin^2(\theta_i)}, \quad (4)$$

where θ_i's indicate the principal angles between the two subspaces spanned by the columns of the matrices $\mathbf{V}$ and $\mathbf{H}_k$ [11]. Since the principal angles are based only on the subspaces spanned by the columns of the matrices, without loss of generality, it is assumed that entries of the codebook $\mathcal{C}$ are unitary matrices, i.e., $\mathbf{V}^H\mathbf{V} = \mathbf{I}_M\ \forall\ \mathbf{V}\ \in\ \mathcal{C}$. The chordal distance given by Equation (4) can be also written as [9]

$$d^2(\mathbf{H}_k, \mathbf{V}) = M - \text{tr}\big(\dot{\mathbf{H}}_k\mathbf{V}\mathbf{V}^H\dot{\mathbf{H}}_k\big), \quad (5)$$

where $\dot{\mathbf{H}}_k$ denotes the orthogonal basis of the subspace of channel $\mathbf{H}_k$.

A. Block diagonalization precoding

BD precoding is employed to null inter-user interference. The precoding matrix for the jth user is designed such that

$$\mathbf{H}_k\ \mathbf{W}_j = 0,\ \forall\ k \neq j, \quad k = 1, 2, ..., |\Gamma|. \quad (6)$$

In the BD precoder, $\mathbf{W}_k = \mathbf{B}_k\mathbf{\Pi}_k$, where $\mathbf{B}_k$ is designed to null the inter-user interference and $\mathbf{\Pi}_k \in \mathbb{C}^{M\times M}$ is employed for power allocation. Thus, Equation (6) is equivalent to $\mathbf{H}_k\ \mathbf{B}_j = 0$ for all $k \neq j$. To determine $\mathbf{B}_k$ for the kth user, an approach based on SVD can be employed. Let us define the aggregate channel matrix of all the other users than the kth user as

$$\bar{\mathbf{H}}_k = \left[\mathbf{H}_1^H\mathbf{H}_2^H...\mathbf{H}_{k-1}^H\mathbf{H}_{k+1}^H...\mathbf{H}_{|\Gamma|}^H\right]^H \in \mathbb{C}^{(|\Gamma|-1)M\times N}. \quad (7)$$

In view of this definition, $\mathbf{B}_k$ should lie in $\mathcal{N}(\bar{\mathbf{H}}_k)$. To find this null space, we employ the SVD of $\bar{\mathbf{H}}_k$ given by

$$\bar{\mathbf{H}}_k = \bar{\mathbf{U}}_k\sum\nolimits_k\big[\bar{\mathbf{V}}_k^{(1)}\bar{\mathbf{V}}_k^{(0)}\big]^H, \quad (8)$$

where $\bar{\mathbf{V}}_k^{(0)}$ forms a basis of $\mathcal{N}(\bar{\mathbf{H}}_k)$, the null space of $\bar{\mathbf{H}}_k$, and $\bar{\mathbf{V}}_k^{(1)}$ consists of M right singular vectors of $\bar{\mathbf{H}}_k$ that span the orthogonal complement of $\mathcal{N}(\bar{\mathbf{H}}_k)$. To nullify inter-user interference, we can choose $\mathbf{B}_k = \bar{\mathbf{V}}_k^{(0)}$ [5]. It is worth mentioning that the columns of $\mathbf{B}_k$ are an orthonormal basis of $\mathcal{R}(\bar{\mathbf{H}}_k)$, i.e., $row(\bar{\mathbf{H}}_k)$ and we must have $N > (|\Gamma| - 1)M$ to employ BD and guarantee the existence of a null space of $(\bar{\mathbf{H}}_k)$ with dimension greater than zero [5].

The aforementioned BD precoder design requires per user SVDs of a complex matrix of size $(|\Gamma|-1)M \times N$. To reduce complexity, we employ an iterative scheme that is instead based on QR decomposition [4], which is described in Part 2 of Algorithm 1. In view of space limitations, details are omitted.

Let L be the number of users that has been scheduled for the DL transmission, i.e., $L = |\Gamma|$. Thus, $L \leq \frac{N}{M}$ in order to employ the BD scheme. In view of this, selecting the best set of L_{max} users in each channel use to maximize the sum-rate (SR) can be expressed as

$$SR = \underset{\Gamma \subset \{1,2,\ldots,K\}}{\arg\max} \sum_{k=1}^{|\Gamma|} R_k, \tag{9}$$

where R_k is the achievable rate for kth user expressed as [10]

$$R_k = \mathbb{E}\ \log_2 \det\left(\mathbf{I}_M + \tilde{\mathbf{H}}_k \Upsilon_k \tilde{\mathbf{H}}_k^H\right), \tag{10}$$

where $\tilde{\mathbf{H}}_k = \mathbf{H}_k \mathbf{B}_k$ and is the effective channel matrix for the kth user and $\Upsilon_k = \mathbf{\Pi}_k \mathbf{\Pi}_k^H$.

Remark 1 : The optimal set of users can, in principle, be found by searching over all possible subsets of K users. However, the complexity of exhaustive search grows as K^L. Thus, we seek to find a suboptimal lower-complexity algorithm that can achieve a significant fraction of the optimal full search performance. We propose a user selection approach that is inspired by an epsilon-greedy search strategy.

Remark 2 : Employing block diagonal precoding to cancel interference is only achieved when there is perfect CSI, i.e., exact knowledge of $\mathbf{H}_k$ at the BS. In the case of imperfect CSI, where there is limited feedback, i.e., the BS is aware of only a quantized version of $\mathbf{H}_k$, namely, $\hat{\mathbf{H}}_k$. Precoding matrices are therefore based on $\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2, ..., \hat{\mathbf{H}}_K$ instead of $\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_K$ while performing BD. Let us denote precoding matrices that use imperfect CSI by $\hat{\mathbf{W}}_1, \hat{\mathbf{W}}_2, ..., \hat{\mathbf{W}}_L$, where each $\hat{\mathbf{W}}_l$ is selected such that $\hat{\mathbf{H}}_i \hat{\mathbf{W}}_l = 0 \ \forall \ l \neq i$. This implies that $\mathbf{H}_i \hat{\mathbf{W}}_l \neq 0$ leading to a loss in throughput from residual interference terms. In view of this, Equation (1) becomes [9]

$$\underline{y}_k = \underbrace{\mathbf{H}_k\ \hat{\mathbf{W}}_k\ \underline{x}_k}_{\text{Desired signal}} + \underbrace{\sum_{l \in \Gamma, l \neq k} \mathbf{H}_k\ \hat{\mathbf{W}}_l\ \underline{x}_l}_{\text{Residual interference}} + \underbrace{\underline{n}_k}_{\text{Noise}}, \tag{11}$$

and the achievable rate for the quantized precoder in Equation (11) for the kth user in the selected group, $1 \leq k \leq L$, can be expressed as [9]

$$R_k^Q = \mathbb{E}\log_2 \frac{\det\left(\mathbf{I}_M + \sum_{j=1}^{L} \mathbf{H}_k \hat{\mathbf{W}}_j \ \Upsilon_k \ \mathbf{H}_k^H \hat{\mathbf{W}}_j^H\right)}{\det\left(\mathbf{I}_M + \sum_{j=1, j\neq k}^{L} \mathbf{H}_k \hat{\mathbf{W}}_j \ \Upsilon_k \ \mathbf{H}_k^H \hat{\mathbf{W}}_j^H\right)}, \tag{12}$$

where Υ_k represents user power allocation, and the expectation is carried out over the channel distribution as well as random codebooks. In the special case of equal power allocation considered here, $\Upsilon_k = \frac{P_t}{N}$, where P_t is the total power budget.

B. Subspace Angles

The angle between subspaces is applied in Section II-C in order to define a pairwise ϵ-orthogonality criterion among the users. We employ the following definition, which is used in the proposed user selection algorithm in Section III:

Definition 1 : Let $\mathcal{W}_k \subset \mathcal{C}^n$ and $\mathcal{V}_j \subset \mathcal{C}^n$ be subspaces with $r_1 = dim(\mathcal{W}_k) \leq dim(\mathcal{V}_j) = r_2$. Between subspaces $\mathcal{W}_{k,1} = \mathcal{W}_k$ and $\mathcal{V}_{j,1} = \mathcal{V}_j$, the principal angles $\theta_{k,j,i} \in [0, \frac{\pi}{2}]$ for $i = 1, ..., r_1$ are recursively defined by [12]

$$\cos\theta_{k,j,i} = \max_{\mathbf{w}_k \in \mathcal{W}_{k,i}, \mathbf{v}_k \in \mathcal{V}_{k,j}, \|\mathbf{w}_k\|_2 = 1, \|\mathbf{v}_j\|_2 = 1} \mathbf{v}_j^H \mathbf{w}_k = \mathbf{v}_{j,i}^H \mathbf{w}_{k,i}, \quad i = 1, ..., r_1 \tag{13}$$

where $\mathbf{w}_{k,i}$ and $\mathbf{v}_{j,i}$ are the vectors that construct the ith principal angle $\theta_{k,j,i}$. These vectors are given by $\|\mathbf{w}_{k,i}\|_2 = 1$, $\|\mathbf{v}_{j,i}\|_2 = 1$, and $\mathcal{W}_{k,i} = \mathcal{W}_{k,i-1} \bigcap \mathbf{w}_{k,i-1}^{\perp}$, $\mathcal{V}_{j,i} = \mathcal{V}_{j,i-1} \bigcap \mathbf{v}_{j,i-1}^{\perp}$. From (13), it can be seen that the cosine of the principal angle is the inner product of two vectors, and the minimum principal angle represents the largest inner product of any vectors in the two subspaces.

Let $\mathcal{W}_O \subset \mathcal{C}^n$ and $\mathcal{V}_O \subset \mathcal{C}^n$ be orthonormal bases of $\mathcal{W}_k = span\{\mathbf{w}_{k,1}, \mathbf{w}_{k,2}, ..., \mathbf{w}_{k,r_1}\}$ and $\mathcal{V}_j = span\{\mathbf{v}_{j,1}, \mathbf{v}_{j,2}, ..., \mathbf{v}_{j,r_2}\}$, respectively. The angle between $\mathcal{W}_k$ and $\mathcal{V}_j$, i.e., $\phi_{k,j} = \measuredangle(\mathcal{W}_k, \mathcal{V}_j)$ is given by [13]

$$\cos^2\phi_{k,j} = \prod_{i=1}^{r_1} \cos^2\theta_{k,j,i}, \tag{14}$$

where $\theta_{k,j,i}$, $\mathbf{w}_{k,i}$ and $\mathbf{v}_{k,j}$ are given in **Definition** 1. Let n and m index users in the pool, where $r_n = \text{rank}(\mathbf{H}_n) \leq \text{rank}(\mathbf{H}_m) = r_m$. The geometric angle between user m's channel, $\mathbf{H}_m$, and user n's channel, $\mathbf{H}_n$, can alternatively be expressed as [6]

$$\cos^2\phi_{n,m} = \frac{\det\left(\mathbf{H}_n \mathbf{H}_m^H \mathbf{H}_m \mathbf{H}_n^H\right)}{\det\left(\mathbf{H}_n \mathbf{H}_n^H\right)}. \tag{15}$$

The value of $\cos^2\phi_{n,m}$ represents the ratio between the volume of the parallelepiped spanned by the projection of the basis vectors of the lower dimension subspace on the higher dimension subspace and the volume of the parallelepiped spanned by the basis vectors of the lower dimension subspace. In (15), $\mathbf{H}_n \mathbf{H}_m^H$ can be interpreted as a type of inner product between channel matrices. We use this to define an ϵ-orthogonality metric the next subsection.

C. Epsilon orthogonality

The concept of ϵ-orthogonality for the case of single-antenna users has been employed previously, e.g., [14] based on spatial orthogonality among channel vectors. Motivated by this, we define an ϵ-orthogonality metric for a multi-antenna group of users based on the $\binom{L}{2}$ angles between channel subspaces.

Definition 2 : Let $\mathsf{G} = \{1, 2, \ldots, L\}$ denote a group of L users chosen from the K-user pool. For any group, G, let its pairwise orthogonality metric

$$\epsilon_L \equiv \min_{j,k \in \mathsf{G}} \phi_{k,j}. \tag{16}$$

A value of $\epsilon_L = 0$ represents maximum spatial orthogonality among all subspaces $\mathcal{W}_k$ and $\mathcal{V}_j$, $j, k, \in \mathsf{G}$. Let the cumulative distribution function (CDF) of ϵ_L over the distribution of random channels be denoted by

$$F_{\epsilon_L}(\epsilon) \equiv P(\epsilon \leq \epsilon_L). \tag{17}$$

We set a threshold, $F_{\epsilon_L}(\epsilon_{th})$, in Equation (17), which determines a value of the metric, ϵ_{th}, such that $P(\epsilon \leq \epsilon_L) \geq F_{\epsilon_L}(\epsilon_{th})$ to characterize the relative orthogonality of user group subspaces. That is, a group is selected if its pairwise orthogonality metric is within the top $1/F_{\epsilon_L}(\epsilon_{th})$ fraction of all realizations over the random channel distribution. Shown in Fig. 1 are the CDFs computed empirically for different group sizes as analytical determination of (17) is difficult.

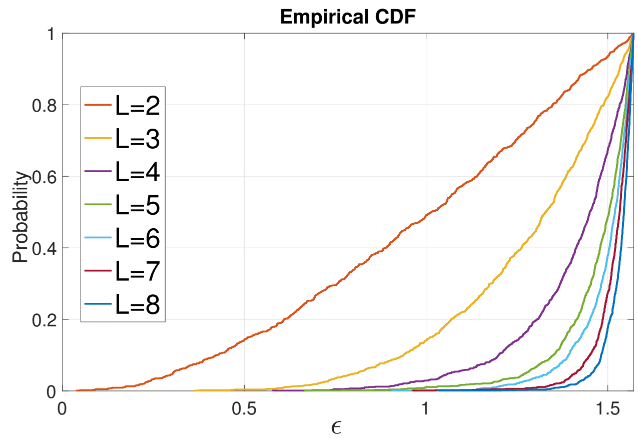

Figure 1: Shown is the CDF of the minimum pairwise subspace angle, Eq. (17), for different group sizes, L.

III. User selection

The combination of exploitation with exploration can improve the performance of a greedy algorithm, and is formulated in terms of reward maximization [7]. However, instead of purely random exploration, a priori problem knowledge can be employed to lower selection complexity and maintain high performance. This criteria, Eq. (16), is based on minimizing the maximum mutual interference among user pairs. This interference can be interpreted as a correlation metric for the corresponding channel matrix subspaces spanned by column vectors. To quantify subspace correlation among the users, the geometric angle between the subspaces is proposed [6]. In principle, selecting the best (highest sum rate) group of L users would require exhaustive search over $\binom{K}{L}$ groups, which requires $O(K^L)$ computation. Since this is computationally prohibitive, we instead propose a random selection scheme that considers only a subset of the groups. User groups are selected until a threshold condition is achieved, based on geometric subspace angles that indicate mutual user interference based on the ϵ_L-metric defined in Section II-C.

A. Complexity reduction

The proposed approach differs from existing approaches in the literature that compute a unique sum rate for each group, which require $O(K^L)$ complicated sum rate computations. In addition to avoiding exhaustive search, the proposed user selection strategy altogether avoids sum rate computation and instead uses a pair-wise selection criterion, whose computation is dominated by a maximum of $\binom{K}{2} = K(K-1)/2$ unique subspace angles that can be reused for different randomly selected user groups rather than the $O(K^L)$ required for group sum rate computation in full search. To further limit computation, a best group random selection strategy is used which, on average, reduces the number of groups searched by the factor $1/F_{\epsilon_L}(\epsilon_{th})$ from Eq. (17), which in the results presented, equals a factor of 10.

B. Proposed user selection algorithm

In this section, we propose a hybrid user selection method in which the random selection process based on a priori knowledge presented in Section III-B1 is combined with a greedy algorithm presented in Section III-B2 in order to maximize the sum rate.

1) Random selection process: In the initial random selection process, we randomly select L_r users from the pool based on subspace angle. The proposed user selection method is given by the randomized and greedy selection processes summarized in Algorithm 1 and Algorithm 2, respectively. First, in Algorithm 1, we randomly select groups L_r users from the pool $\{u_1, u_2, ..., u_K\}$ as the initial users in the selected set and we evaluate its performance based on geometric angle by computing ϵ_{L_r} using (16). If the performance metric meets a given CDF threshold in Equation (17), we have found the initial group and enlarge it using the greedy algorithm described later. Otherwise, we select another group at random and repeat. We note that ϵ_{L_r} acts as a simplified metric that serves as a proxy for sum rate, and is compared to a CDF threshold to assess relative group performance. The lower the value of ϵ_{th}, the stricter the performance and the longer the expected termination time of random group selection.

Then, to design BD precoder matrices for each selected user from Part 1, we employ the iterative method in [4] based on QR decomposition to lower complexity over SVD. In Part 2 of Algorithm 1, the preceding matrix for kth user, $\mathbf{B}_k$, forms an orthonormal basis of $\mathcal{N}(\bar{\mathbf{H}}_k)$ given by (7). Since both columns of $\mathbf{B}_k$ and $\bar{\mathbf{V}}_k^{(0)}$ span $\mathcal{N}(\bar{\mathbf{H}}_k)$, it is easy to verify equivalence to the SVD-based method.

2) A criterion for maximizing sum rate: From (15), it can be seen that when the subspace planes for users k and j are closer, $\cos^2 \phi_{k,j}$ is larger or equivalently $\phi_{k,j}$ is smaller and vice versa. Thus, a simple user-selection metric is needed in order to minimize subspace correlation and consequently minimize interference from nonorthogonality among the selected users. Although this metric measures orthogonality among users, it may not maximize the sum rate among the selected users. Thus, a criterion is employed to maximize the sum rate over the pool of candidate users from the pool that may be added to the group in sequential fashion. In view of this, the change in sum rate (SR) capacity when a new user is added in a selected

Algorithm 1 User selection Based on Randomness

Part 1: Initial Group Selection
Set K, L, L_r, ϵ_{th}.
for $j = 1$ to $\binom{K}{L_r}$ **do**
  Randomly select L_r users from pool.
  Calculate its metric, $\epsilon_{L_r}(j)$, using Eq. (16).
  if $\epsilon_{L_r}(j) \leq \epsilon_{th}$ **then** save user group j. Exit loop.
end for loop.
Part 2: Determine precoder for L_r user group
Set $\mathbf{B}_1^{(1)} = \mathbf{I}_N$, Design precoding matrix:
for i = 1 to $L_r - 1$ **do**
  $\mathbf{B}_{i+1}^{(i+1)} = \mathbf{B}_i^{(i)} \mathcal{N}(\mathbf{H}_i \mathbf{B}_i^{(i)})$
  for $j = 1$ **to** i **do** update precoder
    $\mathbf{B}_j^{(i+1)} = \mathbf{B}_j^{(i)} \mathcal{N}(\mathbf{H}_{i+1} \mathbf{B}_j^{(i)})$
  end for loop.
end for loop.

Algorithm 2 User selection Based on Greediness

Part 3: Adding remaining $L - L_r$ users to selected set
Set $\Phi_{L_r} = \{u_1, u_2, ..., u_K\} - \{u_{s1}, u_{s2}, ..., u_{s(L_r)}\}$,
Set $\Omega_{L_r} = \{u_{s1}, u_{s2}, ..., u_{s(L_r)}\}$ and set $n = L_r$.
Set $\mathbf{Q} = \mathbf{B}_{L_r}^{(L_r)} \times \mathcal{N}(\mathbf{H}_{L_r} \mathbf{B}_{L_r}^{(L_r)})$.
while $n < L$ **do**
  Select user channel with maximum subspace angle
  with $\mathbf{H}_{\mathcal{S}_n} = [\mathbf{H}_1^H \mathbf{H}_2^H ... \mathbf{H}_{|\mathcal{S}_n|}^H]^H$:
  $u_n = \arg \max_{k \in \Phi_n} \det(\mathbf{H}_k \mathbf{H}_k^H) \ \sin^2 \measuredangle(\mathbf{H}_{\mathcal{S}_n}, \mathbf{H}_k)$
  Set $\mathbf{B}_{u_n}^{(n)} = \mathbf{Q}$
  for each $m \in \mathcal{S}_n$ **do**
    Set $\mathbf{G}_{u_n} = \mathcal{N}(\mathbf{H}_{u_n} \mathbf{B}_m^{(n-1)})$
    Update the precoders $\mathbf{B}_m^{(n)} = \mathbf{B}_m^{(n-1)} \mathbf{G}_{u_n}$
  end for loop
  Update $\mathcal{S}_{n+1} = \mathcal{S}_n \bigcup u_n$ and $\Phi_{n+1} = \Phi_n - u_n$.
  Update orthonormal basis for channel matrices
  $\mathbf{Q} = \mathcal{R}(\mathbf{H}_{\mathcal{S}_{(n+1)}})$.
  $n \leftarrow n + 1$
end while loop

user subset can be expressed as [15]

$$\triangle SR = SR_{gain} - SR_{loss}, \tag{18}$$

where SR_{gain} and SR_{loss} denote the gain and loss in sum rate by adding a new user to the set, respectively. SR_{gain} is approximated by [6]

$$SR_{gain} \approx \log_2 \big(\det(\mathbf{H}_k \mathbf{H}_k^H) \sin^2 \measuredangle(\mathbf{H}_{\mathcal{S}_n}, \mathbf{H}_k)\big), \tag{19}$$

where $\measuredangle(\mathbf{H}_{\mathcal{S}_n}, \mathbf{H}_k)$ denotes the geometric angle, as defined by (14), between the range space of the channel matrix of the kth user, $\mathbf{H}_k$, and the aggregate channel matrix of already selected users, $\mathbf{H}_{\mathcal{S}_n}$. Following arguments in [6], for simplicity, we ignore the effect of SR_{loss}, and the greedy user selection metric over a set of users, Φ, becomes

$$\mathcal{M} \approx \arg \max_{k \in \Phi} \ \det(\mathbf{H}_k \mathbf{H}_k^H) \ \sin^2 \measuredangle(\mathbf{H}_{\mathcal{S}_n}, \mathbf{H}_k)\big). \tag{20}$$

This metric is employed in our proposed user selection scheme in Algorithm 2, where the reduced-complexity iterative scheme given in Part 2 of Algorithm 1 is used to compute precoder matrices for the selected user.

IV. Simulation results

In this section, we present the performance and complexity of the proposed method based on 100 Monte Carlo trials each under a set of different parameter choices and compare to those in the MU-MIMO user selection literature [4] and [6]. In [4], user selection is based on the Frobenius norm of the channel. In [6], user selection is based on the angle between subspaces of the channels based on block diagonalization precoding performed in greedy fashion.

First we investigate the performance in terms of the average sum rate. Fig. 2 shows average sum rate of a group of $L = 4$ selected users from the pool of $K = 20$ users by the proposed method (Algorithms 1 and 2), as computed using Eq. (12), versus signal to noise ratio (SNR). Here, $L_r = 2, 3,$ and 4 represent different initial group sizes and the channel is quantized with 4 bits. It can be seen from Fig. 2 that when L_r increases the performance of the proposed method improves as expected since there is more exploration. However, increasing L_r increases complexity due to more randomness in group selection (exponential in L_r) with associated calculations of angles between subspaces. It can be also seen that the performance of the proposed method outperforms the methods presented in [4] and [6] at high SNR for each of the L_r values. Even at low SNRs, the proposed method noticeably outperforms the methods presented in [4] and [6] for $L_r = 4$. At low SNRs, the proposed method outperforms the method presented in [4] and has performance close to that of [6] for $L_r = 2$ and $L_r = 3$.

It can be seen from Fig. 2 that when thresholding given by (16) is not used to select the group of $L_r = 4$ users with minimum subspace angles, the performance outperforms thresholding at both low and high SNRs in terms of the average sum rate, but requires search of all $\binom{K}{4}$ initial groups, as shown by the 'Proposed method (full search, $L_r = 4$)' curve in the legend of Fig. 2. For the CDF threshold ϵ_{th} used, the expected number of random group selections until encountering a top $F_{\epsilon_L}(\epsilon_{th}) \times 100\%$ group is equal to $1/F_{\epsilon_L}(\epsilon_{th})$. Here $F_{\epsilon_L}(\epsilon_{th}) = 0.1$ so only 10 randomly chosen groups on average are required rather than the full search of $\binom{K}{4}$ groups, but at a cost of some performance loss.

We next investigate the performance of the proposed method on different pool sizes. Fig. 3 shows the average sum rate of $L = L_{max} = \frac{N}{M} = 4$ selected users where the user pool size varies from $K = 5$ to $K = 20$ and random selected user group size from $L_r = 2$ to $L_r = 4$. It can be seen that when $L_r = 2$, the performance of the proposed method is close to that provided in [6] and superior to that in [4] for all pool sizes. When L_r increases, the performance of the proposed method significantly improves and outperforms the methods in [4] and [6]. This indicates that by increasing the size of the search space even if randomly sampled, performance sig-

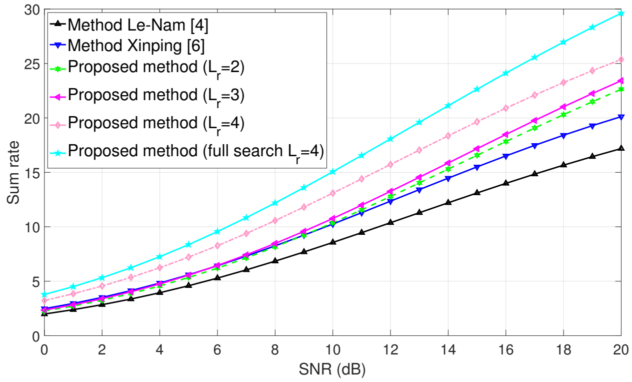

Figure 2: Average sum rate versus SNR for $L_r = 2, 3, 4$, 4 bits quantization, $N = 8$ BS antennas, $M = 2$ antennas per user and a pool size of $K = 20$ users.

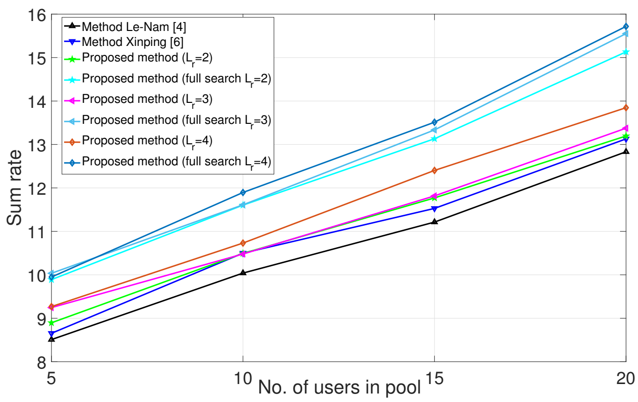

Figure 3: Average sum rate versus pool size, K, for SNR = 10dB, 4 bits quantization, $N = 8$ BS antennas and $M = 2$ antennas per user.

nificantly improves; however this improvement adds selection complexity compared to the purely greedy methods.

It can be also seen in Fig. 3 that for all pool sizes, when thresholding given by (16) is not used and the group of L_{max} users is selected though exhaustive search, the performance of the proposed scheme (with full search) significantly outperforms thresholding methods. As pool size, K, increases, the performance advantage of the proposed scheme becomes more significant. Also, from Fig. 3, by increasing the pool size, the gap between the proposed thresholding method and the full search method is maintained, and performance improves steadily. The proposed approach may therefore be potentially applicable to dense networks and found in IoT applications where the numbers of users served is very large.

V. Conclusion and Future Work

An epsilon-greedy approach to user selection for a downlink MU-MIMO system is proposed where a small group of initial users are selected randomly based on a threshold criterion determined by the CDF of the minimum geometric angle between subspaces of the user channels. Then, the rest of the users are selected based on the subspace angle in a greedy algorithm. The novelty of this approach are the new trade offs that are possible between sum rate performance and implementation complexity.

While these results appear promising, future work will focus on (i) investigating improved low complexity pairwise performance metrics that take both power levels and mutual interference into account, (ii) addressing optimization of the initial random group size L_r and selection threshold ϵ_{th} and (iii) performing a detailed quantitative investigation into complexity and computation cost.

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